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DESIGN OF TESTS FOR PARAMETER EVALUATION WITHIN REMOTE INACCESSIBLE FAULTY SUBNETWORKS

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ABSTRACT

This paper presents the theoretical background for designing tests which are topologically sufficient for evaluation of faulty parameters within inaccessible faulty subnetworks. Nodal voltages are assumed to be obtainable either by measurements or, indirectly, as a result of a nodal fault analysis. A formulation of nodal fault analysis for subnetworks is presented.

INTRODUCTION

Fault diagnosis and automatic testing techniques for analog circuits often require parameter indentification. Recent papers on the subject [1-5] present different techniques of parameter indentification and/or fault region location. For linear analog circuits, necessary and sufficient conditions related to the network topology have been formulated, resulting in the identification of faulty nodes or subnetworks [3-5].

The principal aim of this paper is to develop topologically based necessary and sufficient conditions for the evaluation of faulty elements within a linear subnetwork under test with a reasonably small number of excitations at a single frequency and, thereby, a small number of measurements. The paper extends the results presented by Biernacki and Starzyk [2]. The Coates flow graph representation of network elements is used [6].

LOCATION OF FAULTY NODES AND DESIGN OF NODAL VOLTAGES

Necessary and sufficient conditions for location of faulty nodes have been discussed [4] for linear networks, and more generally [5] for any subnetworks selected during the fault location process in a large network. External voltages and currents of a subnetwork may be measured or designed through identification of nonfaulty parts of a large network [5].

Consider the nodal equations for a nominal subnetwork isolated during a fault location process for a large network as

$$\underset{\sim}{I}^0 = \underset{\sim}{Y}^0 \underset{\sim}{V}^0. \tag{1}$$

Four types of external nodes are associated with this subnetwork: α-nodes, where both voltages and currents are known; β-nodes, where only voltages are known; γ-nodes, where only currents are known; and δ-nodes, where neither voltages nor currents are known. We assume that all the elements spanned over the nodes β and δ have been arbitrarily associated with other subnetworks and they are not represented in (1). See Fig. 1.

For the assumed faulty subnetwork, (1) can be replaced by

$$\underset{\sim}{I} = \underset{\approx}{Y}\underset{\sim}{V} . \tag{2}$$

Let $\underset{\sim}{Z}_{a_1 \ldots a_k,\, b_1 \ldots b_m}$ denote a submatrix of $(\underset{\sim}{Y}^0)^{-1}$ obtained by the intersection of rows $a_1 \cup a_2 \cup \ldots \cup a_k$ and columns $b_1 \cup b_2 \cup \ldots \cup b_m$.

Let $\underset{\sim}{V}^{\alpha\beta}$, $\underset{\sim}{I}^{\alpha\gamma}$, $\underset{\sim}{I}^{\beta\delta}$ be subvectors of vector $\underset{\sim}{V}$ and $\underset{\sim}{I}$ respectively, corresponding to sets of nodes $\alpha \cup \beta$, $\alpha \cup \gamma$ and $\beta \cup \delta$. Let nodes $\eta \subset \alpha \cup \gamma \cup \zeta$ be faulty and card $\alpha >$ card δ + card η.

Solving (1) we obtain

$$\underset{\sim}{V}^{\alpha\beta} - \underset{\sim}{Z}_{\alpha\beta,\alpha\gamma} \underset{\sim}{I}^{\alpha\gamma} = \underset{\sim}{Z}_{\alpha\beta,\beta\delta} \underset{\sim}{I}^{\beta\delta} + \underset{\sim}{Z}_{\alpha\beta,\eta} \underset{\sim}{I}^{\eta} \tag{3}$$

<u>Result 1</u>

If the system of equations (3) is consistent and

$$\text{Rank}[\underset{\sim}{Z}_{\alpha\beta,\beta\delta\eta x}] > \text{Rank}[\underset{\sim}{Z}_{\alpha\beta,\beta\delta}] + \text{card}\ \eta , \tag{4}$$

where $x \in \alpha \cup \gamma \cup \zeta - \eta$, then the only faulty elements can be those spanned over the set of nodes $F = \eta \cup \beta \cup \delta$. These nodes are called <u>faulty nodes</u> although there may be no faulty element incident with β and δ.

Assume that by solving (3) we have evaluated $\underset{\sim}{I}^{\beta}$, $\underset{\sim}{I}^{\delta}$ and $\underset{\sim}{I}^{\eta}$. After solving (3), we can solve (2) to get $\underset{\sim}{V}$. For all independent current excitations we are, therefore, able to calculate voltages in the faulty network if the conditions of Result 1 are satisified. These voltages, which would otherwise have to be measured, are required by the approach presented in [2] for evaluating all the elements of a network. In the present paper we only need to evaluate unknown elements, i.e., those

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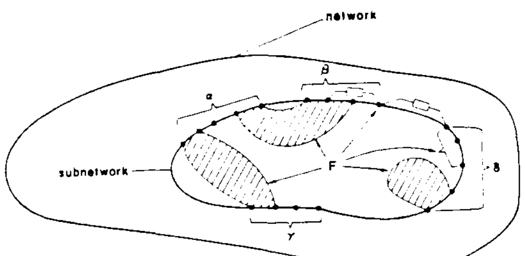

Fig. 1 Remote, inaccessible faulty subnetworks.

which are spanned over the faulty nodes.

ELEMENT EVALUATION FOR SUBNETWORKS SPANNED OVER FAULTY NODES

The elements spanned over faulty nodes may form separate subnetworks within a given subnetwork, as shown in Fig. 1. The subnetworks may be remote and inaccessible from the point of view of direct excitation and measurement. We can formulate conditions for element evaluation within each of these subnetworks separately and combine the results obtained to establish conditions for the whole network. These conditions will show which external nodes should be excited independently to evaluate all faulty elements.

Consider a linear subnetwork spanned over N faulty nodes. Let the N-dimensional vectors I_s and V_s be subsets of I and V, respectively, corresponding to this subnetwork. We can then write

$$Y_s V_s^i = I_s^i , \qquad (5)$$

for the ith excitation. Our goal is to evaluate Y_s and then the element values. For N independent excitations, we can write a matrix equation

$$Y_s V_t = I_t , \qquad (6)$$

or

$$V_t^T Y_s^T = I_t^T , \qquad (7)$$

where the square matrix

$$V_t \triangleq [V_s^1 \;\; V_s^2 \;\; \cdots \;\; V_s^N] \qquad (8)$$

is the matrix of voltage responses and the square matrix

$$I_t \triangleq [I_s^1 \;\; I_s^2 \;\; \cdots \;\; I_s^N] \qquad (9)$$

is the matrix of current excitations. From (6), we find the unknown matrix Y_s as

$$Y_s = I_t V_t^{-1} \qquad (10)$$

provided that V_t is nonsingular.

Using the technique similar to that presented in [2], we may select a sequence of reduced cut-sets. With some of the elements $y_{ij} \in Y_s^T$ known we can formulate a sequence of equations describing reduced cut-sets of the form

$$V_t^T[C_j \mid B_j] \begin{bmatrix} y_{jj_1} \\ \cdot \\ \cdot \\ \cdot \\ y_{jj_k} \end{bmatrix} = \begin{bmatrix} I_{i_1 j} \\ \cdot \\ \cdot \\ \cdot \\ I_{i_k j} \end{bmatrix} , \qquad (11)$$

where the k equations are chosen from (7) in such a way that the square submatrix $V_t^T[C_j \mid B_j]$, obtained as the intersection of rows $C_j = \{i_1, \ldots, i_k\}$ and columns $B_j = \{j_1, \ldots, j_k\}$, is nonsingular. According to relationship (6), the matrix $V_t^T[C_j \mid B_j]$ can be defined as

$$V_t^T [C_j \mid B_j] = I_t^T [C_j \mid \cdot] \, (Y_s^T)^{-1} [\cdot \mid B_j] , \qquad (12)$$

where $I_t^T [C_j \mid .]$ consists of rows C_j from I_t^T and $(Y_s^T)^{-1} [. \mid B_j]$ consists of columns B_j from $(Y_s^T)^{-1}$. On the basis of (12) and the Cauchy-Binet theorem we may formulate the following result.

<u>Result 2</u>

The matrix $V_t^T [C_j \mid B_j]$ is nonsingular if and only if

$$\exists A_j \quad \det I_t^T [C_j \mid A_j] \neq 0, \ \det Y_s (A_j \mid B_j) \neq 0 , \qquad (13)$$

where $Y_s (A_j \mid B_j)$ denotes the submatrix of Y_s obtained by removing rows A_j and columns B_j.

Consider a sequence of sets B_j, $j = j_1, \ldots, j_M$, which corresponds to a sequence of reduced cut-sets of the <u>current graph</u> of the subnetwork. Only those reduced cut-sets will be considered for which external currents, if any, can be specified. Based on (6) and (11), the following result can be summarized.

<u>Result 3</u>

Independent excitations which appear at or are applied to the subset of nodes $A \subset \{1, 2, \ldots, N\}$ are sufficient for the identification of all elements of Y_s if and only if

$$\forall B_j \quad \exists C_j \quad \exists A_j \subset A, \quad \det I_t^T [C_j \mid A_j] \neq 0$$
$$\text{and} \quad \det Y_s(A_j \mid B_j) \neq 0, \qquad (14)$$

where

$$\operatorname{card} A_j = \operatorname{card} B_j = \operatorname{card} C_j . \qquad (15)$$

Nodes A in Result 3 can be chosen from a remote inaccessible subnetwork, therefore we call them <u>injection nodes</u>. For each subnetwork the set A must be a subset of the external nodes of this subnetwork.

<u>Location of Injection Nodes</u>

A very efficient heuristic algorithm which utilizes theoretical aspects discussed at this section was presented in [2]. It allows us to find a nearly minimal set of injection nodes in a time which depends linearly on the subnetwork size. The algorithm localizes injection nodes in such a way that there exist a set of separate paths from injection nodes to the nodes of each reduced cut-set.

In particular cases, when the number of injection nodes is too large because of the subnetwork topology we can reduce them by adding some known elements to the subnetwork under consideration. For evaluation of faulty elements within remote, inaccessible subnetworks, adding the known elements may be equivalent to considering an augmented subnetwork which will contain faulty nodes as well as some nonfaulty ones.

ELEMENT EVALUATION USING EXTERNAL EXCITATION NODES

Let us assume that we have distinct, remote, inaccessible faulty subnetworks $S_1, \ldots, S_f$ spanned over faulty nodes within the subnetwork under investigation (see Fig. 2). According to Result 1, the number of external nodes, where both voltages and external currents are known, have to satisfy the relation

$$\text{card } \alpha > \sum_{i=1}^{f} n_i \,, \qquad (16)$$

where n_i is the number of nodes in the subnetwork S_i. We can identify sets of injection nodes $A^1, \ldots, A^f$ at which independent current excitations could be forced. With independent excitations appearing at injection nodes, we are able to evaluate all elements within $S_1, \ldots, S_f$.

Let T be a subset of the external nodes of the subnetwork S. Let G denote the Coates signal-flow graph of S. Let us assume that we have evaluated faulty currents and designed nodal voltages. Let $k_i = \text{card } A^i$.

Lemma 1

If there exist k_i simultaneous and separate paths in G from T to A^i not incident with other S_i nodes, then all the elements of S_i can be uniquely identified.

Proof is based on the recognition of each cut-set in S_i as a reduced cut-set in S.

Corollary 1

If Lemma 1 is satisfied for all A^i, then T can be chosen as a set of test nodes where independent current excitations are applied, to evaluate all faulty elements in S.

We want to have the cardinality of T as small as possible, to minimize the number of tests and designs of nodal voltages.

Corollary 4

$$\text{card } T \geq \max k_i \,. \qquad (17)$$

The main goal is to find k_i as small as possible, so the technique described guarantees identification of faulty elements effectively. For most practical cases, card T is between 2 and 5.

Remark

For inaccessible subnetworks we design currents flowing into them from the surrounding network using the designed voltages and nominal element values first, and then proceed with element evaluation within each of them as discussed.

Example

Assume that the nominal element values for the network of Fig. 3 are:

$Y_1 = 1,\ Y_2 = 0.5,\ Y_3 = 0.3,\ Y_4 = 0.32,$

$Y_5 = 0.2,\ Y_6 = 0.167,\ Y_7 = 0.143,$

$Y_8 = 0.125,\ Y_9 = 0.1,\ Y_{10} = 0.2,\ Y_{11} = 0.1,$

$Y_{12} = 0.0833,\ Y_{13} = 0.0769,\ Y_{14} = 0.0714,$

$Y_{15} = 0.0667,\ Y_{16} = 0.0625,\ Y_{17} = 0.0588,\ g_m = 8.5.$

Four external points are available for voltage measurements and current excitations at the nodes 1, 3, 4 and 7. Assume for simplicity that all external nodes are of the α type. We have found three faulty nodes, namely, 2, 4, 6 and evaluated currents $\underset{\sim}{I}^n$, $n = \{2, 4, 6\}$. The subnetwork spanned

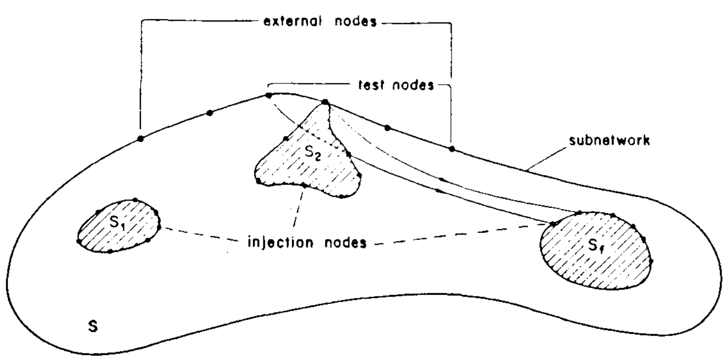

Fig. 2 Faulty subnetwork under consideration.

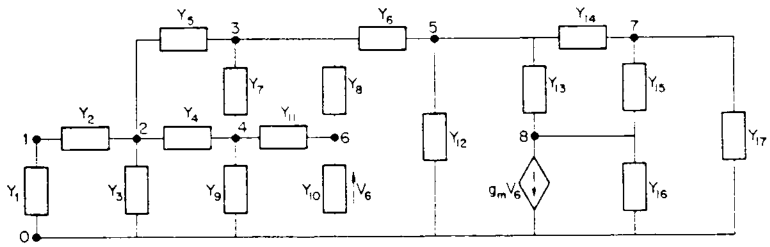

Fig. 3 Faulty network.

TABLE I
NODAL VOLTAGES FOR THE EXAMPLE

Voltage	Excitation at Node 1	Excitation at Node 7
V_1	0.77641	0.016174
V_2	0.32925	0.048524
V_3	-0.0066477	0.175324
V_4	0.14264	0.076466
V_5	-0.57149	0.47525
V_6	0.038418	0.091385
V_7	-0.9163	4.5309
V_8	-2.0943	-2.126

over the faulty nodes is a simple ladder network. We can easily locate nodes 2 and 6 as injection nodes sufficient for evaluation of the ladder elements. According to Lemma 1 external current excitations sufficient for element evaluation can be made at nodes 1 and 7.

Now we simulate the nominal network with independent (unit) excitations at nodes 1 and 7 separately and evaluate currents $\underset{\sim}{I}^n$ from equation (3). With those currents and independent current excitations we excite the nominal network to obtain the current voltages as in Table I. Elements Y_2, Y_5, Y_7 and Y_8 are nominal as they are not spanned over the faulty nodes. Using the voltages from Table I we calculate external currents for the ladder subnetwork spanned over faulty nodes as equal to

$$I_{12} = (V_{11} - V_{12})\, Y_2 + (V_{13} - V_{12})\, Y_5 = 0.1564,$$

$$I_{14} = (V_{13} - V_{14})\, Y_7 = -0.02135,$$

$$I_{16} = (V_{13} - V_{16})\, Y_8 = -0.005633.$$

Similarly we can get

$$I_{72} = 0.009188,\ I_{74} = 0.01414,\ I_{76} = 0.01049.$$

Equation (11) for the first reduced cut-set has the form

$$\begin{bmatrix} V_{12} & V_{14} \\ V_{72} & V_{74} \end{bmatrix} \begin{bmatrix} Y_3 + Y_4 \\ -Y_4 \end{bmatrix} = \begin{bmatrix} I_{12} \\ I_{72} \end{bmatrix}$$

Therefore,

$$\begin{bmatrix} 0.32925 & 0.14264 \\ 0.048524 & 0.076466 \end{bmatrix} \begin{bmatrix} Y_3 + Y_4 \\ -Y_4 \end{bmatrix} = \begin{bmatrix} 0.1564 \\ 0.009188 \end{bmatrix}$$

and we get $Y_3 = 0.333$ and $Y_4 = 0.25$. In the next two reduced cut-sets elements Y_9, Y_{11} and Y_{10} are evaluated respectively, with the help of voltage measurements as well as evaluated and nominal elements.

CONCLUSIONS

The method presented enables us to find a reasonably small number of excitation nodes which are topologically sufficient for the identification of all faulty parameter values of linear analog subnetworks. This can be achieved by searching for a "good" sequence of reduced cut-sets within the subnetworks spanned over faulty nodes, whose elements are consecutively determined from (11). The number of excitations can be reduced by adding external elements or some nominal ones in the case of inaccessible subnetworks.

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