AN OPTIMIZATION APPROACH TO THE BEST ALIGNMENT OF MANUFACTURED AND OPERATING SYSTEMS

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ABSTRACT

This paper formulates the best alignment problem, which arises when a manufactured system or design does not meet its specifications or when an operating system is under emergency conditions and some performance or security constraints are violated. In this case, some elements of the system should be aligned, tuned (if possible), curtailed or replaced to satisfy specifications. A general, nonlinear programming notation is employed.

INTRODUCTION

This paper employs a general notation which facilitates the formulation of the best alignment problem arising in many practical situations when a manufactured system or design does not meet the specifications [1] or when an operating system is under emergency conditions and some constraints (e.g., security constraints [2]) imposed on the system are violated. In this case, some elements of the system should be aligned or tuned (if possible) to satisfy specifications. A set of system variables with lower and upper specifications representing an operating engineering system is proposed. A certain class of alignment problems is presented and an algorithm for selecting best candidates for alignment using the Hald and Madsen minimax optimization method [3-6] is described.

FORMULATION OF THE PROBLEM

Preliminary Concepts

Consider an engineering system represented by

$$\underset{\sim}{x} \overset{\Delta}{=} [\underset{\sim}{x}_1^T \;\; \underset{\sim}{x}_2^T \;\; \ldots \;\; \underset{\sim}{x}_n^T]^T , \tag{1}$$

where T denotes transposition. Let

$$I = \{ 1, 2, \ldots, n \} \tag{2}$$

be the index set for these variables. Each variable can be represented by the elements of the vector $\underset{\sim}{x}_i$, $i \in I$, and the corresponding index set

$$I_i \overset{\Delta}{=} \{ 1, 2, \ldots, n_i \} . \tag{3}$$

If the system is designed and manufactured to meet certain specifications, not all values for each variable are acceptable. Therefore, we define design variable bounds, operating limits and/or security constraints on each variable, i.e.,

$$\underset{\sim}{L}_i(\underset{\sim}{y},\underset{\sim}{x}) \le \underset{\sim}{x}_i(\underset{\sim}{y},\underset{\sim}{x}) \le \underset{\sim}{U}_i(\underset{\sim}{y},\underset{\sim}{x}) , \tag{4}$$

where $\underset{\sim}{L}_i(\underset{\sim}{y},\underset{\sim}{x})$, $\underset{\sim}{U}_i(\underset{\sim}{y},\underset{\sim}{x})$ denote lower and upper specifications, respectively, and $\underset{\sim}{y}$ is a set of independent variables such as frequency, time, temperature, etc. Let us denote the acceptable region (or constraint region) $R_i(\underset{\sim}{y})$ as

$$R_i(\underset{\sim}{y}) \overset{\Delta}{=} \{\underset{\sim}{x} \mid \underset{\sim}{L}_i(\underset{\sim}{y},\underset{\sim}{x}) \le \underset{\sim}{x}_i(\underset{\sim}{y},\underset{\sim}{x}) \le \underset{\sim}{U}_i(\underset{\sim}{y},\underset{\sim}{x})\}, \; i \in I. \tag{5}$$

System variables $\underset{\sim}{x}_i(\underset{\sim}{y},\underset{\sim}{x})$, $i \in I$, may represent design variables and functions of system variables (performance functions, equality constraints in the form of nonlinear equations describing the system behaviour, etc.).

Associated System Variables

Suppose that each variable can be represented as

$$\underset{\sim}{x}_i = \underset{\sim}{x}_i^0 + \underset{\sim}{x}_i^\varepsilon + \underset{\sim}{x}_i^E + \underset{\sim}{x}_i^{t(\varepsilon,E)} + \underset{\sim}{x}_i^{T(\varepsilon,E)} , \tag{6}$$

where superscripts 0, ε, E, t and T denote associated system variables and have the following interpretation:

- 0 signifies a nominal design or normal operating conditions;
- ε signifies a random outcome, small tolerances, first-order changes, uncertainties in the model parameters;
- E signifies large manufacturing errors, faults, large changes in the operating system (contingencies, breakdowns);
- t(•) signifies normal alignment, tuning, operating adjustments and control;
- T(•) signifies large or unusual required changes, repairs, shutdowns or replacements.

This work was supported by the Natural Sciences and Engineering Research Council of Canada under Grants A7239, G0647 and A1708.

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CH1845-7/83/0000-0542 $1.00 © 1983 IEEE

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Each associated variable may have its own bounds, which can be defined similarly to (4). We can define index sets which classify, in part, associated system variables into free to change, fixed at zero, and fixed at constant candidate variables.

We can call constraints (4) the operating limits or security constraints and the set $R(\underset{\sim}{y})$

$$R(\underset{\sim}{y}) \triangleq \bigcap_{i \in I} R_i(\underset{\sim}{y}) \ , \qquad (7)$$

the acceptable constraint set for the problem, characterized by the corresponding set of variables I.

The engineering system problem is

$$\text{minimize } C \text{ subject to } R(\underset{\sim}{y}) \neq \emptyset , \qquad (8)$$

where C is an appropriate, generally nonlinear cost function, for all permissible $\underset{\sim}{y}$, $\underset{\sim}{x}^{\varepsilon}$, $\underset{\sim}{x}^{E}$, and some permissible $\underset{\sim}{x}^{t(\varepsilon,E)}$, $\underset{\sim}{x}^{T(\varepsilon,E)}$. See, for example, [7].

A SPECIAL CLASS OF ALIGNMENT PROBLEMS

Formulation of the Problem

Suppose we have a set of variables representing a manufactured design as in (1), and the index set I for these variables

$$I \triangleq \{ 1, 2, \ldots \ell, \ell+1, \ldots, n_0, n_0+1, \ldots, m, m+1, \ldots, m+\ell, m+\ell+1, \ldots, m+n_0, m+n_0+1, \ldots, 2m, n\} . (9)$$

Let

$$I_p \triangleq \{1, 2, \ldots, m\} \ , \ I_p \subset I \ , \qquad (10)$$

be the index set for the variables

$$\underset{\sim}{x}_i = \underset{\sim}{A} \, \bar{\underset{\sim}{x}}_i + \underset{\sim}{b} \ , \qquad (11)$$

where elements of matrix $\underset{\sim}{A}$ and vector $\underset{\sim}{b}$ are functions of $\underset{\sim}{x}_n$, and $\bar{\underset{\sim}{x}}_i$, $i \varepsilon I_p$, represents data points.

Let

$$I_f \triangleq \{m+1, \ldots, 2m\} \ , \ I_f \subset I \ , \qquad (12)$$

be the index set for variables which are functions of $\underset{\sim}{x}_i$, $i \varepsilon I_p$, as

$$\underset{\sim}{x}_{m+i} = \underset{\sim}{f}_i \, (\underset{\sim}{x}_i) \ , \ i \varepsilon I_p \ . \qquad (13)$$

Let $\underset{\sim}{x}_n$ be an independent variable. Some variables $\underset{\sim}{x}_i$, $i \varepsilon I_p$, may be functions of other variables $\underset{\sim}{x}_j$, $j \neq i$, $j \varepsilon I_p$. We assume that first ℓ-1 variables are only functions of independent variable $\underset{\sim}{x}_n$ and no other variables are related to them. For $\ell \leq i \leq n_0$, we can define the index sets which contain indices of variables related to the ith variable.

We also have bounds on each variable

$$\underset{\sim}{L}_i \leq \underset{\sim}{x}_i \leq \underset{\sim}{U}_i \ , \quad i=1, 2, \quad \ldots, n_0 \ , \ (14)$$

$$\underset{\sim}{L}_i(\underset{\sim}{x}_j) \leq \underset{\sim}{x}_i \leq \underset{\sim}{U}_i(\underset{\sim}{x}_j) \ , \quad i \in I^j , \qquad (15)$$

where

$$\bigcup_j I^j \triangleq \{n_0+1, \ldots, m\}$$

and

$$I^j \bigcap_{j \neq k} I^k = \emptyset \text{ for } j, k \varepsilon \{\ell, \ldots, n_0\},$$

$$\underset{\sim}{x}_i \leq \underset{\sim}{U}_i \ , \ i=m+1, \quad \ldots, 2m \ , \qquad (16)$$

which produce constraint regions given by (5).

Let

$$I^T = I_p \qquad (17)$$

be the index set for all variables which may be considered as candidates for large change tuning and

$$I^{T*} \subset I^T \qquad (18)$$

be the index set for variables which actually have to be tuned to satisfy specifications.

The problem can be now stated as

$$\text{minimize } n^* \triangleq \text{card } (I^{T*})$$

w.r.t. $I^{T*} \varepsilon 2^{I^T}$ subject to

$$R \triangleq \bigcap_{i \varepsilon I_f} R_i \neq \emptyset \ . \qquad (19)$$

where n^* is the cardinality of I^{T*} and 2^{I^T} is the family of all subsets of the set I^T.

Practical Best Alignment Problem

A practical mechanical best alignment problem has been solved [8], in which variables $\underset{\sim}{x}_i$, $i \varepsilon I_p$, represent transformed coordinates of a point in a two-dimensional space with the $\overline{YOX}$ system of coordinates. The transformation corresponding to (11) is given by

$$\begin{bmatrix} x_{i_1} \\ x_{i_2} \end{bmatrix} = \begin{bmatrix} \cos \phi_3 & -\sin \phi_3 \\ \sin \phi_3 & \cos \phi_3 \end{bmatrix} \begin{bmatrix} \bar{x}_{i_1} \\ \bar{x}_{i_2} \end{bmatrix} + \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} , \ (20)$$

where $(\bar{x}_{i_1}, \bar{x}_{i_2})$ are coordinates of a point i in the $\overline{YOX}$ system of coordinates, and (x_{i_1}, x_{i_2}) are coordinates of a point i in the YOX system of coordinates, $\underset{\sim}{\phi}_0 = [\phi_1 \ \phi_2 \ \phi_3]^T$ is a set of variables relating the two systems of coordinates, ϕ_1 and ϕ_2 represent translation parameters while ϕ_3 is the relative rotation. Vector $\underset{\sim}{\phi}_0$ corresponds to independent variable $\underset{\sim}{x}_n$ and to I_i^t, i=n, (the index set for normally tunable parameters).

For each point $\underset{\sim}{x}_i$, $i \in I_p$, there is a corresponding tolerance region R_i, $i \in I_p$, given in the YOX system of coordinates. The regions R_i, $i \in I_p$, may have different shapes (e.g., circular, rectangular), they may be defined using polar coordinates, rectangular coordinates or combined polar and rectangular coordinates. Dimensions of tolerance regions may be given either w.r.t. the main origin of the YOX system of coordinates (for i = 1,2, ..., n_0) or w.r.t. the reference point (for i = n_0+1, ..., m).

Now, the best alignment problem is subject to the constraint

$$\min_{\phi} \max_{i \in I_p} f_i(\phi) \leq 0 \quad , \qquad (21)$$

where ϕ is the vector of optimization variables corresponding to the set I^{T*}. The error function $f_i(\phi)$, indicates whether the point i is in ($f_i \leq 0$) or out ($f_i > 0$) of the tolerance region R_i.

The solution to the best alignment problem consists of two stages. The first stage corresponds to a discrete (or combinatorial) minimization of the number of points which should be deleted from the original set of points, and the second stage is an unconstrained minimax optimization of a set of error functions f_i, $i \in I_p$. The discrete minimization of the first stage is usually implemented as a systematic search of the solution in the family of all subsets of the set I^T. It is convenient to represent this search in the form of a multi-level tree in which the root (level 0) corresponds to the set $I^{T*} = \emptyset$, the level 1 contains all the single element subsets, the level 2 all the subsets of I^T which contain two elements, and so on. The first stage minimization traverses the tree level after level until such a subset I^{T*} is encountered for which the constraint (21) is satisfied. It can be observed, however, that the minimax optimization of the second stage, which is performed for each step of the first stage search, can be used to eliminate those nodes (and their subtrees) of the search tree which cannot influence the solution. In fact, if the minimax constraint corresponding to the subset I^{T*} at a particular level of the search tree is not satisfied then the next level subsets should be derived from the I^{T*} of the previous level by adding only the indices of those points which correspond to the active error functions at the solution ϕ^* of the minimax optimization since the remaining, nonactive error functions do not affect the solution. This observation is the basis of the implemented combinatorial search algorithm, which dynamically creates and traverses the reduced search tree [8].

Illustrative Example

Suppose we have a set of points $P \triangleq \{p_1, p_2, p_3, p_4, p_5\}$ and a set of tolerance regions, $R \triangleq \{R_1, R_2, R_3, R_4, R_5\}$. Fig. 1 illustrates the situation before the alignment. Error functions at the starting point $\phi_0 = [0.0 \;\; 0.0 \;\; 0.0]^T$ are

$$f_1 = 2.0710 \times 10^{-1} \;, \; f_2 = -5.0000 \times 10^{-1} \;,$$

$$f_3 = 5.0000 \times 10^{-1} \;, \; f_4 = -5.0000 \times 10^{-1} \;,$$

$$f_5 = 5.0000 \times 10^{-1} \;.$$

Fig. 2 shows the situation after running the alignment program. The best alignment was found at $\phi_0 = [-2.316 \times 10^{-1} \;\; -2.792 \times 10^{-1} \;\; 4.758 \times 10^{-2}]^T$ with point 5 deleted. Remaining error functions at the solution are

$$f_1 = -1.5400 \times 10^{-1} \;, \; f_2 = -1.2060 \times 10^{-1} \;,$$

$$f_3 = -1.2043 \times 10^{-2} \;, \; f_4 = -1.2043 \times 10^{-2} \;.$$

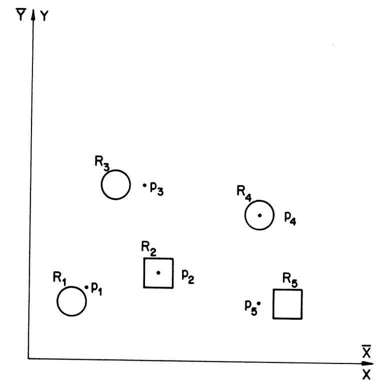

Fig. 1 Points and regions before alignment.

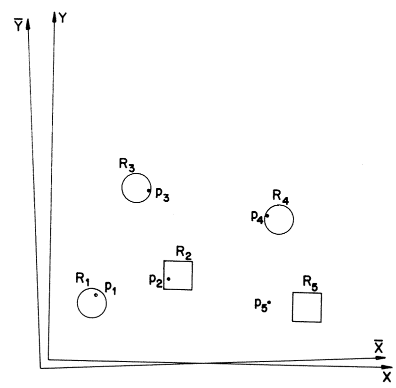

Fig. 2 Results of running the alignment program.

For the circular tolerance region the error function is the difference between the geometrical distance of a point from the center of the tolerance region and its radius. For the rectangular tolerance region the error function results from lower and upper bounds on coordinates of a point.

Test Results on Practical Problems

The algorithm described in the previous section has been tested for seven sets of data supplied by the Woodward Governor Company [9]. The data resulted from practical problems of part alignment in manufactured mechanical systems and have been collected from inspecting actual parts. The points represent holes in one part which have to meet certain specifications when coupled together with another part. Test samples have different numbers of points, varying from 5 to 13 and specified tolerance regions of different shapes. The data as well as the results of running the alignment program for some samples are in [8].

LOAD SHEDDING IN POWER SYSTEMS AS A BEST ALIGNMENT PROBLEM

Formulation of the Problem

Suppose that a power system can be represented by the set of variables of (1) with n = 4, where $\underset{\sim}{x}_1$ is a set of independent (decision variables), $\underset{\sim}{x}_2$ is a set of dependent (state) variables, $\underset{\sim}{x}_3 = \underset{\sim}{0}$ represents power flow equations and $\underset{\sim}{x}_4$ some network variables for which security constraints are explicitly defined.

For the system to be normal and secure in the static sense, it must satisfy

$$\underset{\sim}{L}_i \leq \underset{\sim}{x}_i \leq \underset{\sim}{U}_i \quad , \quad i \in I \triangleq \{ 1, 2, 3, 4 \} , \quad (22)$$

where for i = 3 lower and upper bounds represent accuracy of the solution of nonlinear equations $\underset{\sim}{x}_3(\underset{\sim}{x}) = \underset{\sim}{0}$. We define the security region for each variable as in (5).

There are two types of contingencies [2], both of which can be modeled by (6):

a) a sudden change in the power injection to the network caused by the partial or complete loss of a generator, load or tie;

b) a sudden change in the network's configuration.

If the system is insecure or in an emergency, the control action taken may involve load shedding. In fact, in emergencies the operator may augment the decision vector with certain loads whose values can then be adjusted downwards. The problem can be formulated as an optimization problem (8) with the objective of minimizing load shedding.

Let I^t be the index set for normally tunable power system variables such as generator voltage magnitudes and real generator powers. Let I^T be the index set for all load real powers. Then the load shedding problem can be formulated as

$$\text{minimize } n^* \triangleq \text{card } (I^{T*})$$

for

$$I^{T*} \in 2^{I^T} \quad (23)$$

subject to

$$R = \bigcap_{i \in I} R_i \neq \emptyset ,$$

where $I^{T*} \subset I^T$ is the index set for loads which actually have to be shed. One possible method to select the subset I^{T*} is least pth optimization with p = 1. It finds the minimum number of tunable parameters required to satisfy all constraints.

CONCLUSIONS

This paper formulates the best alignment problem for manufactured and operating systems. A general notation is employed which allows us to formulate this problem as an optimization problem. A concept of a set of system variables with lower and upper specifications representing an operating engineering system is proposed. A special class of alignment problems, which originated from aligning mechanical designs is described. The formulation of the load shedding problem in power systems as a best alignment problem is indicated.

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