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A UNIFIED DECOMPOSITION APPROACH FOR FAULT LOCATION IN LARGE ANALOG CIRCUITS

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1. Abstract

In this paper, a new technique for analog network fault analysis is described. The technique is based mainly on the utilization of network decomposition as well as logical analysis in isolating the faults. The technique is applicable to large networks and closely satisfies proposed criteria for fault analysis methods.

2. Introduction

This paper addresses itself to the problem of fault location in analog circuits. We present a new simulation-after-test method for fault location with the computations and measurements kept to acceptable practical bounds [1].

A nodal decomposition of the network into smaller uncoupled subnetworks is carried out. The measurement nodes must include the nodes of decomposition. The voltage measurements are employed to isolate the faulty subnetworks.

Utilizing the incidence relations between subnetworks and KCL we develop necessary and almost sufficient conditions for a subnetwork or a group of subnetworks to be fault free. Logical analysis of the results of these tests is carried out to identify faulty subnetworks.

After localizing the faults to the subnetwork level, we identify the faulty elements or faulty regions [2] inside the faulty subnetwork using fault verification techniques [3].

The effect of tolerances on the nonfaulty elements is handled by utilizing the weighted-least-squares criterion for solving underdetermined systems of equations. The criterion has a significant probabilistic implication.

3. Network Decomposition and Logical Analysis

In the pre-test stage we perform a nodal decomposition of the network. This results in subnetworks connected by the nodes of decomposition. There should be no mutual coupling between any two subnetworks and the nodes of decomposition should be chosen from the set where voltage measurements can be performed.

Testing conditions are applied to identify the nonfaulty subnetworks. The application of testing conditions is referred to as a test. The outcome of a test is classified simply as pass or fail. The test passes if and only if all subnetworks involved in the test are fault free. The test fails if and only if at least one of these subnetworks is faulty.

The results of different tests are analyzed to identify the faulty and nonfaulty subnetworks. Logical functions are utilized for this purpose. Every subnetwork has associated with it a logical variable σ which takes the value 1 if the subnetwork is good and 0 if it is faulty. Every logical test function (LTF) is equal to the complete product of variables σ_j if the test is a pass

$$T_{J_t} \overset{\Delta}{=} \sigma_{j_1} \cap \sigma_{j_2} \cap \dots \cap \sigma_{j_k} , \tag{1a}$$

where

$$J_t \overset{\Delta}{=} \{j_1, j_2, \dots, j_k\} , \tag{1b}$$

j_i refers to the j_i subnetwork and k is the number of subnetworks involved in the test, or the complete union of complemented variables $\bar{\sigma}_j$

$$T_{J_t} \overset{\Delta}{=} \bar{\sigma}_{j_1} \cup \bar{\sigma}_{j_2} \cup \dots \cup \bar{\sigma}_{j_k} \tag{1c}$$

if the test is a fail.

A logical diagnostic function (LDF) is given by

$$D_\ell \overset{\Delta}{=} (\bigcap_{t=1}^{g} T_{J_t}) \cap (\bigcap_{t=g+1}^{\ell} T_{J_t}), \tag{1d}$$

where the first g LTFs correspond to successful tests and ℓ is the total number of tests. In the LDF, the subnetworks which are represented by $\bar{\sigma}_j$ are faulty and those which are represented by σ_j are nonfaulty. If a subnetwork is not represented in the LDF we assume nothing about its status: more tests are necessary.

4. Application of Testing Conditions to Subnetworks

In this section we give necessary and almost sufficient conditions for a subnetwork or group of subnetworks to be fault-free. The conditions are based on invoking KCL and the topological relations. For analog circuits the effect of two independent faults is highly unlikely to cancel at the measurement nodes. We adopt this reasonable heuristic [4].

The input-output relation for a subnetwork S_i, that is connected to the rest of the network by $m_i + 1$ external nodes, is given by

$$\underset{\sim}{i}^{M_i}(t) = \underset{\sim}{h}^{M_i}(\underset{\sim}{v}^{M_i}(t), \underset{\sim}{\phi}_i), \tag{2}$$

where $\underset{\sim}{\phi}_i$ is the vector of subnetwork parameters and the cardinality of $\underset{\sim}{i}^{M_i}$, $\underset{\sim}{h}^{M_i}$ and $\underset{\sim}{v}^{M_i}$ is m_i. S_i is assumed to be connected. Let

$$M_i = M_{i\alpha} \cup M_{i\beta} \cup M_{i\gamma} \cup M_{i\delta}, \tag{3}$$

where $M_{i\alpha}$ is the set of nodes where both voltages and currents are known, $M_{i\beta}$ is the set of nodes

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where only voltages are known, $M_{i\gamma}$ is the set of nodes where only currents are known, $M_{i\delta}$ is the set of nodes where neither currents nor voltages are known and M_i is the set of m_i nodes. Accordingly we may write (2) as

$$\underset{\sim}{i}^{M_{i\alpha}} = \underset{\sim}{h}^{M_{i\alpha}} (\underset{\sim}{v}^{M_{i\alpha}}, \underset{\sim}{v}^{M_{i\beta}}, \underset{\sim}{v}^{M_{i\gamma}}, \underset{\sim}{v}^{M_{i\delta}}, \underset{\sim}{\phi}_i), \quad (4a)$$

$$\underset{\sim}{i}^{M_{i\beta}} = \underset{\sim}{h}^{M_{i\beta}} (\underset{\sim}{v}^{M_{i\alpha}}, \underset{\sim}{v}^{M_{i\beta}}, \underset{\sim}{v}^{M_{i\gamma}}, \underset{\sim}{v}^{M_{i\delta}}, \underset{\sim}{\phi}_i), \quad (4b)$$

$$\underset{\sim}{i}^{M_{i\gamma}} = \underset{\sim}{h}^{M_{i\gamma}} (\underset{\sim}{v}^{M_{i\alpha}}, \underset{\sim}{v}^{M_{i\beta}}, \underset{\sim}{v}^{M_{i\gamma}}, \underset{\sim}{v}^{M_{i\delta}}, \underset{\sim}{\phi}_i), \quad (4c)$$

$$\underset{\sim}{i}^{M_{i\delta}} = \underset{\sim}{h}^{M_{i\delta}} (\underset{\sim}{v}^{M_{i\alpha}}, \underset{\sim}{v}^{M_{i\beta}}, \underset{\sim}{v}^{M_{i\gamma}}, \underset{\sim}{v}^{M_{i\delta}}, \underset{\sim}{\phi}_i). \quad (4c)$$

If the cardinality of the set $M_{i\alpha}$ is greater than the cardinality of the set $M_{i\delta}$, i.e., $m_{i\alpha} > m_{i\delta}$, a <u>necessary condition</u> for the subnetwork S_i to be fault free is that (4a) and (4c) constitute a consistent system of overdetermined equations with $\underset{\sim}{\phi}_i$ assuming nominal values $\underset{\sim}{\phi}_i^0$. We refer to this condition as the <u>internal-self-testing condition (ISTC)</u>.

When all the voltages of M_i are known and $m_{i\alpha}$ is greater than or equal to one, we can state the following stronger result.

4.1 Lemma 1: Self-Testing Condition (STC)

A necessary and almost sufficient condition for a connected subnetwork S_i with m_i+1 external nodes that do not decompose it further, $m_{i\alpha} \geq 1$ and $m_{i\gamma} = m_{i\delta} = 0$ to be fault-free is that

$$\underset{\sim}{i}^{M_{i\alpha}}(t) - \underset{\sim}{h}^{M_{i\alpha}}(\underset{\sim}{v}^{M_i}(t), \underset{\sim}{\phi}_i^0) = \underset{\sim}{0} \;\forall\; t. \quad (5)$$

Normally, the voltages of the m_i nodes are directly measured. The currents in (5) are not directly measured since it is difficult to do so practically except when they represent the input excitation to the whole network. The application of KCL and topological relations overcomes this difficulty. The currents are not measured: they are computed using the nominal parameter values together with the measured voltages, then KCL is invoked.

4.2 Lemma 2: Mutual-Testing Condition (MTC)

A necessary and almost sufficient condition for subnetworks S_i, $i \varepsilon J_t$, that are incident on the node c to be fault-free is that

$$\sum_{i \varepsilon J_t} h_c^{M_i} (\underset{\sim}{v}^{M_i}(t), \underset{\sim}{\phi}_i^0) = 0 \;\forall\; t, \quad (6)$$

i.e., the currents incident to the common node c computed using the measured voltages and nominal parameter values should satisfy KCL.

4.3 Lemma 3: Generalized-Mutual-Testing Condition (GMTC)

Let E_i, $i \varepsilon J_t$, denote some external nodes of the subnetwork S_i. Each subnetwork S_i is connected and has m_i+1 nodes that do not decompose it further, $E_i \subset M_i$. If the currents incident to E_i, $i \varepsilon J_t$, form a cut set, then a necessary and almost sufficient condition for these subnetworks to be fault-free is that

$$\sum_{i \varepsilon J_t} \sum_{k \varepsilon E_i} h_k^{M_i} (\underset{\sim}{v}^{M_i}(t), \underset{\sim}{\phi}_i^0) = 0 \;\forall\; t. \quad (7)$$

5. Tolerance Considerations

For small tolerances the first-order approximation can be utilized to describe the changes in the network response. Accordingly, we may write (5) as

$$\Delta\underset{\sim}{i}^{M_{i\alpha}} \triangleq \underset{\sim}{i}^{M_{i\alpha}}(t) - \underset{\sim}{h}^{M_{i\alpha}} (\underset{\sim}{v}^{M_{i\alpha}}, \underset{\sim}{\phi}_i^0) \quad (8a)$$

$$= \sum_{j=1}^{p} \frac{\partial \underset{\sim}{h}^{M_{i\alpha}}}{\partial \phi_{ij}} \Delta\phi_{ij}, \quad (8b)$$

where p is the number of subnetwork parameters. At a certain instant t_o of time, (8) is an underdetermined system of linear equations in the variables $\Delta\underset{\sim}{\phi}_i$. The weighted-least-squares solution of (8) is the conditional expected value of the parameters $\Delta\underset{\sim}{\phi}_i$ [5], i.e.,

$$\Delta\underset{\sim}{\phi}_i = E\,[\Delta\underset{\sim}{\phi}_i \mid \Delta\underset{\sim}{i}^{M_{i\alpha}}(t_o)], \quad (9)$$

where E denotes the expectation. Using the probabilistic interpretation we can have a measure of how far (5) is satisfied under the variations caused by the tolerances. If any component of the computed vector $\Delta\underset{\sim}{\phi}_i$ from (9) significantly exceeds its tolerance value we consider the test is unsuccessful. The effect of tolerances on condition (6) and (7) is treated in a similar way.

6. Fault Location Inside Faulty Subnetworks

Further diagnosis is usually necessary to identify faulty elements or at least the faulty region inside a faulty subnetwork. For small subnetworks with few elements, a search for the faulty elements inside the subnetwork is feasible, since the number of different combinations to be considered is very few. For relatively larger subnetworks, we first apply the internal-self-testing condition to find a smaller region inside the subnetwork that contains the faulty elements [6]. The directory approach could be incorporated at this stage by considering faulty models for a prespecified set of possible faulty elements.

7. Examples

Here, we consider the application of the method to a nonlinear video amplifier circuit. The circuit is decomposed into 8 subnetworks using nodes 1, 2, 5, 7 and 10 as the decomposition nodes. The circuit in decomposed form is shown in Fig. 1. DC testing is considered. The nominal values and operating conditions are given in [7]. Different fault situations have been simulated. In this paper, we represent the case when Q_3 is faulty. The measurement voltages are

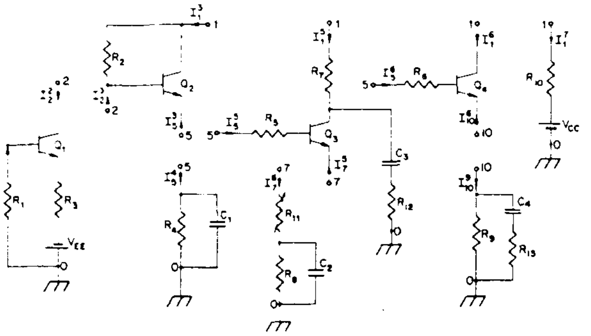

Fig. 1 The video amplified example in decomposed form.

V_1 = 26.1850 V, V_2 = 11.6790 V, V_5 = 10.8809 V, V_7 = 10.8599 V, and V_{10} = 10.1296 V. Table I gives the results when all good elements are kept at nominal. Table II gives the results when the good elements are allowed to vary within ± 10%.

TABLE I

CASE 1 - Q_3 FAULTY

Computed Currents	Diagnosis	Test
$I_2^2 = 4.7437$ mA	$I_2^2 + I_2^3 = 0$	T_{23}
$I_1^3 = 13.9057$ mA	$I_1^3 + I_1^5 + I_1^6 + I_1^7 \neq 0$	T_{3567}
$I_2^3 = -4.7437$ mA	$I_5^3 + I_5^4 + I_5^5 + I_5^6 \neq 0$	T_{3456}
$I_5^3 = -9.1620$ mA	$I_7^5 + I_7^8 \neq 0$	T_{58}
$I_5^4 = 9.0675$ mA	$I_{10}^6 + I_{10}^9 = 0$	T_{69}
$I_1^5 = 5.8736$ mA	$I_2^2 + I_1^7 + I_7^8 + I_5^4$	
$I_5^5 = 0.0593$ mA	$+ I_{10}^9 \equiv 0$	T_{24789}
$I_7^5 = -5.9329$ mA		
$I_1^6 = 3.0389$ mA		
$I_5^6 = 0.0307$ mA		
$I_{10}^6 = -3.0696$ mA		
$I_1^7 = -23.2685$ mA		
$I_7^8 = 6.3882$ mA		
$I_{10}^9 = 3.0696$ mA		

logical diagnostic function $D_6 = (\sigma_2 \cap \sigma_3) \cap (\bar{\sigma}_3 \cup \bar{\sigma}_5 \cup \bar{\sigma}_6 \cup \bar{\sigma}_7) \cap (\bar{\sigma}_3 \cup \bar{\sigma}_4 \cup \bar{\sigma}_5 \cup \bar{\sigma}_6) \cap (\bar{\sigma}_5 \cup \bar{\sigma}_8) \cap (\sigma_6 \cap \sigma_9) \cap (\sigma_2 \cap \sigma_7 \cap \sigma_8 \cap \sigma_4 \cap \sigma_9) = \sigma_2 \cap \sigma_3 \cap \sigma_4 \cap \bar{\sigma}_5 \cap \sigma_6 \cap \sigma_7 \cap \sigma_8 \cap \sigma_9$.

result: S_5 is the only faulty subnetwork

8. Conclusions

The method is applicable to linear and nonlinear networks and is capable of isolating faulty subnetworks, and consequently faulty elements, very efficiently. In applying the method to linear networks we usually follow a hierarchical decomposition approach. This usually results in a systematic way of expediting the testing and with savings in measurements and computations [6].

TABLE II

CASE 2 - Q_3 FAULTY WITH TOLERANCES ON NONFAULTY ELEMENTS

Tests	Maximum Percentage Deviation	Diagnosis
T_{23}	1.983	Pass
T_{3567}	51.44	Fail
T_{3456}	26.99	Fail
T_{58}	70.82	Fail
T_{69}	0.878	Pass
T_{24789}	8.96	Pass

logical diagnostic function $D_6 = T_{23} \cap T_{3567} \cap T_{3456} \cap T_{58} \cap T_{69} \cap T_{24789} = \sigma_2 \cap \sigma_3 \cap \sigma_4 \cap \bar{\sigma}_5 \cap \sigma_6 \cap \sigma_7 \cap \sigma_8 \cap \sigma_9$

result: S_5 is the only faulty subnetwork

9. References

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