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EVALUATION OF FAULTY ELEMENTS WITHIN LINEAR SUBNETWORKS

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SUMMARY

This paper presents a topologically based theoretical background for designing tests for identification of faulty parameter values in linear subnetworks. Nodal voltages are assumed to be obtainable either by measurements or, indirectly, as a result of a nodal fault analysis. A formulation of nodal fault analysis for subnetworks is presented. It is shown how this approach can be used to evaluate faulty elements within inaccessible faulty subnetworks. The objective of this work is the reduction of the number of required current excitations and, thereby, the number of voltage measurements. The Coates flow-graph representation of a network is used.

1. INTRODUCTION

Fault diagnosis and automatic testing techniques for analogue circuits often require parameter identification. Recent papers on the subject[1–17] present different techniques of parameter identification and/or fault region location involving the solution of linear equations. Most of the authors assume voltage measurements, which are more convenient in practice, and consider current excitations only.

A central problem is the formulation of a sufficient number of independent equations subject to a specified number of excitations or voltage measurements. For linear analogue circuits, necessary and sufficient conditions related to the network topology have been formulated, resulting in the identification of faulty nodes or subnetworks.[12–17]

The principal aim of this paper is to develop topologically based conditions for the evaluation of faulty elements within a linear subnetwork under test with a reasonably small number of excitations at a single frequency and, thereby, a small number of measurements. The paper extends the results presented by Biernacki and Starzyk[9] and proposes an efficient approach to the design of test nodes. The Coates flow graph representation of network elements is used.[18]

2. LOCATION OF FAULTY NODES AND DESIGN OF NODAL VOLTAGES

Necessary and sufficient conditions for location of faulty nodes have been discussed[14–16] for linear networks, and more generally[13] for subnetworks selected during the fault location process in a large network. External voltages and currents of a subnetwork may be measured or designed through identification of nonfaulty parts of a large network.[13]

Consider the nodal equations for a nominal subnetwork isolated during a fault location process for a large network as

$$\mathbf{i}^0 = \mathbf{Y}^0\mathbf{v}^0 \qquad (1)$$

where $\mathbf{v}^0$ denotes the response of a nominal subnetwork to a given current excitation $\mathbf{i}^0$.

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Four types of external nodes are associated with this subnetwork: α-nodes, where both voltages and currents are known; β-nodes, where only voltages are known; γ-nodes, where only currents are known; and δ-nodes, where neither voltages nor currents are known.

We assume that all the elements spanned over the nodes β and δ have been arbitrarily associated with other subnetworks, and that they are not represented in (1) (see Figure 1).

Solving (1) we obtain

$$\begin{bmatrix} \mathbf{v}^{\alpha 0} \\ \mathbf{v}^{\beta 0} \\ \mathbf{v}^{\gamma 0} \\ \mathbf{v}^{\delta 0} \\ \mathbf{v}^{\zeta 0} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{\alpha\alpha} & \mathbf{Z}_{\alpha\beta} & \mathbf{Z}_{\alpha\gamma} & \mathbf{Z}_{\alpha\delta} & \mathbf{Z}_{\alpha\zeta} \\ \mathbf{Z}_{\beta\alpha} & \mathbf{Z}_{\beta\beta} & \mathbf{Z}_{\beta\gamma} & \mathbf{Z}_{\beta\delta} & \mathbf{Z}_{\beta\zeta} \\ \mathbf{Z}_{\gamma\alpha} & \mathbf{Z}_{\gamma\beta} & \mathbf{Z}_{\gamma\gamma} & \mathbf{Z}_{\gamma\delta} & \mathbf{Z}_{\gamma\zeta} \\ \mathbf{Z}_{\delta\alpha} & \mathbf{Z}_{\delta\beta} & \mathbf{Z}_{\delta\gamma} & \mathbf{Z}_{\delta\delta} & \mathbf{Z}_{\delta\zeta} \\ \mathbf{Z}_{\zeta\alpha} & \mathbf{Z}_{\zeta\beta} & \mathbf{Z}_{\zeta\gamma} & \mathbf{Z}_{\zeta\delta} & \mathbf{Z}_{\zeta\zeta} \end{bmatrix} \begin{bmatrix} \mathbf{i}^{\alpha 0} \\ \mathbf{i}^{\beta 0} \\ \mathbf{i}^{\gamma 0} \\ \mathbf{i}^{\delta 0} \\ \mathbf{0} \end{bmatrix} \tag{2}$$

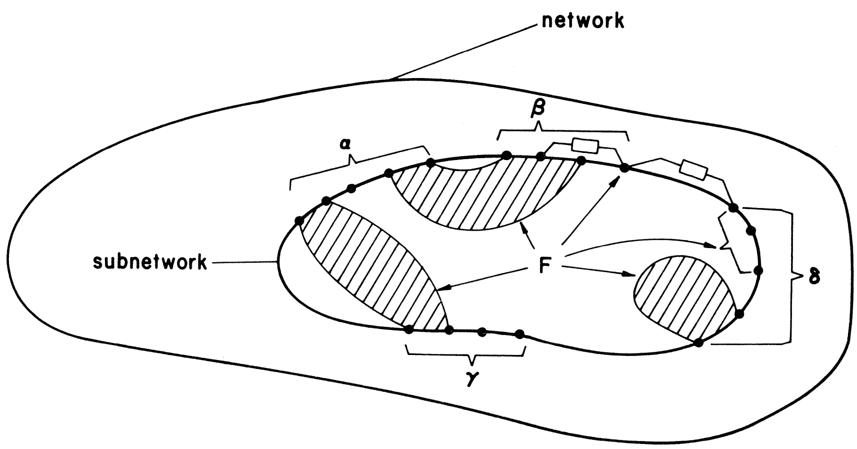


Figure 1. Illustration of remote, inaccessible faulty subnetworks (shaded) spanned over faulty nodes

where ζ represents internal nodes and $\mathbf{Z}_{ab}$ denotes a submatrix of $(\mathbf{Y}^0)^{-1}$ obtained by the intersection of rows a and columns b. The symbol $\mathbf{Z}_{ab}^T$ is defined as $(\mathbf{Z}^T)_{ab}$, symbol $\mathbf{Z}_{ab}^{-1}$ is defined as $(\mathbf{Z}^{-1})_{ab}$, where -1 denotes inversion.

For any subnetwork, with card $\alpha >$ card δ, we obtain an internal-self-testing condition:[13]

$$\begin{bmatrix} \mathbf{v}^{\alpha} \\ \mathbf{v}^{\beta} \end{bmatrix} - \begin{bmatrix} \mathbf{Z}_{\alpha\alpha} & \mathbf{Z}_{\alpha\gamma} \\ \mathbf{Z}_{\beta\alpha} & \mathbf{Z}_{\beta\gamma} \end{bmatrix} \begin{bmatrix} \mathbf{i}^{\alpha} \\ \mathbf{i}^{\gamma} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{\alpha\beta} & \mathbf{Z}_{\alpha\delta} \\ \mathbf{Z}_{\beta\beta} & \mathbf{Z}_{\beta\delta} \end{bmatrix} \begin{bmatrix} \mathbf{i}^{\beta} \\ \mathbf{i}^{\delta} \end{bmatrix} \tag{3}$$

Result 1 (*Fault-free subnetworks*)

If the system of equations (3) is consistent and

$$\text{Rank} \begin{bmatrix} \mathbf{Z}_{\alpha\beta} & \mathbf{Z}_{\alpha\delta} & \mathbf{Z}_{\alpha x} \\ \mathbf{Z}_{\beta\beta} & \mathbf{Z}_{\beta\delta} & \mathbf{Z}_{\beta x} \end{bmatrix} > \text{Rank} \begin{bmatrix} \mathbf{Z}_{\alpha\beta} & \mathbf{Z}_{\alpha\delta} \\ \mathbf{Z}_{\beta\beta} & \mathbf{Z}_{\beta\delta} \end{bmatrix} \tag{4}$$

where $x \in \alpha \cup \gamma \cup \zeta$, then there are no faulty elements incident with nodes x.

According to Result 1, only the elements spanned over the external nodes $\beta \cup \delta$ can be faulty. Because we have associated these elements with other subnetworks we can declare the subnetwork under consideration as fault free. Equation (3) can then be solved for $\mathbf{i}^{\beta}$ and $\mathbf{i}^{\delta}$, hence all the voltages of this subnetwork can be calculated. Consequently, the β- and δ-nodes of this subnetwork become α-nodes of adjacent subnetworks.

Let nodes $\eta \subset \alpha \cup \gamma \cup \zeta$ be faulty, and card $\alpha > (\text{card } \delta) + (\text{card } \eta)$. Let $\mathbf{i}^\eta$ be the vector of node currents representing faults.

Result 2[13] (*Faulty subnetworks*)

If the system of equations

$$\begin{bmatrix} \mathbf{v}^\alpha \\ \mathbf{v}^\beta \end{bmatrix} - \begin{bmatrix} \mathbf{Z}_{\alpha\alpha} & \mathbf{Z}_{\alpha\gamma} \\ \mathbf{Z}_{\beta\alpha} & \mathbf{Z}_{\beta\gamma} \end{bmatrix} \begin{bmatrix} \mathbf{i}^\alpha \\ \mathbf{i}^\gamma \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{\alpha\beta} & \mathbf{Z}_{\alpha\delta} \\ \mathbf{Z}_{\beta\beta} & \mathbf{Z}_{\beta\delta} \end{bmatrix} \begin{bmatrix} \mathbf{i}^\beta \\ \mathbf{i}^\delta \end{bmatrix} + \begin{bmatrix} \mathbf{Z}_{\alpha\eta} \\ \mathbf{Z}_{\beta\eta} \end{bmatrix} \mathbf{i}^\eta \tag{5}$$

is consistent, and

$$\text{Rank} \begin{bmatrix} \mathbf{Z}_{\alpha\beta} & \mathbf{Z}_{\alpha\delta} & \mathbf{Z}_{\alpha\eta} & \mathbf{Z}_{\alpha x} \\ \mathbf{Z}_{\beta\beta} & \mathbf{Z}_{\beta\delta} & \mathbf{Z}_{\beta\eta} & \mathbf{Z}_{\beta x} \end{bmatrix} > \text{Rank} \begin{bmatrix} \mathbf{Z}_{\alpha\beta} & \mathbf{Z}_{\alpha\delta} & \mathbf{Z}_{\alpha\eta} \\ \mathbf{Z}_{\beta\beta} & \mathbf{Z}_{\beta\delta} & \mathbf{Z}_{\beta\eta} \end{bmatrix} \tag{6}$$

where $x \in \alpha \cup \gamma \cup \zeta - \eta$, then the only faulty elements can be those spanned over the set of nodes $F = \eta \cup \beta \cup \delta$. These nodes are called *faulty nodes* although there may be no faulty element incident with β and δ.

Assume that by solving (5) we have evaluated $\mathbf{i}^\beta$, $\mathbf{i}^\delta$ and $\mathbf{i}^\eta$. We can again proceed to evaluate all voltages of the subnetwork under consideration and use the information obtained to analyse the adjacent subnetworks.

For the assumed faulty subnetwork, (1) can be replaced by

$$\mathbf{i}^0 = \mathbf{Y}\mathbf{v} \tag{7}$$

where $\mathbf{v}$ is no longer a nominal response.

If $\mathbf{i}$ is defined as

$$\mathbf{i} = \mathbf{i}^0 + \Delta\mathbf{i} \tag{8}$$

where $\Delta\mathbf{i}$ represents changes in nodal currents due to faulty elements, then we can evaluate nodal voltages in the faulty network $\mathbf{v}$ from the formula

$$\mathbf{i} = \mathbf{Y}^0\mathbf{v} \tag{9}$$

After solving (5), we know the left-hand side of (9) and we can solve (9) to get $\mathbf{v}$.

For all independent current excitations we are, therefore, able to calculate voltages in the faulty network if the conditions of Result 2 are satisfied. These voltages, which would otherwise have to be measured, are required by the approach presented in Reference 9 for evaluating all the elements of a network. In the present paper we only need to evaluate unknown elements, i.e. those which are spanned over the faulty nodes.

3. ELEMENT EVALUATION FOR SUBNETWORKS SPANNED OVER FAULTY NODES

The elements spanned over faulty nodes may form separate subnetworks within a given subnetwork, as shown in Figure 1. The subnetworks may be remote and inaccessible from the point of view of direct excitation and measurement. We can formulate conditions for element evaluation within each of these subnetworks separately and combine the results obtained to establish conditions for the whole network. These conditions will show which external nodes should be excited independently to evaluate all faulty elements.

Consider a linear subnetwork spanned over n faulty nodes. Let the n-dimensional vectors $\bar{\mathbf{i}}$ and $\bar{\mathbf{v}}$ be subsets of $\mathbf{i}^0$ and $\mathbf{v}$, respectively, corresponding to this subnetwork. We can then write

$$\bar{\mathbf{Y}}\bar{\mathbf{v}}^i = \bar{\mathbf{i}}^i \tag{10}$$

for the ith excitation. Our goal is to evaluate $\bar{\mathbf{Y}}$ and then the element values. Although we concentrate our discussion on the nodal equations, it is applicable to any other description based on an independent set of cut-sets.[9]

For n independent excitations, we can write a matrix equation

$$\bar{\mathbf{Y}}\bar{\mathbf{V}}=\bar{\mathbf{I}} \tag{11}$$

where the square matrix

$$\bar{\mathbf{V}}\triangleq[\bar{\mathbf{v}}^1 \quad \bar{\mathbf{v}}^2 \quad \cdots \quad \bar{\mathbf{v}}^n] \tag{12}$$

is the matrix of voltage responses and the square matrix

$$\bar{\mathbf{I}}\triangleq[\bar{\mathbf{i}}^1 \quad \bar{\mathbf{i}}^2 \quad \cdots \quad \bar{\mathbf{i}}^n] \tag{13}$$

is the matrix of current excitations. From (11), we find the unknown matrix $\bar{\mathbf{Y}}$ as

$$\bar{\mathbf{Y}}=\bar{\mathbf{I}}\bar{\mathbf{V}}^{-1} \tag{14}$$

provided that $\bar{\mathbf{V}}$ is non-singular. As a consequence of equations (11) and (14), the following result provides sufficient conditions for the evaluation of $\bar{\mathbf{Y}}$.

Result 3[9]

If a given linear subnetwork can be described by the nodal equation (10) and the current excitations are chosen in such a way that $\bar{\mathbf{I}}$ is a non-singular matrix, then $\bar{\mathbf{V}}$ is also non-singular and the solution (14) exists.

Proof of this result follows from equation (11) since

$$n=\operatorname{rank}\bar{\mathbf{I}}\leqslant\operatorname{rank}\bar{\mathbf{V}}\leqslant n$$

Thus, in order to identify the values of all elements of $\bar{\mathbf{Y}}$, we could arrange for n independent current excitations, design or measure all nodal voltages and then apply equation (14).

In order to perform the least number of tests, however, we must obviously eliminate whole columns of $\bar{\mathbf{V}}$. We propose a systematic way which enables us to identify tests necessary for component evaluation. The method assumes that all components have non-zero values.

Numerical and topological conditions

Equation (11) can be rewritten in the form

$$\bar{\mathbf{V}}^{\mathrm{T}}\bar{\mathbf{Y}}^{\mathrm{T}}=\bar{\mathbf{I}}^{\mathrm{T}} \tag{15}$$

Consider the product of $\bar{\mathbf{V}}^{\mathrm{T}}$ and the jth column of $\bar{\mathbf{Y}}^{\mathrm{T}}$. We have

$$\bar{\mathbf{V}}^{\mathrm{T}}\hat{\mathbf{y}}=\begin{bmatrix}\bar{\mathbf{v}}^{1\mathrm{T}}\\ \bar{\mathbf{v}}^{2\mathrm{T}}\\ \vdots\\ \bar{\mathbf{v}}^{n\mathrm{T}}\end{bmatrix}\begin{bmatrix}y_{j1}\\ y_{j2}\\ \vdots\\ y_{jn}\end{bmatrix}=\hat{\mathbf{i}} \tag{16}$$

where $\hat{}$ is used to represent a column of the appropriate transposed matrix (or, equivalently, transposed row of the corresponding original matrix).

Let the k unknown elements of $\hat{\mathbf{y}}$ be identified by the set of indices $C=\{j_1,\ldots,j_k\}$. We call the set of elements y_{ji}, $i\in C$, a *reduced cut-set*. Transferring the known terms from the left-hand side to the right-hand side of (16) and adjusting $\hat{\mathbf{i}}$ appropriately we rewrite the equation as

$$\bar{\mathbf{V}}^{\mathrm{T}}\begin{bmatrix}0\\ \vdots\\ 0\\ y_{jj_1}\\ \vdots\\ y_{jj_k}\\ \vdots\\ 0\end{bmatrix}=\bar{\mathbf{V}}^{\mathrm{T}}_{NC}\hat{\mathbf{y}}^{C}=\begin{bmatrix}I_{1j}\\ I_{2j}\\ \vdots\\ I_{nj}\end{bmatrix} \tag{17}$$

where $\bar{\mathbf{V}}^{\mathrm{T}}_{NC}$ consists of columns C from $\bar{\mathbf{V}}^{\mathrm{T}}$, N is the set of subnetwork nodes, and I_{ij} is the equivalent external current for a reduced cut-set at the jth node due to the ith current excitation.

In order to determine the elements $y_{jj_1}, \ldots, y_{jj_k}$, we can solve a subsystem of (17) given by

$$\bar{\mathbf{V}}^{\mathrm{T}}_{BC}\hat{\mathbf{y}}^{C}=\begin{bmatrix}I_{i_1 j}\\ \vdots\\ I_{i_k j}\end{bmatrix} \tag{18}$$

where the k equations are chosen from (17) in such a way that the square submatrix $\bar{\mathbf{V}}^{\mathrm{T}}_{BC}$ obtained as the intersection of rows $B=\{i_1, \ldots, i_k\}$ and columns C, is non-singular. See Figure 2 for an illustration. According to relationship (15), the matrix $\bar{\mathbf{V}}^{\mathrm{T}}_{BC}$ can be defined as

$$\bar{\mathbf{V}}^{\mathrm{T}}_{BC}=\bar{\mathbf{I}}^{\mathrm{T}}_{BN}(\bar{\mathbf{Y}}^{\mathrm{T}})^{-1}_{NC} \tag{19}$$

where $\bar{\mathbf{I}}^{\mathrm{T}}_{BN}$ consists of rows B from $\bar{\mathbf{I}}^{\mathrm{T}}$ and $(\bar{\mathbf{Y}}^{\mathrm{T}})^{-1}_{NC}$ consists of columns C from $(\bar{\mathbf{Y}}^{\mathrm{T}})^{-1}$. On the basis of (19) and the Cauchy–Binet theorem[19] we may formulate the following result.

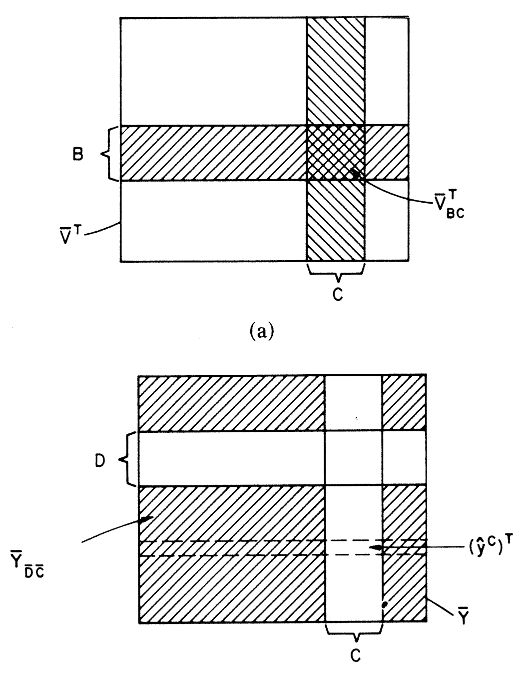


Figure 2. Illustrations of equation (18) and Result 4

Result 4

If the matrix $\bar{\mathbf{V}}_{BC}^{\mathrm{T}}$ is non-singular then

$$\exists D\colon \det \bar{\mathbf{I}}_{BD}^{\mathrm{T}} \neq 0 \quad \text{and} \quad \det \bar{\mathbf{Y}}_{\bar{D}\bar{C}} \neq 0 \tag{20}$$

where $\bar{D} = N - D$, $\bar{C} = N - C$ (see Figure 2).

Consider a sequence of sets C_j, $j = j_1, \ldots, j_M$, which corresponds to a sequence of reduced cut-sets of the *current graph*[20] of the subnetwork. Only those reduced cut-sets will be considered for which external currents, if any, can be specified. Based on (11) and (18), the following result can be summarized.

Result 5

If independent excitations which appear at or are applied to the subset of nodes $A \subset N$ are sufficient for the identification of all elements of $\bar{\mathbf{Y}}$ then

$$\forall C_j \exists B_j \subset A \quad \text{and} \quad \exists D_j\colon \det \bar{\mathbf{I}}_{B_j D_j}^{\mathrm{T}} \neq 0 \quad \text{and} \quad \det \bar{\mathbf{Y}}_{\bar{D}_j \bar{C}_j} \neq 0 \tag{21}$$

where

$$\operatorname{card} B_j = \operatorname{card} C_j = \operatorname{card} D_j \tag{22}$$

Nodes A in Result 5 can be chosen from a remote inaccessible subnetwork, therefore we call them *injection nodes.* For each subnetwork the set A must be a subset of the external nodes of this subnetwork.

As a consequence of (22), we have the following corollary.

Corollary 1

$$\operatorname{card} A \geqslant \max_j \operatorname{card} C_j \tag{23}$$

It is seen from (23) that the choice of the sequence of C_j is crucial for the minimization of the number of sufficient tests.

Now, in order to characterize D_j feasible for a given C_j, we consider topological equations for the nodal admittance matrix.

$$\bar{\mathbf{Y}} = \boldsymbol{\lambda}_- \mathbf{Y}_{\mathrm{e}} \boldsymbol{\lambda}_+^{\mathrm{T}} \tag{24}$$

where the element ij of $\boldsymbol{\lambda}_-$ is equal to 1 if the jth edge is directed towards the ith node, otherwise zero; and the element ij of $\boldsymbol{\lambda}_+$ is equal to 1 if the jth edge is directed away from the ith node, otherwise zero; $\mathbf{Y}_{\mathrm{e}}$ is a diagonal matrix of edge admittances.

The submatrix of $\bar{\mathbf{Y}}$ obtained by removing columns C_j can be expressed as

$$\bar{\mathbf{Y}}_{\mathrm{N}C_j} - \boldsymbol{\lambda}_- \mathbf{Y}_{\mathrm{e}} \boldsymbol{\lambda}_+'^{\mathrm{T}} \tag{25}$$

where $\boldsymbol{\lambda}_+'$ is obtained from $\boldsymbol{\lambda}_+$ by removing rows C_j. In the Coates graph, this corresponds to deleting all the edges outgoing from nodes C_j.

Similarly,

$$\bar{\mathbf{Y}}_{\bar{D}_j \bar{C}_j} = \boldsymbol{\lambda}_-' \mathbf{Y}_{\mathrm{e}} \boldsymbol{\lambda}_+'^{\mathrm{T}} \tag{26}$$

where $\boldsymbol{\lambda}_-'$ is obtained from $\boldsymbol{\lambda}_-$ by removing rows D_j. In the Coates graph, this corresponds to deleting all the edges incoming to nodes D_j.

Let G denote a directed Coates graph[18] and let P denote a set of node pairs of G, namely $P = \{(v_{s1}, v_{e1}), \ldots, (v_{sk}, v_{ek})\}$, where $v_{pl} \neq v_{nm}$ for $l \neq m$ $(p, n = s, e)$.

Definition[21]

A k-connection of a graph G is a subgraph c_P of the graph, such that elements of c_P form a set of k node-disjoint directed paths and node-disjoint directed circuits incident with all graph nodes. The starting point and the endpoint of the paths are indicated by the pairs of P.

Let us consider the Coates graph $G(D|C)$ obtained from the graph of the given subnetwork after deleting all the edges incoming to nodes D and all the edges outgoing from nodes C. The following theorem can be proved on the basis of the Cauchy–Binet theorem[19] and the concept of the k-connection.[21]

Theorem 1

If $\det \bar{\mathbf{Y}}_{\bar{D}\bar{C}} \neq 0$, there exists in $G(D|C)$ at least one k-connection c_P (see Figure 3), where

$$P = \{(v_s, v_e) | v_s \in D, v_e \in C\} \tag{27}$$

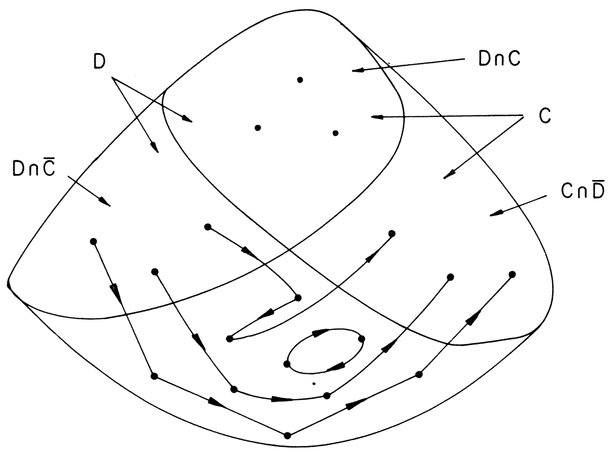


Figure 3. Example of required 3-connections

and

$$k = \text{card } P = \text{card } D = \text{card } C \tag{28}$$

(v_s, v_e) represents a path directed from the node v_s to the node v_e or isolated node when $v_s = v_e$.

Proof. According to the Cauchy–Binet theorem and relation (26), we have

$$\det \bar{\mathbf{Y}}_{\bar{D}\bar{C}} = \sum \det \mathbf{K}^- \det \mathbf{K}^+ \tag{29}$$

where $\mathbf{K}^-$ is a major submatrix of $\boldsymbol{\lambda}'_-\mathbf{Y}_e$ with order equal to $(n - \text{card } D)$ and $\mathbf{K}^+$ is the corresponding major submatrix of $\boldsymbol{\lambda}'^{T}_{+}$. If $\det \bar{\mathbf{Y}}_{\bar{D}\bar{C}} \neq 0$, then there exists at least one pair of corresponding determinants, both different from zero. A major determinant of $\boldsymbol{\lambda}'_-\mathbf{Y}_e$ is different from zero if and only if there exists one non-zero element in every row of the chosen submatrix (chosen set of columns). This corresponds to the set of $(n - \text{card } D)$ edges, such that every edge has a different endpoint, belonging to the set of nodes $(N - D)$. The corresponding submatrix is different from zero if the same edges have different origins, belonging to the same set of nodes $(N - C)$. Now it is easy to check that these edges form a k-connection, as stated in Theorem 1.

Remark

If rank $\bar{\mathbf{I}}^{T}_{AN} = \text{card } A$, where $\bar{\mathbf{I}}^{T}_{AN}$ consists of rows A from $\bar{\mathbf{I}}^{T}$, then

$$\forall B_j \subset A \qquad \exists D_j\colon \det \bar{\mathbf{I}}^{T}_{B_j D_j} \neq 0 \tag{30}$$

As a consequence of Theorem 1 and the Remark, we have an important corollary.

Corollary 2

From Result 5 it follows that we should find a set B_j such that, after deleting all the edges outgoing from nodes C_j and after deleting all the edges incoming to nodes D_j, there are no isolated nodes in the set $\overline{D_j \cap C_j}$.

Definition

A node is said to be a *corner* if there exists a complete subgraph containing all the edges incoming to the node as well as the edges having the same weight as any of the incoming ones.

The order of this complete subgraph is not defined. In particular, it may be a complete graph of zero order—see Figure 4(a), in which vertex v is a corner. Also, there may exist edges outgoing from a corner to other parts of the graph of the network which are not part of the complete subgraph—see Figure 4(b), in which both vertices labelled v are corners. The remaining two vertices are not corners simply because complete subgraphs that contain all edges incoming to these nodes do not exist. Vertex x in Figure 4(c) is not a corner, although the complete subgraph containing all the edges incoming to x exists, but it does not contain another edge of weight α.

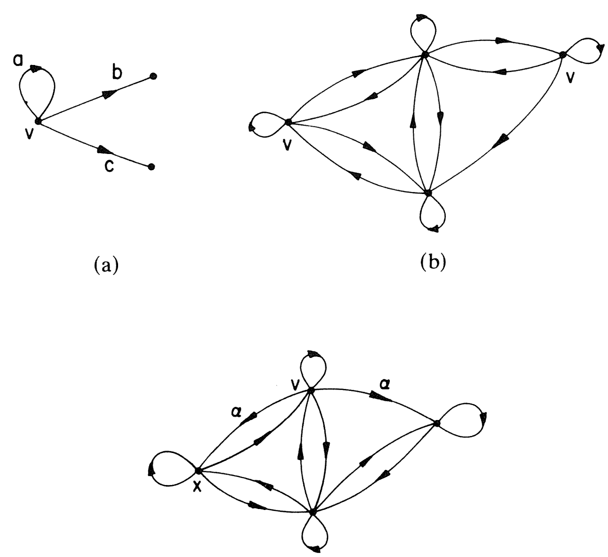


Figure 4. Examples of corners. Corners are denoted by v

In practice, if a vertex is not a corner it follows that we do not have to provide independent excitations at this node to solve for elements of a reduced cut-set at that node. This arises from the following theorem.

Theorem 2

All the corners must be injection nodes.

Proof. Assume that a corner is not an injection node. If we identify an edge within the subnetwork incident with the corner, then every reduced cut-set containing the edge must contain all the nodes of the complete subgraph. After deleting all the edges outgoing from the nodes of this reduced cut-set, the corner will be an isolated node, and if it is not an injection node, we obtain an isolated node in the set $\overline{D_j \cap C_j}$ and a contradiction to Corollary 2.

Thus, the number of corners influences the minimal cardinality of A. In order to estimate the cardinality of A, the following remarks may be helpful.

Remark 1

card $A \geq$ order of the maximal complete subgraph.

Remark 2

card $A \geq$ minimal incoming degree in the remaining subgraph after deleting all edges incident with corners.

The incoming degree of a vertex is the number of edges incoming to this vertex.

Location of injection nodes

An optimal selection of injection nodes could be done in a combinatorial way, where different sets of reduced cut-sets are considered and then different combinations of injection nodes are checked. However, for large networks, it may be quite tedious to check the conditions of Theorem 1, even if reduced cut-sets and a set A are known.

An efficient heuristic algorithm, which can be adopted to find injection nodes, was presented in Reference 9. It allows us to find a nearly minimal set of injection nodes in a time which depends linearly on the subnetwork size. Since the conditions stated in Theorem 1 must be satisfied, the algorithm localizes injection nodes in such a way that there exists a set of separate paths from injection nodes to the nodes of each reduced cut-set, as illustrated in Figure 5.

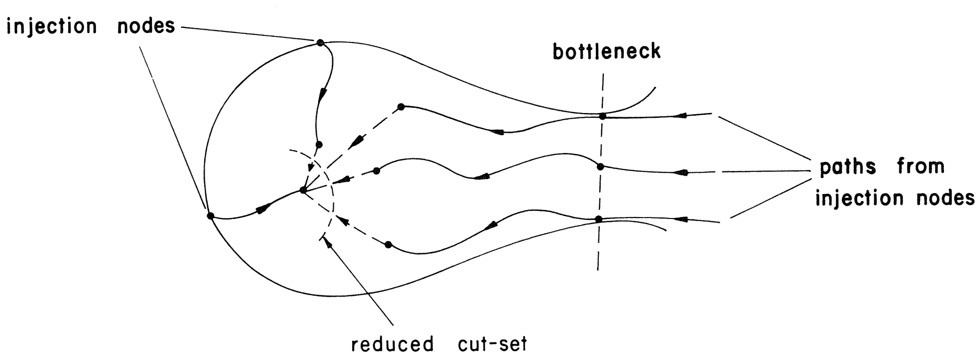


Figure 5. Illustration of the paths required from injection nodes to a reduced cut-set

In particular cases, when the number of injection nodes is too large because of the subnetwork topology we can reduce them by adding some known elements to the subnetwork under consideration. The same argument holds when we have too many corners in the subnetwork (Figure 6). These remarks concern the case when we identify elements of a given network using voltage measurements at all nodes[9] as well as evaluation of faulty elements within remote, inaccessible subnetworks. In the latter case, adding the known elements may be equivalent to considering an augmented subnetwork which will contain faulty nodes as well as some non-faulty ones.

The following examples explain how to use the results obtained from the test finding algorithm to identify all network elements.

Example 1

The subnetwork, whose parameters we want to design, and its Coates graph are shown in Figure 7 (node 0 is chosen as the reference node). Let us assume for simplicity that the independent current

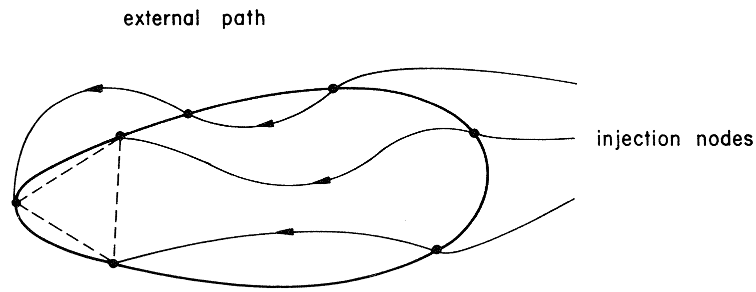
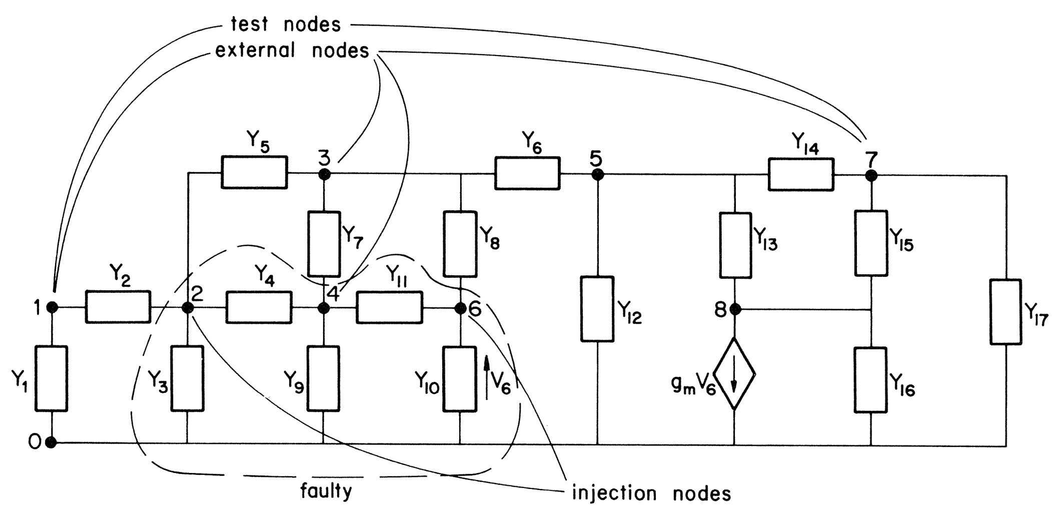
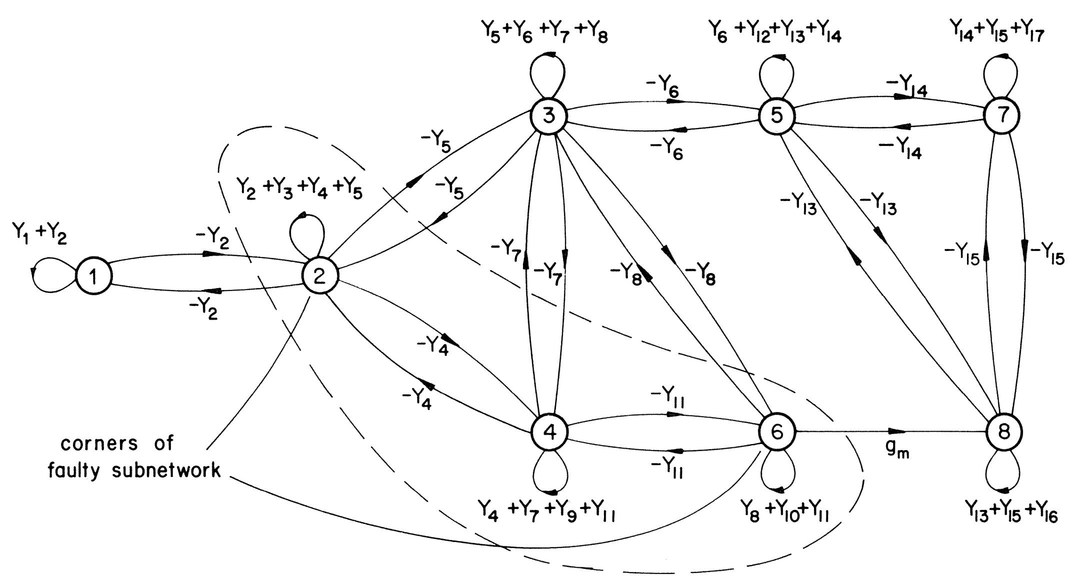


Figure 6. External path from an injection node to a corner



(a)



(b)

Figure 7. (a) Faulty network with faulty subnetwork spanned over faulty nodes, (b) the corresponding Coates graph

excitation $\bar{\mathbf{I}}=\mathbf{1}$. This can be easily achieved when elements are identified through direct voltage measurements. There are 3 corners in this network—nodes 1, 6 and 7. We find that they constitute a sufficient set of injection nodes for this network. Table I illustrates the reduced cut-sets considered and elements associated with them. For identification of network elements, we apply excitations at nodes 1, 6 and 7. The nodal voltages measured with unit excitations at different nodes are shown in Table II. We formulate equations (18) for successive reduced cut-sets and compute element values. The first equation is as follows:

$$\begin{bmatrix} V_{11} & V_{12} \\ V_{61} & V_{62} \end{bmatrix} \begin{bmatrix} Y_1+Y_2 \\ -Y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 0\cdot77641 & 0\cdot32925 \\ -0\cdot38775 & -1\cdot1633 \end{bmatrix} \begin{bmatrix} Y_1+Y_2 \\ -Y_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and we obtain $Y_1=1$, $Y_2=0\cdot5$.

Table I. Reduced cut-sets

Step i	Nodes in reduced cut-set	Elements in the reduced cut-set to be found
1	1, 2	Y_1, Y_2
2	2, 3, 4	Y_3, Y_4, Y_5
3	3, 4, 6	Y_7, Y_9, Y_{11}
4	3, 6	Y_8, Y_{10}
5	3, 5	Y_6
6	5, 7, 8	Y_{12}, Y_{13}, Y_{14}
7	7, 8	Y_{15}, Y_{17}
8	6, 8	Y_{16}, g_m

Table II. Nodal voltages for example 1

Excitation at node no.	Voltage at node no.							
	1	2	3	4	5	6	7	8
1	0·77641	0·32925	−0·0066477	0·14264	−0·57149	0·038418	−0·91631	−2·0943
6	−0·38775	−1·1633	−4·6575	−1·4699	−15·751	0·89959	−22·757	−50·343
7	0·016174	0·048524	0·17534	0·076466	0·47525	0·091385	4·5309	−2·126

The second equation

$$\begin{bmatrix} V_{11} & V_{12} & V_{13} & V_{14} \\ V_{61} & V_{62} & V_{63} & V_{64} \\ V_{71} & V_{72} & V_{73} & V_{74} \end{bmatrix} \begin{bmatrix} -Y_2 \\ Y_2+Y_3+Y_4+Y_5 \\ -Y_5 \\ -Y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

can be transformed, because Y_2 is now known, to

$$\begin{bmatrix} V_{12} & V_{13} & V_{14} \\ V_{62} & V_{63} & V_{64} \\ V_{72} & V_{73} & V_{74} \end{bmatrix} \begin{bmatrix} Y_3+Y_4+Y_5 \\ -Y_5 \\ -Y_4 \end{bmatrix} = \begin{bmatrix} (V_{11}-V_{12})Y_2 \\ (V_{61}-V_{62})Y_2 \\ (V_{71}-V_{72})Y_2 \end{bmatrix}$$

or

$$\begin{bmatrix} 0\cdot32925 & -0\cdot0066477 & 0\cdot14264 \\ -1\cdot1633 & -4\cdot6575 & -1\cdot4699 \\ 0\cdot048524 & 0\cdot17534 & 0\cdot076466 \end{bmatrix} \begin{bmatrix} Y_3+Y_4+Y_5 \\ -Y_5 \\ -Y_4 \end{bmatrix} = \begin{bmatrix} 0\cdot22358 \\ 0\cdot38778 \\ -0\cdot016175 \end{bmatrix}$$

and we obtain $Y_3=0\cdot333$, $Y_4=0\cdot25$, $Y_5=0\cdot2$.

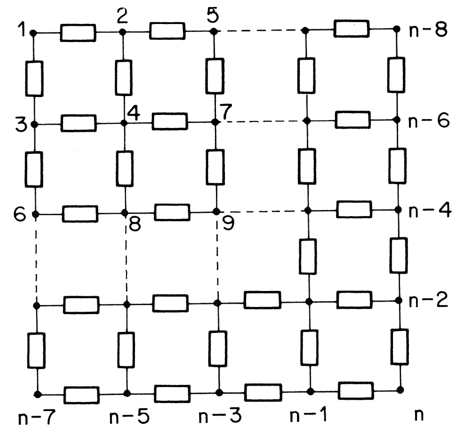


Figure 8. Grid circuit example

Continuing the procedure we design all the other network elements as

$$Y_6=0{\cdot}167,\quad Y_7=0{\cdot}143,\quad Y_8=0{\cdot}125,\quad Y_9=0{\cdot}111,\quad Y_{10}=0{\cdot}1$$
$$Y_{11}=0{\cdot}0909,\quad Y_{12}=0{\cdot}0833,\quad Y_{13}=0{\cdot}0769,\quad Y_{14}=0{\cdot}0714$$
$$Y_{15}=0{\cdot}0667,\quad Y_{16}=0{\cdot}0625,\quad Y_{17}=0{\cdot}0588,\quad g_m=8{\cdot}5$$

Example 2

We apply the algorithm proposed to the passive grid circuit shown in Figure 8. In such circuits, the number of nodes $n=k^2$ and number of passive elements $e=2k^2-2k$, where $k=2,3,\ldots,$ We assume that the voltage at each node is known. We find that, no matter what the size of the grids, three tests at a single frequency are sufficient for determining all the element values.

4. ELEMENT EVALUATION USING EXTERNAL EXCITATION NODES

Let us assume that we have distinct, remote, inaccessible faulty subnetworks $S_1,\ldots,S_f$ spanned over faulty nodes within the subnetwork under investigation (see Figure 9). According to Result 2, the number

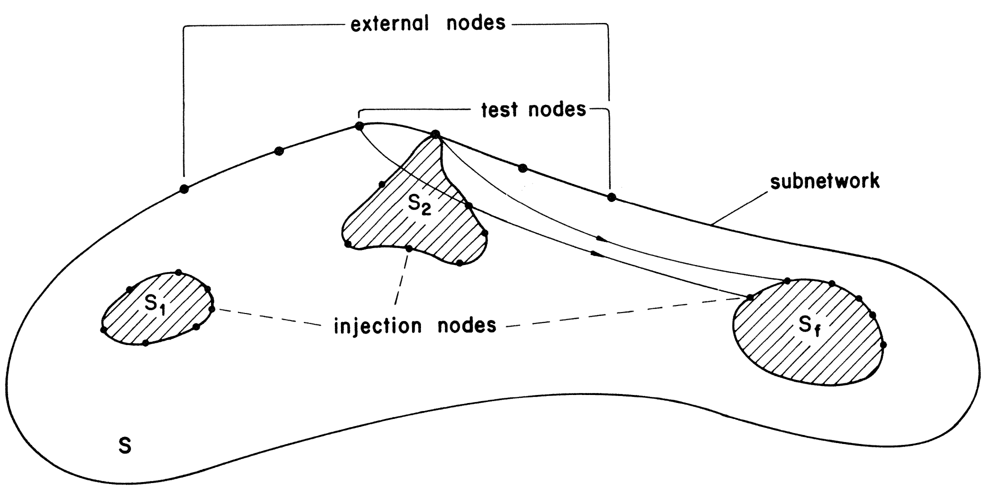


Figure 9. Inaccessible faulty subnetworks (shaded) spanned over faulty nodes

of external nodes, where both voltages and external currents are known, have to satisfy the relation

$$\text{card}\,\alpha > \sum_{i=1}^{f} n_i \tag{31}$$

where n_i is the number of nodes in the subnetwork S_i. We can apply the approach discussed in Section 3 to each subnetwork $S_1, \ldots, S_f$ separately to identify sets of injection nodes $A^1, \ldots, A^f$ at which independent current excitations could be forced. To be able to evaluate all elements within $S_1, \ldots, S_f$, independent excitations must appear at injection nodes.

Let T be a subset of the external nodes of the subnetwork S, which is defined by (2). Let G denote the Coates signal-flow graph of S. Let us assume that we have evaluated faulty currents and designed nodal voltages as discussed in Section 2. Let $k_i = \text{card}\, A^i$.

Lemma 1

To evaluate all the elements of S_i there must exist k_i simultaneous and separate paths in G from T to A^i not incident with other S_i nodes.

Proof is based on the recognition of each cut-set in S_i as a reduced cut-set in S.

Corollary 3

To evaluate all faulty elements in S Lemma 1 must be satisfied for all A^i. Then T can be chosen as a set of test nodes where independent current excitations are applied.

We are interested in having the cardinality of T as small as possible, to minimize the number of tests and designs of nodal voltages.

Corollary 4

$$\text{card}\, T \geq \max k_i \tag{32}$$

The main goal of the approach presented is to find k_i as small as possible, so the technique described guarantees the identification of faulty elements effectively. For most practical cases, card T is between 2 and 5.

Remark

For the identification of faulty elements within remote inaccessible subnetworks we design currents flowing into these subnetworks from the surrounding network using the designed voltages and nominal element values first, and then proceed with element evaluation within each of them, as discussed.

Example 3

Assume that the nominal element values for the network from Figure 7 are as follows:

$$Y_1 = 1, \quad Y_2 = 0{\cdot}5, \quad Y_3 = 0{\cdot}3, \quad Y_4 = 0{\cdot}32$$

$$Y_5 = 0{\cdot}2, \quad Y_6 = 0{\cdot}167, \quad Y_7 = 0{\cdot}143$$

$$Y_8 = 0{\cdot}125, \quad Y_9 = 0{\cdot}1, \quad Y_{10} = 0{\cdot}2, \quad Y_{11} = 0{\cdot}1$$

$$Y_{12} = 0{\cdot}0833, \quad Y_{13} = 0{\cdot}0769, \quad Y_{14} = 0{\cdot}0714$$

$$Y_{15} = 0{\cdot}0667, \quad Y_{16} = 0{\cdot}0625, \quad Y_{17} = 0{\cdot}0588, \quad g_m = 8{\cdot}5$$

Four external points are available for voltage measurements and current excitations at the nodes 1, 3, 4 and 7. Assume for simplicity that all external nodes are of the α type. Using the approach discussed in Section 2 we have found three faulty nodes, namely, 2, 4, 6 and evaluated currents $\mathbf{i}^{\eta}$, $\eta = \{2, 4, 6\}$. The subnetwork spanned over the faulty nodes is a simple ladder network. With the help of the method discussed in Section 3 we can locate nodes 2 and 6 as injection nodes sufficient for evaluation of the ladder elements. According to Lemma 1 external current excitations for element evaluation can be made at nodes 1 and 7.

Now we simulate the nominal network with independent (unit) excitations at nodes 1 and 7 separately and evaluate currents $\mathbf{i}^7$ from equation (5). With those currents and independent current excitations we excite the nominal network to obtain the current voltages as in rows 1 and 3 of the Table II. Elements Y_2, Y_5, Y_7 and Y_8 are nominal as they are not spanned over the faulty nodes. Using the voltages from Table II we calculate external currents for the ladder subnetwork spanned over faulty nodes as equal to

$$I_{12}=(V_{11}-V_{12})Y_2+(V_{13}-V_{12})Y_5=0{\cdot}1564$$

$$I_{14}=(V_{13}-V_{14})Y_7=-0{\cdot}02135$$

$$I_{16}=(V_{13}-V_{16})Y_8=-0{\cdot}005633$$

Similarly, we can get

$$I_{72}=0{\cdot}009188, \qquad I_{74}=0{\cdot}01414, \qquad I_{76}=0{\cdot}01049$$

Equation (18) for the first reduced cut-set has the form

$$\begin{bmatrix} V_{12} & V_{14} \\ V_{72} & V_{74} \end{bmatrix} \begin{bmatrix} Y_3+Y_4 \\ -Y_4 \end{bmatrix} = \begin{bmatrix} I_{12} \\ I_{72} \end{bmatrix}$$

Therefore,

$$\begin{bmatrix} 0{\cdot}32925 & 0{\cdot}14264 \\ 0{\cdot}048524 & 0{\cdot}076466 \end{bmatrix} \begin{bmatrix} Y_3+Y_4 \\ -Y_4 \end{bmatrix} = \begin{bmatrix} 0{\cdot}1564 \\ 0{\cdot}009188 \end{bmatrix}$$

and we get $Y_3=0{\cdot}333$ and $Y_4=0{\cdot}25$. In the next two reduced cut-sets elements Y_9, Y_{11} and Y_{10} are evaluated, respectively, with the help of a voltage measurement as well as evaluated and nominal elements.

5. CONCLUSIONS

The method presented helps us find, on the basis of network topology, a reasonably small number of excitation nodes for the identification of all faulty parameter values of linear analogue subnetworks. This can be achieved by searching for a 'good' sequence of reduced cut-sets within the subnetworks spanned over faulty nodes, whose elements are consecutively determined from (18). The element evaluation approach, as presented in Sections 3 and 4, is easy to program and gives a linear dependence of computational effort on the size of the network. The notion of corner is particularly important, since it influences the number of necessary injection nodes independently of a sequence of cut-sets. The number of excitations can be reduced by adding external elements or some nominal ones in the case of inaccessible subnetworks.

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