

RECENT ADVANCES IN FAULT LOCATION OF ANALOG NETWORKS

J.W. Bandler and A.E. Salama

Simulation Optimization Systems Research Laboratory and the  
Department of Electrical and Computer Engineering  
McMaster University, Hamilton, Canada L8S 4L7

**ABSTRACT**

This paper serves as an evaluation of the state of the art of the fault location problem in analog circuits. The emphasis is on the recent developments in the subject. The problems that are associated with the application of the simulation-before-test and the simulation-after-test techniques are identified. We summarize the basic characteristics of the different techniques in a comparative table.

**INTRODUCTION**

For more than two decades, the subject of fault location in analog circuits has been of interest to researchers in the circuits and systems society. In recent years this interest has intensified and a number of promising developments has emerged [1-20]. We review these recent developments and identify the obstacles that hinder the practical application of the different fault location techniques.

The fault location techniques are classified here according to the stage in the testing process at which simulation of the circuit under test occurs [1]. We consider the simulation-before-test approach, as well as the simulation-after-test approach for fault location.

For the simulation-before-test approach, we present the recent developments in the fault dictionary methods. For the simulation-after-test approach we consider parameter identification techniques, algebraic invariance techniques for fault isolation and approximation techniques which utilize optimization. Recent important theoretical developments in both parameter identification techniques and algebraic invariance techniques are emphasized. The basic characteristics of the different techniques are finally summarized and compared with the ideal goals.

**FAULT DICTIONARY METHODS**

The before-test effort involves faults definition, where the most likely faults are anticipated, and dictionary construction using an optimum set of diagnostic measurements. At the time of testing, a fault isolation criterion is utilized to identify the fault circuit to one of the prestored faults.

The basic problems that are faced by the approach are limited fault situations, large size of stored data, unavailability of fault models, ambiguity due to parameter tolerances, difficulties in very large scale testing, applicability to soft fault isolation, diagnostic measurements selection and the fault isolation criteria.

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Definition of Faults

Early fault dictionaries considered only hard faults (short or open circuit element faults). Specific single faults corresponding to element deviations have been added to the dictionary [2] to overcome the difficulties in isolating soft faults (element value deviations from nominal not reaching the bounds). Most dictionaries do not consider multiple faults so as to limit the size of the dictionary to a practical size.

Dictionary Construction

Features (diagnostic measurements) selection is the most important part of the fault dictionary technique. A few features with a high degree of distinguishability are required. Different techniques have been proposed for features selection. In [3] the hypothesized faults are grouped into ambiguity sets according to their simulated responses. Logical analysis is then used to eliminate noninformative and redundant features. A heuristic optimization technique is proposed in [2] to find a set of optimum measurements without performing an exhaustive search. A performance index has been introduced as a measure of the effectiveness of the measurements in fault isolation.

An information theoretic measure of the expected information deficit (compared to perfect certainty) when we have performed a set of measurements  $V^m$  is given by [4]

$$J(V^m) \triangleq - \int \text{Prob}(\Delta\phi | V^m) \text{Log}(\text{Prob}(\Delta\phi | V^m)) d\Delta\phi, \quad (1)$$

where  $\Delta\phi$  is the change of the element values from nominal. Integrating over all possible outcomes of  $V^m$  each weighted by its respective probability and assuming a normal distribution we arrive at a criterion that is similar to the D-optimal design criterion in the statistical design of experiments problem.

In most cases, the dictionary takes the form of a look-up table with entries  $d_{ij}$ ,  $i = 1, 2, \dots, n_m$ ,  $j = 1, 2, \dots, n_f$ , where  $n_f$  is the number of hypothesized faults and  $n_m$  is the number of measurements used in constructing the dictionary.

Fault Isolation

Probabilistic measures are introduced in the decision criterion to indicate the most probable faulty elements [4]. The measures take into consideration the reliability history of circuit under test, the uncertainties in the measurements and the tolerance effects. The isolation criterion can be expressed as choosing the fault case  $f$  that minimizes

$$(V^m - d^f)^T (\Lambda_{mm}^f)^{-1} (V^m - d^f) + \text{Log}[\Lambda_{mm}^f / (\text{Prob}(f))^2], \quad (2)$$

where  $d^f$  corresponds to the dictionary entries  $d_{if}$ ,  $i = 1, 2, \dots, n_m$  and  $\Lambda_{mm}^f$  is the covariance matrix.

Under special conditions (2) is reduced to the widely used nearest-neighbor rule [2]. Definite improvements in isolation resulted by using criteria similar to (2) [5].

Efficient Fault Simulation Methods

Excessive computation time is required to develop a

fault dictionary for large networks. Efficient algorithms have been proposed for the simulation of multiple faults [6-7]. Furthermore, theoretically based fault response bounds are obtained without the need of a full worst-case analysis [8].

Let the linear network be described by the nodal equations

$$\mathbf{Y}_n \mathbf{V}^n = \mathbf{I}^n, \quad (3)$$

where  $\mathbf{Y}_n$  is the nodal admittance matrix,  $\mathbf{V}^n$  is the vector of nodal voltages and  $\mathbf{I}^n$  is the vector of the current sources.

For  $n_f$  faults which have changed simultaneously the change in the nodal admittance can be expressed as

$$\Delta \mathbf{Y}_n = \mathbf{A}_f \Delta \mathbf{Y}_f \bar{\mathbf{A}}_f^T, \quad (4)$$

where  $\mathbf{A}_f, \bar{\mathbf{A}}_f$  are transformation matrices and  $\Delta \mathbf{Y}_f$  is a diagonal matrix of order  $n_f$ . Utilizing Householder's formula, the changes in the nodal voltages are given by

$$\Delta \mathbf{V}^n = -\mathbf{Y}_n^{-1} \mathbf{A}_f [\Delta \mathbf{Y}_f^{-1} + \bar{\mathbf{A}}_f^T \mathbf{Y}_n^{-1} \mathbf{A}_f]^{-1} \bar{\mathbf{A}}_f^T \mathbf{V}^n, \quad (5)$$

which results in substantial computational savings.

Fault models [9] for some elements and devices have been proposed for the analytic construction of faulty network responses. The emphasis is to utilize the available models in any simulation routine after modifications to the values of model parameters not to the basic topology of the model. Many other fault models for solid state devices need to be developed.

### SIMULATION-AFTER-TEST APPROACH

The simulation-after-test approach for fault location has been the most active area of research in recent years. The advances in technology have made it feasible to have on-line powerful computational capabilities. Therefore, simulation-after-test has become more feasible. Nevertheless, the on-line computational requirement is still the major problem faced by the approach.

Depending on the number of available independent measurements, either all network elements could be identified or faulty elements are searched for and identified.

#### Parameter Identification Techniques

For a sufficient number of measurements obtained by multiple excitations, the fault location problem is treated as a parameter identification problem.

For dc testing of nonlinear networks [10], let the input-output model of the network be given by

$$\mathbf{V}^m = \mathbf{h}(\mathbf{I}^m, \boldsymbol{\phi}), \quad (6)$$

where, without loss of generality, we assume that the outputs are the nodal voltages  $\mathbf{V}^m$  and the inputs are the nodal currents  $\mathbf{I}^m$ .  $\boldsymbol{\phi}$  is the  $n_\phi$ -vector of network elements.

Assuming that  $\mathbf{h}(\mathbf{I}^m, \boldsymbol{\phi})$  is analytic in  $\mathbf{I}^m$  and  $\boldsymbol{\phi}$ , we check the following matrix for the diagnosability of a regular parameter point  $\boldsymbol{\phi}^*$  [10].

$$\mathbf{R}(\boldsymbol{\phi}^*) = \sum_{i=1}^{n_\phi} [\nabla_{\boldsymbol{\phi}} \mathbf{h}^T(\mathbf{I}_i^m, \boldsymbol{\phi}^*)][\nabla_{\boldsymbol{\phi}} \mathbf{h}^T(\mathbf{I}_i^m, \boldsymbol{\phi}^*)]^T, \quad (7)$$

where  $\nabla_{\boldsymbol{\phi}}$  indicates the gradients of  $\mathbf{h}$  w.r.t.  $\boldsymbol{\phi}$  and the subscript  $i$  refers to the  $i$ th excitation. If the rank of  $\mathbf{R}(\boldsymbol{\phi}^*)$ ,  $\rho^*$ , is equal to  $n_\phi$  then the circuit is locally diagnosable and any  $n_\phi$  randomly chosen inputs can be used to solve uniquely for the element values of the circuit. The measure  $\mu^* = n_\phi - \rho^*$  is considered as a measure of testability of the circuit under test. Similar results

have been developed for time domain testing of nonlinear networks and multifrequency linear networks testing [11].

The evaluation of network parameter values is carried out by solving the set of equations

$$\mathbf{h}(\mathbf{I}_i^m, \boldsymbol{\phi}) - \mathbf{V}_i^m = \mathbf{0}, \quad i = 1, 2, \dots, n_\phi, \quad (8)$$

using fast converging iterative methods [12].

For linear networks, if the internal nodes could be restored as explained in [13], only a linear system of equations is solved to obtain the network parameters.

Similarly, under the assumption that all network nodes are accessible we may construct a system of linear equations utilizing more than one excitation and solve for the network parameters uniquely [14].

#### Fault Verification Techniques

For a limited number of measurements, the emphasis is on fault element isolation assuming that most of the network elements are within tolerance bounds. Most techniques utilize a single current excitation and nodal voltage measurements.

In linear networks, either the faulty elements are isolated directly or, more easier, the faulty nodes are identified. Recalling (3) we may write, using the theory of perturbation, the nodal equations of the faulty network as

$$\mathbf{Y}_n \Delta \mathbf{V}^n = \Delta \mathbf{I}^{nf}, \quad (9)$$

where  $\Delta \mathbf{I}^{nf}$  represents the faulty nodal currents. A node  $i$  is faulty if and only if the  $i$ th component of  $\Delta \mathbf{I}^{nf}$  is nonzero [15].

For  $n_f$  faulty nodes we may write (9) after appropriate rearrangements as

$$\begin{bmatrix} \Delta \mathbf{V}^m \\ \Delta \mathbf{V}^{n-m} \end{bmatrix} = \mathbf{Y}_n^{-1} \Delta \mathbf{I}^{nf} = \begin{bmatrix} \mathbf{Z}_{mf} & \mathbf{Z}_{m,n-f} \\ \mathbf{Z}_{n-m,f} & \mathbf{Z}_{n-m,n-f} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{I}^f \\ \mathbf{0} \end{bmatrix}. \quad (10a)$$

Hence,

$$\Delta \mathbf{V}^m = \mathbf{Z}_{mf} \Delta \mathbf{I}^f, \quad (10b)$$

where  $\Delta \mathbf{I}^f$  is the nonzero part of  $\Delta \mathbf{I}^{nf}$  and consists of  $n_f$  components. If the number of measurements  $n_m$  is greater than the number of faulty nodes (10b) will be an overdetermined system of equations. Eliminating  $\Delta \mathbf{I}^f$ , we get

$$[\mathbf{Z}_{mf} (\mathbf{Z}_{mf}^T \mathbf{Z}_{mf})^{-1} \mathbf{Z}_{mf}^T - \mathbf{I}] \Delta \mathbf{V}^m = \mathbf{0}. \quad (11)$$

(11) provides a necessary condition for locating the faulty nodes. It is independent on the values of the faulty currents  $\Delta \mathbf{I}^f$  and depends only on their location.

The  $n_f$  faulty nodes are uniquely located if the following rank test, which is known as the  $n_f$ -node-fault testability condition is satisfied [15].

$$\text{Rank} \{ \mathbf{Z}_{mq} \} = n_f + 1 \quad (12)$$

for all possible  $q$ , where  $q$  refers to  $(n_f + 1)$  columns of  $\mathbf{Z}_{mn}$ .

Topological conditions are derived [15] to characterize testability condition (12). Similar results are derived for  $n_f$ -branch-fault testability [16] as well as faulty region location [17]. For many linear networks  $n_f + 1$  measurement nodes are necessary and sufficient for the isolation of  $n_f$  faulty nodes or branches. Also, all computations are linear and rely only on the nominal network parameters [18].

In nonlinear networks the problem of the unique diagnosability of a single fault has been characterized [10]. For the multiple fault case, the theorems can be easily extended.

### Failure Bounds Technique

For large networks, the number of combinations that are considered to check (11) is enormous and the computations will be prohibitive. Under the assumption that the maximum number of faults is bounded and is less than the number of measurements and the assumption that the effect of two independent analog failures will never cancel, the heuristic assumption, an efficient heuristic technique has been proposed to isolate faulty elements in linear and nonlinear networks [19]. The technique requires partitioning of the set of network elements into two subsets and testing the components of one subset using the nominal characteristics of the other subset.

### Network Decomposition Approach [20]

A large network could be viewed as a set of mutually uncoupled subnetworks that are connected at the nodes of decomposition. The input-output description of the  $i$ th subnetwork  $S_i$  is given by

$$I^{mi} = h^{mi}(V^{mi}, \phi_i), \quad (13)$$

where  $\phi_i$  is the vector of the subnetwork parameters and  $I^{mi}$  and  $V^{mi}$  are the currents and voltages of the external nodes of the subnetwork.

A necessary and almost sufficient condition for subnetworks  $S_i$ ,  $i \in J_c$ , that are incident on a common node  $c$  to be fault-free is that

$$\sum_{i \in J_c} h_c^{mi}(V^{mi}, \phi_i^0) = 0, \quad (14)$$

where  $\phi_i^0$  indicates the nominal parameter values. This condition is known as the mutual-testing condition. Other testing conditions for subnetworks have been derived [20].

### Other Techniques

Other techniques have been proposed for fault location. Approximation techniques that utilize either an optimization technique or probabilistic technique for isolating the most likely faulty elements from a limited number of measurements have been devised. These methods are characterized with the need of large computational requirements. Also, an attempt has been taken to apply artificial intelligence to the problem of fault location.

## DISCUSSION AND COMPARISON

A number of different points are important regarding the practical application of any of the above mentioned techniques. We consider the following criteria: on-line computational requirements, off-line computational requirements, test points, robustness, type of faults, network types, network models, diagnosis resolution and in-situ testing. In Table I the goals for an ideal algorithm are summarized and the degree to which the various techniques achieve these goals is indicated.

The recent theoretical developments in the simulation-after-test approach have suggested regarding fault analysis as a third branch of network theory. A successful practical technique for fault isolation is still the objective of many current researchers, but the recently developed techniques have been incorporated in some analog automatic test program generation (AATPG) schemes.

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TABLE I

COMPARISON OF FAULT LOCATION TECHNIQUES WITH IDEAL GOALS

	On-line Compu- tation	Off-line Compu- tation	Test Nodes	Robustness	Types of Faults	Network Types	Network Models	Diagnosis Level	In-Situ Testing
Fault Dictionary	Minimal	High	Limited	No	Single	Linear/ Nonlinear	Fault	Set	No
Parameter Identification	High	Minimal	Limited	Yes	Multiple	Linear/ Nonlinear	Nominal	Element	No
Fault Verification	Moderate	Minimal	Limited	Yes/No	Multiple	Linear/ Nonlinear	Nominal	Element	Yes
Failure Bounds	Moderate	Minimal	Limited	Yes/No	Multiple	Linear/ Nonlinear	Nominal	Element	Yes
Network Decomposition	Minimal	Minimal	Limited	Yes	Multiple	Linear/ Nonlinear	Nominal	Subnetwork	Yes
Approx. Techniques	High	Minimal	Limited	Yes/No	Mostly Single	Linear/ Nonlinear	Nominal	Element	Yes
Ideal Goals	Minimal	Moderate	Limited	Yes	Multiple	General	Fault/ Nominal	Module/ Parameter	Yes