Proceedings of ISCAS 85

A SUPERLINEARLY CONVERGENT ALGORITHM FOR NONLINEAR ℓ_1 OPTIMIZATION WITH CIRCUIT APPLICATIONS

J.W. Bandler*, W. Kellermann* and K. Madsen**

* Simulation Optimization Systems Research Laboratory
and Department of Electrical and Computer Engineering
McMaster University, Hamilton, Canada L8S 4L7
** Institute for Numerical Analysis
Technical University of Denmark
DK-2800 Lyngby, Denmark

ABSTRACT

This paper presents a new and highly efficient algorithm for nonlinear ℓ_1 optimization. The algorithm is similar to a minimax algorithm originated by Hald and Madsen. It is a combination of a first-order method that approximates the solution by successive linear programming and a quasi-Newton method using approximate second-order information to solve the system of nonlinear equations resulting from the first-order necessary conditions for optimum. The new ℓ_1 algorithm is particularly useful in fault location methods using the ℓ_1 norm. Another important application of the algorithm is the parameter identification problem in multi-coupled cavity narrow band-pass filters.

INTRODUCTION

Let

$$f_j(\mathbf{x}) \triangleq f_j(x_1, x_2, \ldots, x_n), \quad j=1, \ldots, m, \tag{1}$$

be a set of m nonlinear, continuously differentiable functions. The vector $\mathbf{x} \triangleq [x_1 \; x_2 \; \ldots \; x_n]^T$ is a set of n parameters to be optimized. We consider the optimization problem

$$\underset{\mathbf{x}}{\text{minimize}} \quad F(\mathbf{x}) \triangleq \sum_{j=1}^{m} |f_j(\mathbf{x})| \tag{2}$$

This is called the unconstrained ℓ_1 optimization problem.

We present an iterative algorithm for solving (2) which requires the user to supply function and gradient values of the nonlinear functions f_j. The algorithm also uses some second-order information, i.e., some information about the second order derivatives of the functions. This is approximated from the user supplied gradients.

The algorithm is similar to that of Hald and Madsen in [1]. It has been reported by Hald in [2], which describes and lists a Fortran subroutine implementing a version of the algorithm. Hald and Madsen [3] have demonstrated that the algorithm has sure convergence properties.

The algorithm is a two stage one. It always starts in Stage 1, which is a first-order trust region method similar to that of Madsen [4]. Often this method has quadratic final convergence but in some cases (called singular, see Madsen and Schjaer-Jacobsen [5]) the final convergence is slow. Therefore, Stage 2 is introduced. Here a quasi-Newton method is used to solve a set of nonlinear equations that necessarily hold at a local solution of (2). If the Stage 2 iteration is unsuccessful, then a switch is made back to Stage 1. Several switches between the two stages are allowed. The switching criteria ensure that the global convergence properties of the Stage 1 iteration are not wasted by the Stage 2 iteration. Experiments show that usually very few switches are performed.

The algorithm presented here considers, for simplicity, only the unconstrained ℓ_1 optimization problem. The full description and actual implementation takes full advantage of linear equality and inequality constraints.

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada under Grants A7239 and G1135.

DESCRIPTION OF THE ALGORITHM

The Stage 1 Iteration

At the kth stage of the iteration we have an approximation $\mathbf{x}_k$ of the solution and a local bound Λ_k. We wish to use the gradient information at $\mathbf{x}_k$ to find a better approximation $\mathbf{x}_{k+1}$. Therefore, we find the increment $\mathbf{h}_k$ as a solution of the linearized ℓ_1 problem

$$\underset{\mathbf{h}}{\text{minimize}} \quad \bar{F}(\mathbf{x}_k, \mathbf{h}) \triangleq \sum_{i=1}^{m} |f_i(\mathbf{x}_k) + \mathbf{f}_i'(\mathbf{x}_k)^T \mathbf{h}|$$

$$\text{subject to} \quad \|\mathbf{h}\|_\infty \le \Lambda_k , \tag{3}$$

where $\mathbf{f}_i'$ is the gradient of f_i w.r.t. $\mathbf{x}$. The subproblem is solved using linear programming.

The next iterate is found by the formula

$$\mathbf{x}_{k+1} = \begin{cases} \mathbf{x}_k + \mathbf{h}_k & \text{if } F(\mathbf{x}_k + \mathbf{h}_k) < F(\mathbf{x}_k) \\ \mathbf{x}_k & \text{otherwise} \end{cases} \tag{4}$$

Finally, the local bound, which is intended to be a measure of the goodness of the linear approximations, is updated using comparisons of the decrease $D_k(\mathbf{h}) = F(\mathbf{x}_k) - F(\mathbf{x}_k + \mathbf{h})$ and the predicted decrease $PD_k(\mathbf{h}) = F(\mathbf{x}_k) - \bar{F}(\mathbf{x}_k, \mathbf{h})$,

$$\Lambda_{k+1} = \begin{cases} 2\,\Lambda_k & \text{if } D_k(\mathbf{h}_k) \ge 0.75 \; PD_k(\mathbf{h}_k) \\ 0.5\,\Lambda_k & \text{if } D_k(\mathbf{h}_k) \le 0.25 \; PD_k(\mathbf{h}_k) \\ \Lambda_k & \text{otherwise} \end{cases} \tag{5}$$

The Stage 2 Iteration

At a local solution $\mathbf{x}$ of (2) the following equations hold (see, e.g., Charalambous [6]), with $|\delta_j| \le 1$,

$$\sum_{j \notin Z} \text{sign}(f_j(\mathbf{x}))\, \mathbf{f}_j'(\mathbf{x}) + \sum_{j \in Z} \delta_j \mathbf{f}_j'(\mathbf{x}) = \mathbf{0},$$
$$f_j(\mathbf{x}) = 0 , \qquad j \in Z . \tag{6}$$

The set $Z = Z(\mathbf{x})$, called the active set at $\mathbf{x}$, corresponds to those functions that are 0 at $\mathbf{x}$.

The Stage 2 iteration is a quasi-Newton method for solving (6) with Z being replaced by

$$Z_k = \{ j \mid |f_j(\mathbf{x}_k)| < \varepsilon \} . \tag{7}$$

Conditions for Switching to Stage 2

The Stage 2 iteration is started when it seems reasonable to assume that the estimate (7) corresponds to a solution **x**. Therefore, we require that the active set as defined in (7) has stabilized before we start a Stage 2 iteration. It is required that

$$Z_k = Z_{k-j_1} = \dots = Z_{k-j_v} , \tag{8}$$

i.e., the active set must have been constant for $(v+1)$ consecutive Stage 1 iterates. Secondly, we require that the first-order multiplier estimates are in the prescribed ranges

$$-1 \le (\delta_k)_j \le 1 , \quad j \in Z_k . \tag{9}$$

Causes for Switching Back to Stage 1

The rules of this section are set up in order to ensure that if a Stage 2 iteration is started with an improper active set then a switch back to Stage 1 will take place. The rules for continuing in Stage 2 are the following.

It is required that the active set Z_k remains constant and that no inactive function changes sign. It is required that the sign restrictions (9) hold on every iteration. Finally it is required that the residuals $\mathbf{r}(\mathbf{x},\boldsymbol{\delta})$ corresponding to equation (6) decrease in every iteration in the sense

$$\| \mathbf{r}(\mathbf{x}_{k+1},\boldsymbol{\delta}_{k+1}) \|_2 \le 0.99 \| \mathbf{r}(\mathbf{x}_k,\boldsymbol{\delta}_k) \|_2 . \tag{10}$$

It has been shown in [3] that the method can converge only to stationary points. Further, it has been shown that the final rate of convergence is either quadratic or superlinear, depending on whether the solution is regular or not. When a solution **x** is regular, n functions (at least) are 0 at **x**. Finally, it has been shown that when the active set is correctly chosen the Stage 2 iteration generates the same sequence of points as would be obtained if Powell's sequential quadratic programming method [7] were applied to a nonlinear programming formulation of (2).

Several numerical examples, with n ranging between 2 and 8 and m ranging between 3 and 60, have been solved. In all cases a local minimum was found to more than 10 digits and the number of function evaluations ranged between 5 and 27.

FAULT ISOLATION USING THE ℓ_1 NORM

Formulation of the Problem

This application of the new ℓ_1 optimization algorithm deals with fault isolation in linear analog circuits under an insufficient number of independent voltage measurements. The ℓ_1 norm is used to isolate the most likely faulty elements. Practically, the faulty components are very few and the relative change in their values is significantly larger than in the nonfaulty ones [8]. The method presented here is a modification of the method utilizing multiple test vectors to obtain the measurements [9]. For k different excitations applied to the faulty network we consider the following optimization problem.

$$\underset{\mathbf{x}}{\text{Minimize}} \sum_{i=1}^{n} |\Delta x_i / x_i^0| \tag{11a}$$

subject to

$$\mathbf{V}_1^{c} - \mathbf{V}_1^{m} = \mathbf{0} ,$$

$$\vdots \tag{11b}$$

$$\mathbf{V}_k^{c} - \mathbf{V}_k^{m} = \mathbf{0} ,$$

where $\mathbf{x} \triangleq [x_1 \; x_2 \; \dots \; x_n]^T$ is a vector of network parameters, $\mathbf{x}^0$ represents the nominal parameter values, $\Delta x_i \triangleq x_i - x_i^0$, i = 1, 2, ..., n, represent the deviations in network parameters from nominal values, $\mathbf{V}_k^{m}$ is a p-dimensional vector of voltage measurements performed at the accessible nodes for the kth excitation and $\mathbf{V}_k^{c}$ is a p-dimensional vector of voltages at accessible nodes calculated using the vector **x** as parameter values.

The corresponding nonlinear ℓ_1 problem can be formulated based on an exact penalty function [6] as follows.

$$\underset{\mathbf{x}}{\text{Minimize}} \sum_{j=1}^{n+kxp} |f_j(\mathbf{x})| , \tag{12}$$

where

$$f_i(\mathbf{x}) \triangleq \Delta x_i / x_i^0 , \qquad 1, 2, \dots, n, \tag{13}$$

$$f_{n+i}(\mathbf{x}) \triangleq \beta_i (V_i^{c} - V_i^{m}) , \qquad i = 1, 2, \dots, kxp , \tag{14}$$

and β_i , i=1, 2, ..., kxp, are appropriate multipliers (satisfying certain conditions stated in [6]).

Mesh Network Example [9]

Consider the resistive network shown in Fig. 1 with the nominal values of elements G_i = 1.0 and tolerances ε_i = ± 0.05, i = 1, 2, ..., 20. All outside nodes are assumed to be accessible with node 12 taken as the reference node. Nodes 4, 5, 8 and 9 are assumed internal, where no measurements can be performed.

Two faults are assumed in the network in elements G_2 and G_{18}. For Case 1 we applied the new ℓ_1 algorithm to optimization problem (12) with a single excitation at node 1. For Case 2 we considered two excitations applied at nodes 3 and 6 sequentially. The results of both optimization problems are summarized in Table I. The nominal component values have been used as a starting point since just a few elements change significantly from nominal.

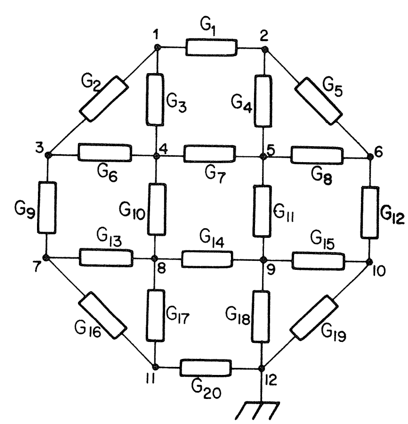

Fig. 1 The resistive mesh network.

In both cases the actual faulty elements have been identified, but in Case 2, the estimated changes in the faulty elements are closer to their true values. Also some of the changes in the nonfaulty components approach better their true values in Case 2. The estimated changes in the faulty elements are much closer to the actual changes as compared to the results reported in [9].

TABLE I

RESULTS FOR THE MESH NETWORK EXAMPLE

Element	Nominal Value	Actual Value	Percentage Deviation		
			Actual	Case 1	Case 2
G_1	1.0	0.98	-2.0	0.00	0.13
G_2	1.0	0.50	-50.0*	-48.78	-49.44
G_3	1.0	1.04	4.0	0.00	3.60
G_4	1.0	0.97	-3.0	0.00	0.00
G_5	1.0	0.95	-5.0	-2.26	-1.71
G_6	1.0	0.99	-1.0	0.00	0.00
G_7	1.0	1.02	2.0	0.00	0.00
G_8	1.0	1.05	5.0	0.00	0.00
G_9	1.0	1.02	2.0	2.80	0.97
G_{10}	1.0	0.98	-2.0	0.00	0.00
G_{11}	1.0	1.04	4.0	0.00	0.00
G_{12}	1.0	1.01	1.0	3.45	2.08
G_{13}	1.0	0.99	-1.0	0.00	-0.44
G_{14}	1.0	0.98	-2.0	0.00	0.00
G_{15}	1.0	1.02	2.0	0.00	1.55
G_{16}	1.0	0.96	-4.0	-2.42	-5.71
G_{17}	1.0	1.02	2.0	0.00	2.67
G_{18}	1.0	0.50	-50.0*	-52.16	-48.94
G_{19}	1.0	0.98	-2.0	0.00	-1.95
G_{20}	1.0	0.96	-4.0	-3.67	-4.88
Number of Function Evaluations				8	8
Execution Time (secs) on Cyber 170/815				3.0	3.9

* Faults

PARAMETER IDENTIFICATION USING THE ℓ_1 NORM

Formulation of the Problem

In this application we deal with multi-coupled cavity narrow band-pass filters used in microwave communication systems (see Fig. 2).

A narrow-band lumped model of an unterminated multi-cavity filter has been given by Atia and Williams [10] as

$$\mathbf{Z}\,\mathbf{I} = \mathbf{V}, \tag{15}$$

where

$$\mathbf{Z} = j(s\mathbf{1} + \mathbf{M}), \tag{16}$$

$$s = \frac{\omega_0}{\Delta\omega}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right), \tag{17}$$

$\mathbf{1}$ denotes an nxn identity matrix and $\mathbf{M}$ an nxn coupling matrix whose (i, j) element represents the normalized coupling between the ith and jth cavities.

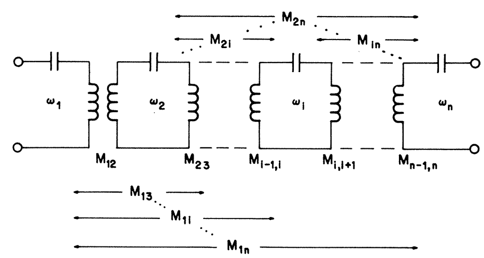

Fig. 2 Unterminated coupled-cavity filter illustrating the coupling coefficients.

In practice it is often desired to determine the actual filter couplings based on response (return loss or insertion loss) measurements. The problem can be formulated as an optimization problem with the ℓ_1 objective function.

In this example we have used reflection coefficient as the filter response. The formulation is as follows.

$$\underset{\mathbf{x}}{\text{Minimize}} \sum_{j=1}^{m} |f(\mathbf{x}, \omega_j)| , \tag{18}$$

where

$$f(\mathbf{x},\omega_j) \triangleq w(\omega_j)\,(F^c(\mathbf{x}, \omega_j) - F^m(\omega_j)), \tag{19}$$

$\mathbf{x}$ is the vector of filter couplings to be identified, F^c is the response calculated using the current parameter values, F^m is the measured response and w is a positive weighting factor.

The filter response and its sensitivities are calculated using the formulas given in [11].

6th Order and 10th Order Filter Examples

A 6th order filter centered at 12000 MHz with 40 MHz bandwidth is considered. Optimally designed filter parameters have been perturbed and the filter has been simulated. Reflection coefficient at 23 frequency points is used as the specification (measured response). The optimization problem (18) has been solved using optimal filter couplings as starting values. The results of parameter identification are summarized in Table II.

An optimally designed 10th order filter in the 12 GHz region with parameters perturbed from optimal is considered as a second example of parameter identification. Using reflection coefficient at 38 frequency points as the specifications all filter couplings have been identified with accuracy sufficient to produce the same response as the perturbed system. The results of parameter identification are shown in Fig. 3.

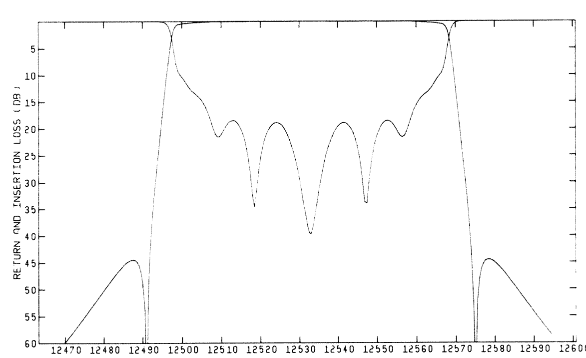

Fig. 3 10th order, 12 GHz filter responses with coupling parameters identified by the ℓ_1 algorithm.

TABLE II

RESULTS FOR THE 6TH ORDER FILTER EXAMPLE

Coupling	Percentage Deviation Actual	Identified
M_{12}	2.0	2.0
M_{23}	-1.0	-1.0
M_{34}	5.0	5.0
M_{16}	5.0	5.0
M_{25}	-4.0	-4.0
M_{45}	-1.0	-1.0
M_{56}	2.0	2.0
Number of Function Evaluations		24
Execution Time (secs) on Cyber 170/815		6.2

CONCLUSIONS

We have described a new and highly efficient algorithm for nonlinear ℓ_1 optimization problems. The algorithm combines linear programming methods with quasi-Newton methods and the convergence is at least superlinear.

The importance of the algorithm presented stems from the fact that in approximation problems with data containing a few wild points or gross errors the ℓ_1 norm residual is superior to using other norms ℓ_p with $p > 1$.

We have demonstrated that the new ℓ_1 algorithm is very successful in methods for fault isolation in linear analog circuits under an insufficient number of independent voltage measurements. We have presented a formulation using the ℓ_1 norm for model parameter identification problems and illustrated it with 6th order and 10th order multi-coupled cavity narrow band-pass filters.

REFERENCES

[1] J. Hald and K. Madsen, "Combined LP and quasi-Newton methods for minimax optimization", Mathematical Programming, vol. 20, 1981, pp. 49-62.

[2] J. Hald, "A 2-stage algorithm for nonlinear ℓ_1 optimization", Report No. NI-81-03, Inst. for Num. Analysis, Tech. University of Denmark, 1981.

[3] J. Hald and K. Madsen, "Combined LP and quasi-Newton methods for nonlinear ℓ_1 optimization", SIAM J. on Numerical Analysis, to be published.

[4]. K. Madsen, "An algorithm for minimax solution of overdetermined systems of nonlinear equations", J.IMA, vol. 16, 1975, pp. 321-328.

[5] K. Madsen and H. Schjaer-Jacobsen, "Singularities in minimax optimization of networks", IEEE Trans. Circuits and Systems, vol. CAS-23, 1976, pp. 456- 460.

[6] C. Charalambous, "On conditions for optimality of the nonlinear ℓ_1 problem", Mathematical Programming, vol. 17, 1979, pp. 123-135.

[7] M.J.D. Powell, "The convergence of variable metric methods for nonlinearly constrained optimization calculations", Report DAMTP 77/NA3, University of Cambridge, 1977.

[8] H.M. Merrill, "Failure diagnosis using quadratic programming", IEEE Trans. Reliability, vol. R-22, 1973, pp. 207-213.

[9] J.W. Bandler, R.M. Biernacki, A.E. Salama and J.A. Starzyk, "Fault isolation in linear analog circuits using the ℓ_1 norm", Proc. IEEE Int. Symp. Circuits and Systems (Rome, Italy, 1982), pp. 1140-1143.

[10] A.E. Atia and A.E. Williams, "Narrow-bandpass waveguide filters", IEEE Trans. Microwave Theory Tech., vol. MTT-20, 1972, pp. 258-265.

[11] J.W. Bandler, S.H. Chen, S. Daijavad and W. Kellermann, "Optimal design of multi-cavity filters and contiguous-band multiplexers", Proc. 14th European Microwave Conference (Liège, Belgium, 1984), pp. 863-868.