COMPUTER AIDED DESIGN OF BRANCHED CASCADED NETWORKS

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ABSTRACT

A new and attractive theory is presented for computer oriented simulation, sensitivity analysis and design of branched cascaded circuits. The forward and reverse analysis approach developed by Bandler et al. for cascaded circuit analysis is extended and applied to general branched cascaded circuits. This theory permits an efficient and fast analytical and numerical investigation of responses and sensitivities of all functions of interest w.r.t. any variable parameter, including frequency.

INTRODUCTION

The implementation of a gradient-based optimization technique in the design of branched cascaded networks, e.g., contiguous or non-contiguous band multiplexers, requires a robust and efficient algorithm for simulation and sensitivity analysis. In this paper, we present a new and elegant approach to the simulation of such responses as common port and branch output port return losses, insertion loss and group delay between source and branch output ports and their first-order sensitivities w.r.t. all network parameters as well as frequency.

The basic components of the structure considered are 2-port equivalents or 3-port junctions. The fundamental requirement for the approach is that the transmission matrix description of all basic components and their derivatives, if they contain variables, are provided. The presentation includes the evaluation of various responses and sensitivities for an arbitrarily constructed illustrative example as well as the optimal design of a 12-channel 12 GHz contiguous-band multiplexer using gradient-based optimization techniques.

CASCADED ANALYSIS

To apply forward and reverse analysis [1], 3-port junctions are reduced to 2-port representations so that the cascaded analysis can be readily carried through these junctions in different desired directions. Consider the 3-port network shown in Fig. 1. To carry the analysis through the junction along the main cascade, we terminate port 3 and represent the transmission matrix between ports 1 and 2 by **A**. The linear combination between the voltages and currents at ports 2 and 3 can be expressed as

$$\boldsymbol{\alpha}^T \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \boldsymbol{\beta}^T \begin{bmatrix} V_3 \\ -I_3 \end{bmatrix} . \qquad (1)$$

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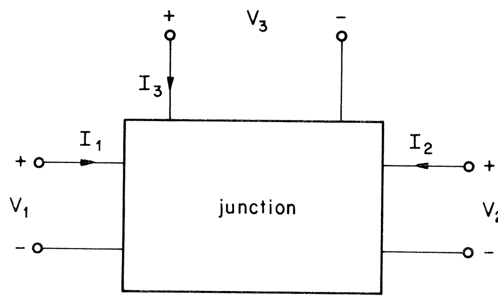

Fig. 1 A 3-port network in which ports 1 and 2 are considered along a main cascade and port 3 represents a branch of the main cascade.

The analysis can also be carried through the junctions into any desired branch by terminating port 2 and denoting the transmission matrix between ports 1 and 3 by **D**.

Consider a network consisting of N sections, as shown in Fig. 2. A typical section has a junction, n(k) cascaded elements of branch k and a subsection along the main cascade. All reference planes in the entire network are defined uniformly and numbered consecutively beginning from the main cascade termination, which is designated reference plane 1. The source port is designated reference plane 2N+2. The termination of the kth branch is called reference plane $\iota(k)$ and the branch main cascade connection is reference plane $\sigma(k)$, k = 1, 2, ..., N, where

$$\begin{aligned} \iota(1) &= 2N + 3, & & \\ \sigma(k) &= \iota(k) + n(k), & k &= 1, 2, ..., N , \\ \iota(k) &= \sigma(k-1) + 1, & k &= 2, 3, ..., N . \end{aligned} \qquad (2)$$

Two-port matrix and vector representations **A**, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and **D** are calculated for each branch/junction combination and are denoted as $\mathbf{A}_{2k}$, $\boldsymbol{\alpha}_{2k}$, $\boldsymbol{\beta}_{2k}$ and $\mathbf{D}_{2k}$ for the kth junction. Elements in every branch and subsection in every section are represented by chain matrices $\mathbf{A}_i$, where i is the index of the reference plane at the output of the corresponding element or subsection.

Let

$$I_r = \{1, 2, 3, ..., \sigma(N)\} \qquad (3)$$

be the index set containing indexes of all reference planes and

$$I = \{i \mid i \in I_r, i \neq 2N+2, i \neq \sigma(k), \quad k = 1, 2, ..., N\} \qquad (4)$$

be the index set containing subscripts of all **A** matrices which can logically be defined using the subscript of the associated output reference plane.

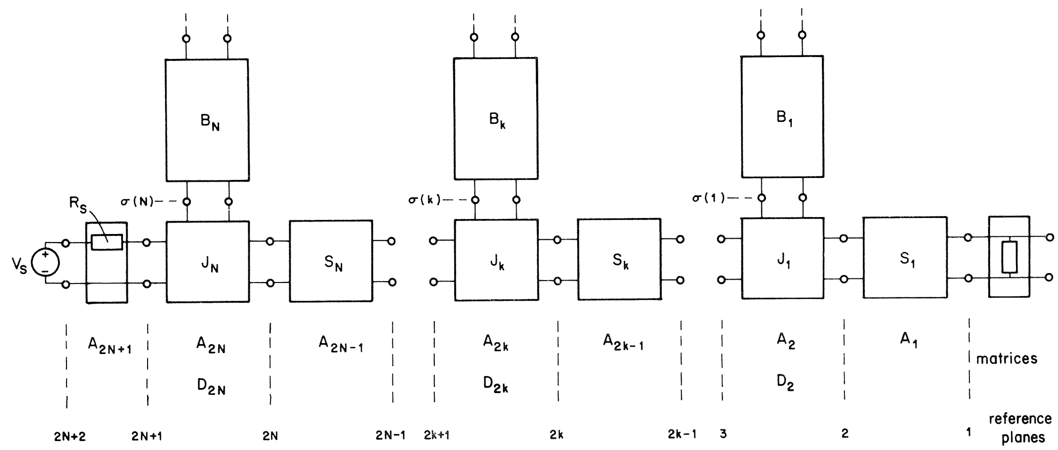

Fig. 2 Illustration of principal concepts involved in branched cascaded network simulation showing reference planes and transmission matrices.

The forward analysis $(\mathbf{u}^{xi})^T$ (reverse analysis $\mathbf{v}^{ix}$) is the result of a row (column) vector initialized at reference plane x as either $[1\ \ 0]$, $[0\ \ 1]$ ($[1\ \ 0]^T$, $[0\ \ 1]^T$) or a suitable linear combination and successively premultiplying (postmultiplying) each corresponding chain matrix by the resulting row (column) vector until reference plane i is reached.

The result of the analysis between reference planes i and j is defined as

$$\mathbf{Q}_{ij} \triangleq [\mathbf{p}_{ij}\ \ \mathbf{q}_{ij}] \triangleq \begin{bmatrix} A_{ij} & B_{ij} \\ C_{ij} & D_{ij} \end{bmatrix}, \tag{5}$$

where

$$\mathbf{p}_{ij} \triangleq \begin{bmatrix} A_{ij} \\ C_{ij} \end{bmatrix}, \quad \mathbf{q}_{ij} \triangleq \begin{bmatrix} B_{ij} \\ D_{ij} \end{bmatrix} \tag{6}$$

and where A_{ij}, B_{ij}, C_{ij} and D_{ij} are the equivalent chain matrix elements between reference planes i and j and are expressed in the form $\mathbf{u}^T\, \mathbf{A}\, \mathbf{v}$ to facilitate sensitivity, first-order change, and large change analysis [1]. For example, we have

$$\frac{\partial Q_{ij}}{\partial \phi} = \sum_{\ell \in I_\phi} \frac{\partial Q_{ij}^{\ell}}{\partial \phi}, \tag{7}$$

where I_ϕ is an index set whose elements identify the chain matrices between reference planes i and j containing the variable parameter ϕ and Q represents A, B, C or D.

VARIOUS FREQUENCY RESPONSE AND SENSITIVITY FORMULAS

Having performed the appropriate forward and reverse analysis, the branch output responses and their sensitivities are readily calculated. For example, for a short-circuit main cascade termination (at reference plane 1), we can calculate the output voltage of the kth branch and its sensitivities as

$$V = \frac{\boldsymbol{\alpha}^T \mathbf{q}_{2k,1} V_S}{\boldsymbol{\beta}^T \mathbf{p}_{\sigma\iota} B_{2N+2,1}} \tag{8}$$

and

$$V' = \frac{(\boldsymbol{\alpha}^T \mathbf{q}_{2k,1} V_S)' - (\boldsymbol{\beta}^T \mathbf{p}_{\sigma\iota} B_{2N+2,1})' V}{\boldsymbol{\beta}^T \mathbf{p}_{\sigma\iota} B_{2N+2,1}}. \tag{9}$$

Similar response and sensitivity formulas are also available for different excitations and terminations [2].

The branch output voltages can be utilized to compute the insertion loss between output and common ports. Sensitivities of insertion loss for each branch output w.r.t. all variables are also computed using the sensitivities of the corresponding branch output voltage. In particular, the sensitivities w.r.t. frequency can result in the exact calculation of group delay and gain slope for each branch [3-4].

The group delay from the source port to the kth branch output port, namely, T_G^k and the gain slope S_G^k are calculated as

$$T_G^k = -\mathrm{Im}(K) \quad \text{and} \quad S_G^k = -\frac{20}{\ln 10}\,\mathrm{Re}(K), \tag{10}$$

where

$$K = \left\{ \frac{(\boldsymbol{\alpha}^T \mathbf{q}'_{2k,1} + \mathbf{q}^T_{2k,1} \boldsymbol{\alpha}')}{\boldsymbol{\alpha}^T \mathbf{q}_{2k,1}} - \frac{(\boldsymbol{\beta}^T \mathbf{p}'_{\sigma\iota} + \mathbf{p}^T_{\sigma\iota} \boldsymbol{\beta}')}{\boldsymbol{\beta}^T \mathbf{p}_{\sigma\iota}} - \frac{B'_{2N+2,1}}{B_{2N+2,1}} \right\} \tag{11}$$

and $\boldsymbol{\beta} \equiv \boldsymbol{\beta}_{2k}$, $\boldsymbol{\alpha} \equiv \boldsymbol{\alpha}_{2k}$, $\sigma \equiv \sigma(k)$, $\iota \equiv \iota(k)$ and $\partial/\partial\omega$ is denoted as $'$.

The common port and branch output port return loss responses are evaluated using the Thevenin equivalent approach originated by Bandler et al. [1]. Denoting the Thevenin equivalent voltages and impedances at reference planes i and j by V_S^i, Z_S^i, V_S^j and Z_S^j, we have

$$V_S^j = \frac{V_S^i}{A_{ij} + Z_S^i C_{ij}} \quad \text{and} \quad Z_S^j = \frac{B_{ij} + Z_S^i D_{ij}}{A_{ij} + Z_S^i C_{ij}}. \tag{12}$$

The sensitivities are obtained as

$$(V_S^j)' = \frac{(V_S^i)' - [A'_{ij} + Z_S^i C'_{ij} + (Z_S^i)' C_{ij}] V_S^j}{A_{ij} + Z_S^i C_{ij}} \tag{13}$$

and

$$(Z_S^j)' = \frac{[1 \;\; Z_S^i]\, \mathbf{Q}'_{ij} \begin{bmatrix} -Z_S^j \\ 1 \end{bmatrix} + (Z_S^i)' (D_{ij} - Z_S^j C_{ij})}{A_{ij} + Z_S^i C_{ij}} \; . \qquad (14)$$

If the reflection coefficient at the branch output port is to be calculated (evaluation of branch output return loss), then we have the special cases

$$Z_S^{\iota+1} = \frac{B_{2N+2,\iota+1}}{A_{2N+2,\iota+1}} \qquad (15)$$

and

$$(Z_S^{\iota+1})' = \frac{B'_{2N+2,\iota+1} - A'_{2N+2,\iota+1} Z_S^{\iota+1}}{A_{2N+2,\iota+1}} \; . \qquad (16)$$

Norton equivalent admittances and current sources are calculated similar to the Thevenin equivalents. The Norton equivalent admittance at the common port, given by

$$Y_L^{2N+2} = \frac{D_{2N+2,1}}{B_{2N+2,1}} \; , \qquad (17)$$

is used in computation of common port reflection coefficient and return loss [2].

EXAMPLES

The theory discussed above has been implemented into a computer program for simulation and sensitivity analysis of branched cascaded networks with an arbitrary number of sections and arbitrary number of branch elements. Exact sensitivity analysis can be performed w.r.t. any variable, including frequency.

Consider an arbitrary 4-branch cascaded circuit depicted in Fig. 3. All element values are normalized. The normalized frequency is 1 Hz. Tables I and II show responses and some selected sensitivities. Table III gives numerical values for the gain slope and group delay for selected branches. The units for all quantities are SI units except as noted.

A practical application of the theory which we have presented, is the optimal design of contiguous-band multiplexers. We have used our simulation and sensitivity formulas in conjunction with the powerful gradient-based minimax optimization procedure of Hald and Madsen [5] to optimize a 12-channel, 12 GHz multiplexer without dummy channels [6]. The structure under consideration consists of synchronously and asynchronously tuned multi-coupled cavity filters distributed along a waveguide manifold. Waveguide spacings, input and output transformer ratios, cavity resonant frequencies as well as intercavity couplings are used as optimization variables. A lower specification of 20 dB on return loss has been imposed. The filters are assumed lossy and dispersive; waveguide junctions are assumed nonideal. The results of optimization are shown in Fig. 4.

CONCLUSIONS

We have presented a new approach for simulation and sensitivity analysis of branched cascaded networks. The approach facilitates the evaluation of various frequency responses and their derivatives w.r.t. network parameters as well as frequency at arbitrarily chosen reference planes in the network. The new theory has proved most beneficial in the design of contiguous or non-contiguous band multiplexers.

TABLE I
NUMERICAL VALUES OF THE RESPONSES FOR THE 4-BRANCH CASCADED NETWORK OF FIG. 3

Type of Response	Branch 1†	Branch 4
output voltage	0.03624 −j0.07487	−15.00361 +j1.16405
Thevenin equivalent voltage*	0.03008 −j0.07785	−15.65346 −j2.31876
Thevenin equivalent impedance*	0.00003 −j0.08225	0.02515 +j0.23408
insertion loss (dB)	55.57892	10.42942
branch port return loss (dB)	0.00055	0.41430

common port return loss = 0.41243 dB

† Branch 1 is the furthest from the common port.
* Thevenin equivalents for each branch are evaluated at the reference plane just before the load corresponding to that branch.

TABLE II
SENSITIVITIES OF INSERTION LOSS W.R.T. VARIABLE PARAMETERS FOR THE CIRCUIT OF FIG. 3

Variable	Branch 2	Branch 4
ϕ_1	2.09034	−0.05475
ϕ_2	0.01270	−0.00059
ϕ_6 (per Gm)	−114.77168	−1.22074

TABLE III
GAIN SLOPE AND GROUP DELAY FOR THE CIRCUIT OF FIG. 3

Type of response	Branch 3	Branch 4
gain slope (dB/Hz)	390.162	6.579
group delay (s)	0.32862	0.50006

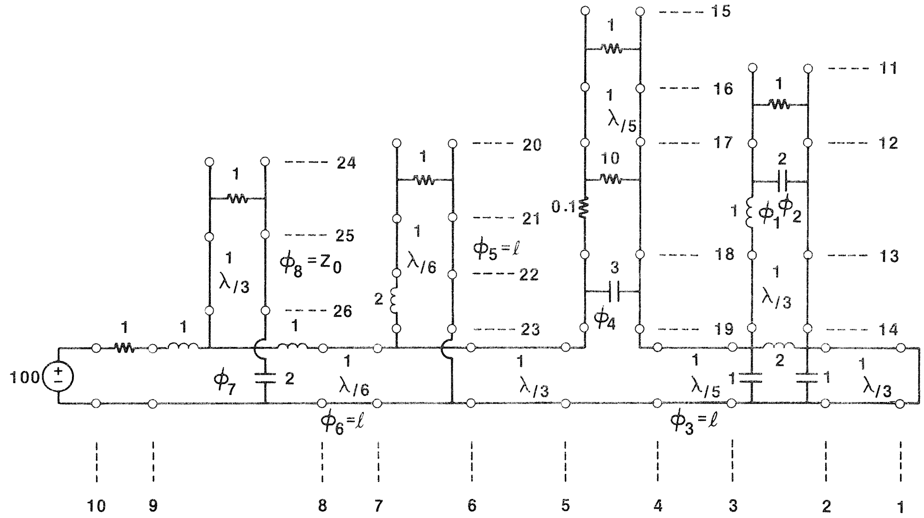

Fig. 3 Illustration of an arbitrary 4-branch cascaded circuit with short-circuit termination of the main cascade. Lossy elements as well as transmission lines are included.

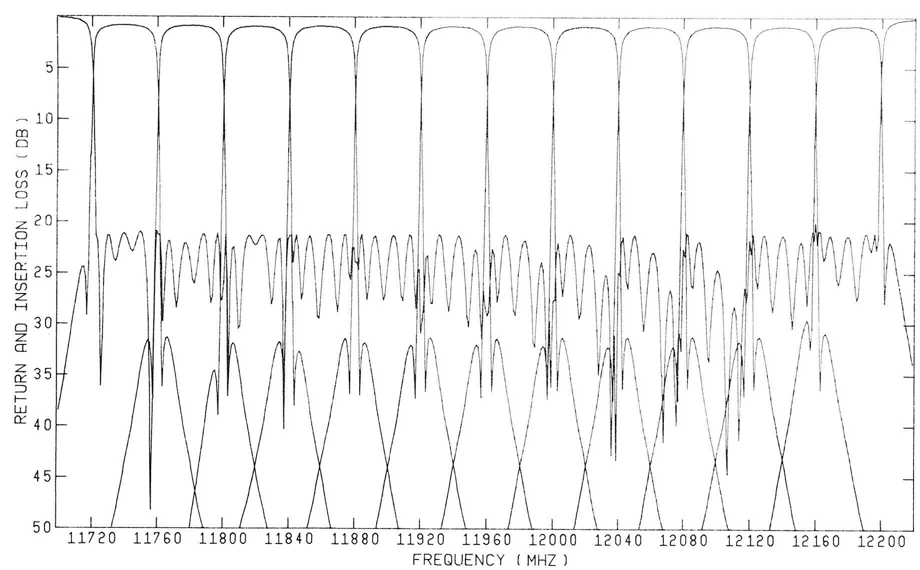

Fig. 4 Responses of a 12-channel multiplexer without dummy channels with optimized spacings, input-output transformer ratios, cavity resonances and coupling parameters.

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