EXACT SENSITIVITY ANALYSIS AND OPTIMIZATION FOR MULTI-CAVITY MICROWAVE COMMUNICATION FILTERS

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ABSTRACT

This paper describes an efficient approach to the simulation and exact sensitivity evaluation of multi-coupled cavity filters. A filter model which takes into account many nonideal factors such as losses, frequency dependent coupling parameters and stray couplings is used. The formulation also treats synchronously or asynchronously tuned structures in a unified manner. Explicit tables of first- and second-order sensitivities w.r.t. all variables of interest, including frequency, are presented.

INTRODUCTION

The application of multi-coupled cavity microwave filters in modern communication systems has received increasing attention. The theory originated by Atia and Williams [1] has inspired many advances in this area. See, for example, Atia and Williams [1,2], Chen et al. [3], Cameron [4] and Kudsia [5].

This paper describes a systematic and efficient approach to the simulation and exact sensitivity evaluation of narrow-band multi-coupled cavity microwave filters. The responses of interest are reflection coefficient, return loss, insertion loss, transducer loss, gain slope and group delay. We have used our approach to solve three problems of current interest in manufacturing of multi-cavity filters. A 10th order filter is considered for all three cases. The first problem involves the simultaneous optimization of amplitude and delay responses, i.e., design of self-equalized filters. The second problem is the prediction of responses for a filter which takes into account a nonideal but realistic effect, dissipation in this case, by simulation of the ideal filter. The third problem involves parameter identification of the filter from simulated measurements on its responses.

BASIC MODEL AND SENSITIVITIES

A narrow-band model of an unterminated filter is given by

$$j\mathbf{Z}\,\mathbf{I} = \mathbf{V}, \tag{1}$$

where

$$j\mathbf{Z} \triangleq j(s\,\mathbf{1} + \mathbf{M}) + r\,\mathbf{1}\,. \tag{2}$$

$\mathbf{1}$ denotes an nxn identity matrix and s is the normalized variable given by

$$s \triangleq \frac{\omega_0}{\Delta\omega}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\Delta\omega}\right), \tag{3}$$

ω_0 and $\Delta\omega$ being the synchronously tuned cavity resonant frequency and the bandwidth parameter, respectively. We assume uniform dissipation for all cavities indicated by parameter r. In equation (2), $\mathbf{M}$ is the coupling matrix whose (i, j) element represents the normalized coupling between the ith and jth cavities and the diagonal entries M_{ii} represent the deviations from synchronous tuning. Element M_{ij} does not necessarily correspond to a desirable and designable coupling. It may as well represent a stray coupling which is excluded from the nominal electrical equivalent circuit. Dispersion effects on the filter can be modelled by a frequency dependent $\mathbf{M}$ matrix.

The ideal model, namely the non-dispersive and lossless filter of Atia and Williams [1], is recovered by considering a frequency independent $\mathbf{M}$ matrix and letting r be zero.

The unterminated filter can be reduced to a two-port model whose parameters and sensitivity expressions can be obtained by solving

$$\mathbf{Z}\mathbf{p} = \mathbf{e}_1\,, \tag{4}$$

$$\mathbf{Z}\mathbf{q} = \mathbf{e}_n\,, \tag{5}$$

$$\mathbf{Z}\bar{\mathbf{p}} = \mathbf{p} \tag{6}$$

and

$$\mathbf{Z}\bar{\mathbf{q}} = \mathbf{q}\,, \tag{7}$$

where $\mathbf{e}_1 \triangleq [1\ 0\ 0\ \dots\ 0]^T$ and $\mathbf{e}_n \triangleq [0\ 0\ \dots\ 0\ 1]^T$. The two-port model, including the input and output transformers, is given by

$$\begin{bmatrix} I_1 \\ I_n \end{bmatrix} = -j\,\mathbf{y}\begin{bmatrix} V_1 \\ V_n \end{bmatrix} = -j\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_n \end{bmatrix}, \tag{8}$$

where

$$\mathbf{y} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} n_1^2 p_1 & n_1 n_2 p_n \\ n_1 n_2 p_n & n_2^2 q_n \end{bmatrix}, \tag{9}$$

n_1 and n_2 being the input and output transformer ratios, respectively. Utilizing the solutions of (4)-(7), first-order and second-order sensitivity expressions of $\mathbf{y}$ have been derived. Tables I and II summarize the first- and second-order sensitivity expressions for variables of interest in multi-coupled cavity filters.

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada under Grant G1135.

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The tables correspond to the case in which **M** is frequency independent.

TABLE I
FIRST-ORDER SENSITIVITY EXPRESSIONS

Variable (ϕ)	$\mathbf{y}_\phi = \partial \mathbf{y}/\partial \phi$
$M_{\ell k}$ †	$c_1 \begin{bmatrix} 2n_1^2 p_\ell p_k & n_1 n_2 (p_\ell q_k + p_k q_\ell) \\ n_1 n_2 (p_\ell q_k + p_k q_\ell) & 2n_2^2 q_\ell q_k \end{bmatrix}$
$\omega, \Delta\omega, \omega_0$ ††	$-s_\phi \begin{bmatrix} n_1^2 \mathbf{p}^T \mathbf{p} & n_1 n_2 \mathbf{p}^T \mathbf{q} \\ n_1 n_2 \mathbf{p}^T \mathbf{q} & n_2^2 \mathbf{q}^T \mathbf{q} \end{bmatrix}$
r	$j \begin{bmatrix} n_1^2 \mathbf{p}^T \mathbf{p} & n_1 n_2 \mathbf{p}^T \mathbf{q} \\ n_1 n_2 \mathbf{p}^T \mathbf{q} & n_2^2 \mathbf{q}^T \mathbf{q} \end{bmatrix}$
n_1, n_2	$\begin{bmatrix} 2n_1 p_1 & n_2 p_n \\ n_2 p_n & 0 \end{bmatrix}, \begin{bmatrix} 0 & n_1 p_n \\ n_1 p_n & 2 n_2 q_n \end{bmatrix}$

† $c_1 \triangleq \begin{cases} -0.5 & \text{if } \ell = k \\ -1 & \text{otherwise} \end{cases}$ †† $s_\phi \triangleq \dfrac{\partial s}{\partial \phi}$

These tables show that solving four systems of equations, namely, (4)-(7), provides sufficient information for filter simulation and sensitivity analysis. From a computational point of view, this means one LU factorization of matrix **Z**, followed by four forward and backward substitutions. For a lossless filter **Z** is a real matrix. Obviously, solving real systems of equations enjoys significant computational advantages over the complex calculations.

FREQUENCY RESPONSES OF A TERMINATED FILTER

Consider a filter terminated by a load Z_L and a normalized voltage source $E = 1$ V with an impedance Z_S. The input and output currents, namely, I_1 and I_n, can be obtained by solving eqn. (8) subject to the terminating conditions

$$V_1 = 1 - Z_S I_1 , \tag{10}$$

and

$$V_n = -Z_L I_n . \tag{11}$$

Denoting

$$\mathbf{T} \triangleq \begin{bmatrix} Z_S & 0 \\ 0 & Z_L \end{bmatrix} , \tag{12}$$

we have

$$[I_1 \; I_n]^T = \mathbf{I}_p = -j(1 - j\,\mathbf{y}\,\mathbf{T})^{-1}\,\mathbf{y}\,\mathbf{e}_1 = -j\,\mathbf{H}\,\mathbf{y}\,\mathbf{e}_1 , \tag{13}$$

where

$$\mathbf{H} \triangleq (1 - j\,\mathbf{y}\,\mathbf{T})^{-1} . \tag{14}$$

Furthermore, first- and second-order sensitivities of $\mathbf{I}_p$ can be evaluated as

$$\partial \mathbf{I}_p/\partial\phi = (\mathbf{I}_p)_\phi = j\mathbf{H}[\mathbf{y}\,\mathbf{T}_\phi\,\mathbf{I}_p - \mathbf{y}_\phi(\mathbf{e}_1 - \mathbf{T}\,\mathbf{I}_p)] \tag{15}$$

and

$$\partial^2 \mathbf{I}_p/(\partial\phi\partial\omega) = (\mathbf{I}_p)_{\phi\omega} = j\mathbf{H}\{\mathbf{y}_\omega[\mathbf{T}_\phi\,\mathbf{I}_p + \mathbf{T}(\mathbf{I}_p)_\phi] + \mathbf{y}_\phi[\mathbf{T}_\omega\,\mathbf{I}_p + \mathbf{T}(\mathbf{I}_p)_\omega] + \mathbf{y}[\mathbf{T}_\phi(\mathbf{I}_p)_\omega + \mathbf{T}_\omega(\mathbf{I}_p)_\phi] + \mathbf{y}\,\mathbf{T}_{\phi\omega}\,\mathbf{I}_p - \mathbf{y}_{\phi\omega}\,\mathbf{V}_p\} . \tag{16}$$

In practice, the performance of filters is often evaluated via some conventionally defined frequency responses such as the reflection coefficient and corresponding return loss, transducer loss, insertion loss, gain slope and group delay. These frequency responses and their sensitivities can be calculated utilizing formulas (13), (15) and (16). Table III summarizes various frequency responses of interest and their sensitivities.

APPLICATIONS

Three examples of significant practical value are selected to illustrate the direct application of the approach presented. A 10th order multi-coupled cavity filter with a center frequency of 4 GHz and a bandwidth of 40 MHz is considered. The first example describes a nonminimum-phase self-equalized design achieved by simultaneous optimization of the amplitude and group delay responses. For the second example, the sensitivities w.r.t. cavity dissipation are utilized to predict the amplitude response of a lossy filter. The parameter identification of the filter from simulated measurements is described in the third example.

A Quasi-Elliptic Self-Equalized Filter

A 10th order quasi-elliptic, self-equalized filter has been obtained from simultaneous optimization of the amplitude and delay responses. A powerful gradient-based minimax optimization method [6] is employed. The objective functions to be optimized are formulated from the filter responses including the reflection coefficient for both the passband and the stopband and the relative group delay for the passband. Fig. 1 shows the amplitude and group delay responses of the filter designed.

Prediction of the Effect of Cavity Dissipation

Ideal lossless models are often used to obtain nominal designs. In reality, the actual devices are subject to certain imperfections such as dissipation. An efficient method of predicting the non-ideal response is utilizing the sensitivities to obtain a first-order estimation. We have used this method to compute the response of the 10th order filter given in the first example with Q = 10,000. Fig. 2 shows the predicted passband insertion loss, which is indistinguishable from the exact simulation of the lossy filter (the numerical difference is less than 0.001 dB).

Parameter Identification Using Simulated Measurements

Identification of network parameters from external measurements provides the necessary guidance for post-production tuning. The objective of such identification is to obtain by optimization an electrical equivalent circuit capable of reproducing frequency responses which correlate accurately with the measurements. It is desired to identify the parameters of the filter that deviate from their nominal values, using the amplitude responses shown in Fig. 3 as simulated measurements. Employing the ℓ_1 optimization technique [7], all parameters are accurately identified. The responses after identification are indistinguishable from the responses of Fig. 3.

TABLE II

SECOND-ORDER SENSITIVITY EXPRESSIONS

Variable (ϕ)	$\mathbf{y}_{\phi\omega} = \partial^2 \mathbf{y}/(\partial\phi\partial\omega)$
$M_{\ell k}$ †	$c_2 \begin{bmatrix} 2n_1^2(p_\ell \bar{p}_k + p_k \bar{p}_\ell) & n_1 n_2(p_\ell \bar{q}_k + p_k \bar{q}_\ell + \bar{p}_\ell q_k + \bar{p}_k q_\ell) \\ n_1 n_2(p_\ell \bar{q}_k + p_k \bar{q}_\ell + \bar{p}_\ell q_k + \bar{p}_k q_\ell) & 2n_2^2(q_\ell \bar{q}_k + q_k \bar{q}_\ell) \end{bmatrix}$
$\omega, \Delta\omega, \omega_0$ ††	$2 s_\phi s_\omega \begin{bmatrix} n_1^2 \mathbf{p}^T \bar{\mathbf{p}} & n_1 n_2 \mathbf{p}^T \bar{\mathbf{q}} \\ n_1 n_2 \mathbf{p}^T \bar{\mathbf{q}} & n_2^2 \mathbf{q}^T \bar{\mathbf{q}} \end{bmatrix} - s_{\phi\omega} \begin{bmatrix} n_1^2 \bar{p}_1 & n_1 n_2 \bar{p}_n \\ n_1 n_2 \bar{p}_n & n_2^2 \bar{q}_n \end{bmatrix}$
r	$-2j\, s_\omega \begin{bmatrix} n_1^2 \mathbf{p}^T \bar{\mathbf{p}} & n_1 n_2 \mathbf{p}^T \bar{\mathbf{q}} \\ n_1 n_2 \mathbf{p}^T \bar{\mathbf{q}} & n_2^2 \mathbf{q}^T \bar{\mathbf{q}} \end{bmatrix}$
n_1, n_2	$-s_\omega \begin{bmatrix} 2n_1 \bar{p}_1 & n_2 \bar{p}_n \\ n_2 \bar{p}_n & 0 \end{bmatrix}, \quad -s_\omega \begin{bmatrix} 0 & n_1 \bar{p}_n \\ n_1 \bar{p}_n & 2n_2 \bar{q}_n \end{bmatrix}$

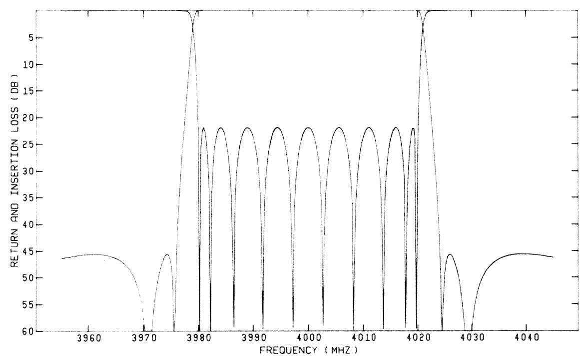
† $c_2 \triangleq \begin{cases} \frac{1}{2} s_\omega \text{ if } \ell = k \\ s_\omega \text{ otherwise} \end{cases}$, †† $s_\phi = \dfrac{\partial s}{\partial \phi}$, $s_\omega = \dfrac{\partial s}{\partial \omega}$, $s_{\phi\omega} = \dfrac{\partial^2 s}{\partial\phi\partial\omega}$

TABLE III

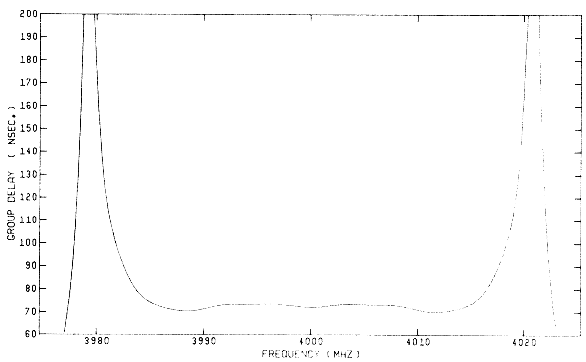
SENSITIVITY EXPRESSIONS FOR VARIOUS FREQUENCY RESPONSES

Response Type	Formula	Expression For Sensitivity w.r.t. ϕ
Reflection Coefficient	$1 - 2R_S I_1$	$-2[R_S (I_1)_\phi + (R_S)_\phi I_1]$
Insertion Loss	$-20 \log_{10} \lvert Z_T I_n \rvert$	$c\,\mathrm{Re}\left[\dfrac{(I_n)_\phi}{I_n} + \dfrac{(Z_T)_\phi}{Z_T} \right]$
Gain Slope	$c\,\mathrm{Re}\left[\dfrac{(I_n)_\omega}{I_n} + \dfrac{(Z_T)_\omega}{Z_T} \right]$	$c\,\mathrm{Re}\left[\dfrac{(I_n)_{\phi\omega}}{I_n} + \dfrac{(I_n)_\phi (I_n)_\omega}{I_n^2} + \dfrac{(Z_T)_{\phi\omega}}{Z_T} - \dfrac{(Z_T)_\phi (Z_T)_\omega}{Z_T^2} \right]$
Group Delay	$-\mathrm{Im}\left[\dfrac{(I_n)_\omega}{I_n} + \dfrac{(Z_L)_\omega}{Z_L} \right]$	$-\mathrm{Im}\left[\dfrac{(I_n)_{\phi\omega}}{I_n} - \dfrac{(I_n)_\phi (I_n)_\omega}{I_n^2} + \dfrac{(Z_L)_{\phi\omega}}{Z_L} - \dfrac{(Z_L)_\phi (Z_L)_\omega}{Z_L^2} \right]$

$R_S \triangleq \mathrm{Re}(Z_S)$ $R_L \triangleq \mathrm{Re}(Z_L)$ $Z_T \triangleq Z_S + Z_L$ $c \triangleq -\dfrac{20}{\ell n 10}$



(a) Return loss and insertion loss response.



(b) Group delay response.

Fig. 1 Responses of the 10th order quasi-elliptic self-equalized filter showing optimized amplitude and group delay.

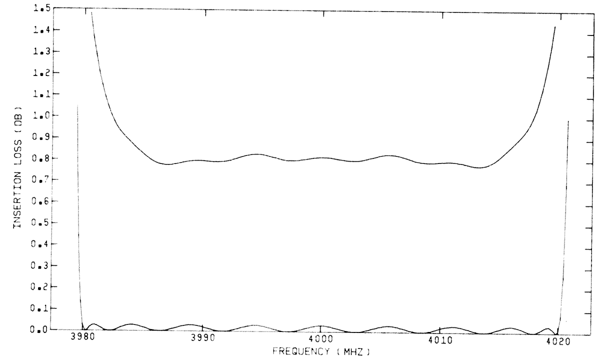


Fig. 2 Passband insertion loss of the 10th order filter example for Q = 10,000 and Q = ∞.

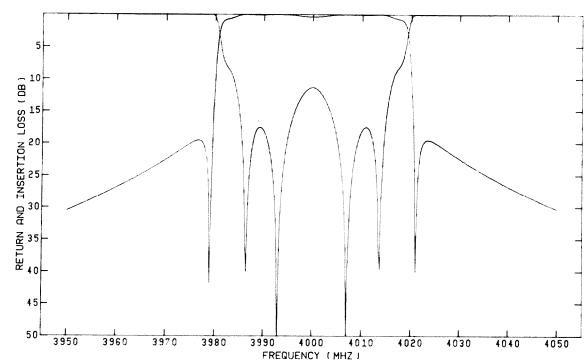


Fig. 3 Return loss and insertion loss responses of a 10th order detuned filter. The parameters capable of reproducing such responses are identified.

CONCLUSIONS

An efficient and flexible approach to the simulation and exact sensitivity analysis of multi-coupled cavity filters has been presented. Illustrative examples of practical engineering problems solved by the actual implementation of our approach have also been provided. With its computational efficiency and structural flexibility, the approach presented provides, for the first time, a basis for the development of more advanced CAD software for automatic design, modelling and tuning of multi-coupled cavity filters and multiplexing networks [8]. Such a prospect makes this work extremely attractive.

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