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CENTERING, TOLERANCING AND TUNING IN COMPUTER-AIDED DESIGN OF ENGINEERING SYSTEMS[1]

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ABSTRACT

This paper presents practical engineering design problems which have been solved using concepts and techniques developed for design centering, tolerancing and tuning (DCTT). A brief review of the mathematical concepts involved in optimal DCTT is given. This includes the formulation of some important special cases such as the zero tolerance problem (centering), the fixed tolerance problem, the variable tolerance problem and the tuning problem. Formulations are fully supported by implementations of new and highly efficient algorithms for nonlinear minimax and ℓ_1 optimization due to Hald and Madsen. An important practical extension of the problems discussed is the best alignment problem for which a mathematical formulation is given and an algorithm using minimax optimization is proposed. The presentation is complemented by descriptions of software for implementation in a high technology environment.

INTRODUCTION

The increasing size and complexity of physical man-made engineering systems necessitate the use of computers in all aspects of the design, production and maintenance processes. A corresponding need has developed for efficient and powerful computer-aided techniques for thorough study and optimal realization of the above mentioned processes.

Computer-aided design (CAD) techniques are now well established for design centering, tolerance optimization, post-production tuning, yield maximization, cost minimization and the rapidly increasing range of applications includes electronic circuits, power systems, microwave systems and mechanical systems.

Computer-aided design is often treated together with computer-aided manufacturing (CAM). We are not including CAM in this paper, since CAM starts from data, preferably machine-readable data, produced in the design process, but CAM is not part of the design process itself.

An important practical problem is optimal design subject to tolerances. Generally, the problem is to ensure that a design, when manufactured, will satisfy specifications. Over the past 15 years this has been a problem of significant interest amongst both electrical and mechanical engineers. A general mathematical theory of the DCTT problem with electronic circuit applications has been given by Bandler et al. [1] in 1976. This approach was implemented in the area of mechanical design by Michael and Siddall [2,3].

The development of new design procedures and techniques can be, in general, characterized as an attempt to include in the design process as many factors which may influence the performance of a manufactured design as possible. With readily available and ever increasing computing power at hand, computer-aided designers are dealing with more realistic problems.

In the classical design problem we are interested in finding one single point in the design parameter space which satisfies the design specifications. This kind of solution is impractical from the manufacturing point of view since there are factors which influence the performance of a manufactured design.

Phenomena associated with the design of circuits, for example, and which can be considered are [4]

(a) manufacturing tolerances (i.e., the actual value of the design variable outcome may be within an interval with a certain probability density function);
(b) model uncertainties; equivalent circuits are used to model actual circuits and the parameters of equivalent circuits usually have uncertainties associated with them;
(c) parasitic effects; these parasitics can substantially alter the ideal circuit performance and should be taken into consideration where possible; they are marked in many analog electrical circuits (active, high frequency, etc.);
(d) environmental uncertainties; some circuits have to meet stringent specifications for a variety of different environmental conditions, military and telephone equipment, for example, often has to be designed for extreme temperatures;
(e) mismatched terminations; network terminations or loads may be substantially different from ideal;
(f) material uncertainty; uncertainties exist in the materials used to fabricate the circuits.

Taking into account in the design process the above mentioned factors, if at all possible, is usually in conflict with the feasible or acceptable computational effort involved. Therefore, a successful design procedure is usually a compromise between the complexity of the model and the computational cost to produce a design satisfying all specifications.

In the next section we consider the relevant fundamental concepts and definitions commonly used in the DCTT literature.

FUNDAMENTAL CONCEPTS AND DEFINITIONS [1]

A design consists of design data of the nominal point ϕ^0, the tolerance vector ε and the tuning vector t where, for n parameters,

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$$\phi^0 \triangleq \begin{bmatrix} \phi_1^0 \\ \phi_2^0 \\ \cdot \\ \cdot \\ \cdot \\ \phi_n^0 \end{bmatrix}, \quad \varepsilon \triangleq \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_n \end{bmatrix}, \text{ and } \quad t \triangleq \begin{bmatrix} t_1 \\ t_2 \\ \cdot \\ \cdot \\ \cdot \\ t_n \end{bmatrix}. \tag{1}$$

Some of the parameters can be set to zero or held constant.

An outcome $\{\phi^0, \varepsilon, \mu\}$ of a design $\{\phi^0, \varepsilon, t\}$ implies a point

$$\phi = \phi^0 + E\mu, \tag{2}$$

where

$$E \triangleq \begin{bmatrix} \varepsilon_1 & & & & \\ & \varepsilon_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \varepsilon_n \end{bmatrix} \tag{3}$$

and $\mu \in R_\mu$. R_μ is a set of multipliers determined from realistic situations of the tolerance spread. We consider

$$R_\mu \triangleq \{\mu \mid -1 \leq \mu_i \leq 1, i \in I_\phi\}, \tag{4}$$

$$I_\phi \triangleq \{1, 2, ..., n\}. \tag{5}$$

The tolerance region R_ε is a set of points described by (2) for all $\mu \in R_\mu$. In the case of $-1 \leq \mu_i \leq 1$, $i \in I_\phi$,

$$R_\varepsilon \triangleq \{\phi \mid \phi_i = \phi_i^0 + \varepsilon_i \mu_i, -1 \leq \mu_i \leq 1, i \in I_\phi\}, \tag{6}$$

which is a convex regular polytope of n dimensions with sides of length $2\varepsilon_i$, $i \in I_\phi$, and centered at ϕ^0.

The extreme points of R_ε are obtained by setting $\mu_i = \pm 1$. Thus, the set of vertices may be defined as

$$R_v \triangleq \{\phi \mid \phi_i = \phi_i^0 + \varepsilon_i \mu_i, \ \mu_i \in \{-1, 1\}, i \in I_\phi\}. \tag{7}$$

The number of points in R_v is 2^n. Let each of these points be indexed by ϕ^i, $i \in I_v$, where

$$I_v \triangleq \{1, 2, ..., 2^n\}. \tag{8}$$

The tuning region $R_t(\mu)$ is defined as the set of points

$$\phi = \phi^0 + E\mu + T\rho \tag{9}$$

for all $\rho \in R_\rho$, where

$$T \triangleq \begin{bmatrix} t_1 & & & & \\ & t_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & t_n \end{bmatrix}, \tag{10}$$

$$R_\rho \triangleq \{\rho \mid -1 \leq \rho_i \leq 1, i \in I_\phi\}. \tag{11}$$

The constraint region R_c is given by

$$R_c \triangleq \{\phi \mid g_i(\phi) \geq 0, \text{for all } i \in I_c\}, \tag{12}$$

where

$$I_c \triangleq \{1, 2, ..., m_c\}. \tag{13}$$

See Fig. 1 for an illustration of the concepts and definitions.

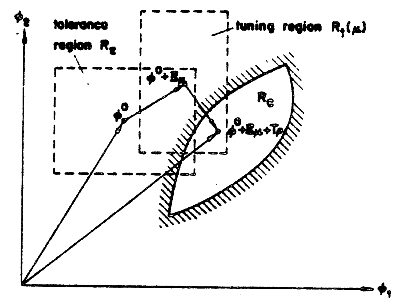

Fig. 1 Illustration of concepts in design centering, tolerancing and tuning.

TWO FORMULATIONS OF THE OPTIMAL DCTT PROBLEM

The first general formulation of the optimal DCTT problem was given by Bandler, Liu and Tromp [1]. The problem was stated as follows: obtain a set of optimal design values $\{\phi^0, \varepsilon, t\}$ such that any outcome $\{\phi^0, \varepsilon, \mu\}$, $\mu \in R_\mu$, may be tuned into R_c for some $\rho \in R_\rho$.

It was formulated as the nonlinear programming problem:

$$\text{minimize } C(\phi^0, \varepsilon, t)$$

subject to

$$\phi \in R_c \text{ (or } g(\phi) \geq 0), \tag{14}$$

where ϕ is defined by (9), and constraints $\phi^0, \varepsilon, t \geq 0$, for all $\mu \in R_\mu$ and some $\rho \in R_\rho$. C is an appropriate function chosen to represent a reasonable approximation to known component cost data.

Stated in an abstract sense, the worst-case solution of the problem must satisfy

$$R_t(\mu) \cap R_c \neq \varnothing, \tag{15}$$

for all $\mu \in R_\mu$, where $\varnothing$ denotes an empty set.

They also discussed the geometrical structure of the problem and introduced some important special cases obtained by separating the components into effectively tuned and effectively toleranced parameters. They proved that the solution of the reduced problem is the solution of the original one under certain conditions.

Polak and Sangiovanni-Vincentelli [5] formulated the DCTT problem as a mathematical programming problem in the form

$$\text{minimize } C(\phi^0, \varepsilon, t)$$

subject to

$$\min_{i \in I_c} \ \min_{\mu \in R_\mu} \ \max_{\rho \in R_\rho} \ g_i(\phi) \geq 0 \tag{16}$$

and the constraints $\phi^0, \varepsilon, t \geq 0$, where ϕ is given by (9). They demonstrated that their formulation is equivalent to the one of Bandler et al. [1]. They suggested a new algorithm which deals with the nondifferentiable constraints (16). The algorithm solves the problem as a sequence of approximating problems with $R_\mu^{\ i} \subset R_\mu$ as a discrete set. They showed that, under certain conditions, the accumulation points of the sequence of stationary points of the approximating problems are stationary points of the original problem.

SPECIAL CASES AND THE OBJECTIVE FUNCTIONS

Several objective functions (or cost functions) have been proposed [6]-[10]. In practice, a suitable modeling problem would have to be solved to determine the cost-tolerance relationship. We assume that the nominal parameter values, tolerances and tuning ranges (either absolute or relative) are the main variables and that the cost of the design is the sum of the cost of the individual components.

Suitable objective functions will be, for example, of the form

$$C(\phi^0, \varepsilon, t) = \sum_{i=1}^{n} \left(c_i \frac{\phi_i^0}{\varepsilon_i} + c'_i \frac{t_i}{\phi_i^0} \right), \tag{17}$$

where the c_i and c'_i are nonnegative constants. These may be set to zero if the corresponding element is not to be toleranced or tuned, respectively.

The special cases considered may be defined mathematically as a zero tolerance problem (ZTP), a fixed tolerance problem (FTP) and a variable tolerance problem (VTP) (see [10]). Schjaer-Jacobsen and Madsen [10] define the problems in terms of a set of m nonlinear differentiable functions of n real variables. In this presentation we define those problems using notation and concepts directly related to the design problem.

Centering Problem (Zero Tolerance Problem)

In this problem we have $\varepsilon = 0$ and $t = 0$. We want to find the nominal design ϕ^0 satisfying the design specifications $g(\phi) \geq 0$, where $\phi = \phi^0$. The problem is a pure centering problem in which a feasible, centered nominal design is found if $R_c \neq \emptyset$. The solution may be useful at the initial stage of a design process when the designer has no prior experience with the problem and an initial rough approximation gives some insight (e.g., the order of magnitude of the elements).

The problem can be conveniently formulated in minimax form as

$$\underset{\phi^0}{\text{minimize}} \; F(\phi^0), \tag{18}$$

where

$$F(\phi^0) = \max_{i \in I_c} (-g_i(\phi^0)), \tag{19}$$

subject to

$$\phi^0 \geq 0. \tag{20}$$

Fixed Tolerance Problem

Here we have $\varepsilon = \text{const} \neq 0$, $t = 0$. We want to find ϕ^0, the center of the tolerance region R_ε, when the manufacturing tolerances on the components are fixed. The problem is basically a centering problem and can be formulated in minimax form as

$$\underset{\phi^0}{\text{minimize}} \; F(\phi^0), \tag{21}$$

subject to

$$\phi^0 \geq 0, \tag{22}$$

where

$$\phi = \phi^0 + E\mu \quad \text{for all } \mu \in R_\mu, \text{ and} \tag{23}$$

$$F(\phi^0) = \max_{i \in I_c} (-g_i(\phi)). \tag{24}$$

Under certain assumptions (one-dimensional convexity of R_c) it is sufficient to choose only the vertices of R_ε to form appropriate minimax functions.

Variable Tolerance Problem

In this problem we have $\varepsilon \neq \text{const}$, $t = 0$. The manufacturing tolerances are considered as variables instead of fixed.

The design problem can be formulated as

$$\underset{\phi^0, \varepsilon}{\text{minimize}} \; C(\phi^0, \varepsilon) \tag{25}$$

subject to

$$\phi^0 \geq 0, \tag{26}$$

$$\varepsilon \geq 0, \tag{27}$$

$$g_i(\phi) \geq 0, \quad i \in I_c, \tag{28}$$

where ϕ is given by (23).

The objective function C is directly related to the component cost, and generally possesses the properties

$$C(\phi^0, \varepsilon) \rightarrow \text{const. as } \varepsilon \rightarrow \infty, \tag{29}$$

$$C(\phi^0, \varepsilon) \rightarrow \infty \text{ as } \varepsilon_i \rightarrow 0. \tag{30}$$

A common form of this objective is

$$\sum_{i=1}^{n} c_i \frac{\phi_i^0}{\varepsilon_i}, \tag{31}$$

where the c_i's are positive constant weights.

Tuning Problem

It is often necessary to introduce tuning parameters in order to obtain a feasible design. Usually this is the case when tolerances are fixed and there is no solution to the FTP satisfying all design specifications.

We can distinguish two cases depending on the nominal design being fixed or variable.

Case 1: Pure tuning problem.

Here we have $\phi^0 = \text{const}$, $\varepsilon = \text{const} \neq 0$ and $t \neq \text{const}$. If the tolerances are fixed the suitable objective function minimizing the weighted sum of tuning ranges could be for $c_i \geq 0$

$$C = \sum_{i=1}^{n} c_i t_i. \tag{32}$$

The problem is then

$$\underset{t, \rho^r}{\text{minimize}} \; C(t) \tag{33}$$

subject to

$$g(\phi) \geq 0, \tag{34}$$

$$t \geq 0, \tag{35}$$

$$-1 \leq \rho_i^r \leq 1, \; i \in I_\phi, \; r \in I_v, \tag{36}$$

where

$$\phi_i = \phi_i^0 + \varepsilon_i \mu_i^r + t_i \rho_i^r \quad \text{for all } \mu^r \in R_\mu. \tag{37}$$

Case 2: Centering-tuning problem.

In this case we have $\phi^0 \neq \text{const.}$, $\varepsilon = \text{const} \neq 0$, $t \neq \text{const.}$ and we want to find ϕ^0 and necessary tuning parameters (and their ranges) with the objective of minimizing the cost of introducing tunable parameters. Ideally we would like to find the minimum number of tunable parameters which are necessary to satisfy the specifications.

The suitable objective could be of the form (32). The problem statement, however, is slightly different, namely,

$$\underset{t, \phi^0, \rho^r}{\text{minimize}} \; C(t) \tag{38}$$

subject to

$$g(\phi) \geq 0, \tag{39}$$

$$t \geq 0, \tag{40}$$

$$\phi^0 \geq 0, \tag{41}$$

$$-1 \le \rho_i^r \le 1, \quad i \in I_\phi, \quad r \in I_v, \tag{42}$$

where

$$\phi_i = \phi_i^0 + \varepsilon_i \mu_i^r + t_i \rho_i^r \quad \text{for all } \mu^r \in R_\mu \tag{43}$$

and some $\rho^r \in R_\rho$.

SOFTWARE FOR OPTIMAL DESIGN CENTERING, TOLERANCING AND TUNING

This section reviews the practical implementation of recent optimization techniques for optimal design centering, tolerancing and tuning. The discussion is focused on four nonlinear programming codes including linearly constrained minimax optimization, linearly constrained ℓ_1 optimization and optimization with general constraints.

Linearly Constrained Minimax Optimization (the MMLC Package [11])

Given a set of nonlinear differentiable residual functions $f_i(\mathbf{x})$, $i=1,2,...,m$, of n variables $\mathbf{x} = [x_1\ x_2\ ...\ x_n]^T$, it is the purpose of the package to find a local minimum of the minimax objective function

$$F(\mathbf{x}) \triangleq \max_{1 \le i \le m} f_i(\mathbf{x}) \tag{44}$$

subject to linear constraints

$$\begin{aligned} \mathbf{c}_i^T \mathbf{x} + b_i &= 0, \quad i = 1, 2, ..., \ell_{eq}, \\ \mathbf{c}_i^T \mathbf{x} + b_i &\ge 0, \quad i = \ell_{eq}+1, ..., \ell, \end{aligned} \tag{45}$$

where $\mathbf{c}_i$ and b_i, $i=1,2,...,\ell$, are constants.

The MMLC package [11] is based on the method described by Hald and Madsen [12]. It is an extension and modification of the MMLA1Q package due to Hald [13]. First derivatives of all functions with respect to all variables are assumed to be known. The solution is found by an iteration that uses either linear programming applied in connection with first-order derivatives or a quasi-Newton method applied in connection with first-order and approximate second-order derivatives.

Han-Powell Algorithm (the MFNC Package [14])

The purpose of the package is to minimize the objective function $F(\mathbf{x})$ of n variables, $\mathbf{x} = [x_1\ x_2\ ...\ x_n]^T$, subject to general equality and inequality constraints

$$\begin{aligned} f_j(\mathbf{x}) &= 0, \quad j = 1, ..., \ell_{eq}, \\ f_j(\mathbf{x}) &\ge 0, \quad j = \ell_{eq}+1, ..., \ell, \end{aligned} \tag{46}$$

where the objective and the constraint functions are differentiable.

The MFNC package [14] is an extension and modification of a set of subroutines from the Harwell Subroutine Library [15]. The method implemented was presented by Han [16] and Powell [17]. First derivatives of all functions with respect to all variables are assumed to be available. The solution is found by an iteration that minimizes a quadratic approximation of the objective function subject to linearized constraints.

Augmented Lagrangian (the MINOS/AUGMENTED System [18])

The MINOS/AUGMENTED system [18] is a general purpose programming system to solve large-scale optimization problems involving sparse linear and nonlinear constraints. Any nonlinear functions appearing in the objective or the constraints must be continuous and smooth. MINOS/AUGMENTED employs a projected augmented Lagrangian algorithm to solve problems with nonlinear constraints presented by Murtagh and Saunders [19]. This involves a sequence of sparse, linearly constrained subproblems, which are solved by a reduced-gradient algorithm.

The problem to be solved must be expressed in the following standard form [18]

$$\text{minimize } f_0(\mathbf{x}) + \mathbf{c}^T\mathbf{x} + \mathbf{d}^T\mathbf{y} \tag{47}$$

subject to

$$\mathbf{f}(\mathbf{x}) + \mathbf{A}_1\mathbf{y} = \mathbf{b}_1, \tag{48}$$

$$\mathbf{A}_2\mathbf{x} + \mathbf{A}_3\mathbf{y} = \mathbf{b}_2, \tag{49}$$

$$\boldsymbol{\ell} \le \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} \le \mathbf{u}, \tag{50}$$

where

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1(\mathbf{x}) \\ \cdot \\ \cdot \\ \cdot \\ f_m(\mathbf{x}) \end{bmatrix} \tag{51}$$

and the functions $f_i(\mathbf{x})$ are smooth and have known gradients. The components of $\mathbf{x}$ are called the nonlinear variables, and they must be the first set of unknowns. Similarly constraints (48) are called the nonlinear constraints and they must appear before the linear constraints (49).

Linearly Constrained ℓ_1 Optimization (the L1NONL Package [20])

Let $f_j(\mathbf{x}) \equiv f_j(x_1, x_2, ..., x_n)$, $j=1,2,...,m$, be a set of m nonlinear, continuously differentiable functions. The vector $\mathbf{x} = [x_1\ x_2\ ...\ x_n]^T$ is a set of n parameters to be optimized. The linearly constrained ℓ_1 optimization problem is

$$\underset{\mathbf{x}}{\text{minimize}} \quad F(\mathbf{x}) \triangleq \sum_{j=1}^{m} |f_j(\mathbf{x})| \tag{52}$$

subject to

$$\begin{aligned} \mathbf{a}_i^T \mathbf{x} + b_i &= 0, \quad i = 1, 2, ..., \ell_{eq}, \\ \mathbf{a}_i^T \mathbf{x} + b_i &\ge 0, \quad i = \ell_{eq}+1, ..., \ell, \end{aligned} \tag{53}$$

The algorithm is similar to the minimax algorithm of Hald and Madsen [12]. It has been reported by Hald in [20], which describes and lists a Fortran subroutine implementing a version of the algorithm. Hald and Madsen [21] have demonstrated that the algorithm has sure convergence properties. The algorithm is a combination of a first-order method that approximates the solution by successive linear programming and a quasi-Newton method using approximate second-order information to solve a system of nonlinear equations resulting from the first-order necessary conditions for optimum.

APPLICATIONS

Contiguous Band Microwave Multiplexer

We formulate the design of a contiguous band microwave multiplexer structure for satellite communications as a centering problem [22].

The objective function to be minimized is given by

$$F(\mathbf{x}) \triangleq \max_{j \in J} f_j(\mathbf{x}) \tag{54}$$

where **x** is a vector of optimization variables (e.g., section or spacing lengths, channel input and output couplings and filter coupling parameters) and $J \triangleq \{1,2,...,m\}$ is an index set. The minimax functions $f_j(\mathbf{x}), j \in J$, can be of the form

$$w_{Uk}^1(\omega_i)(F_k^1(\mathbf{x},\omega_i) - S_{Uk}^1(\omega_i)), \quad (55)$$

$$-w_{Lk}^1(\omega_i)(F_k^1(\mathbf{x},\omega_i) - S_{Lk}^1(\omega_i)), \quad (56)$$

$$w_U^2(\omega_i)(F^2(\mathbf{x},\omega_i) - S_U^2(\omega_i)), \quad (57)$$

$$-w_L^2(\omega_i)(F^2(\mathbf{x},\omega_i) - S_L^2(\omega_i)), \quad (58)$$

where $F_k^1(\mathbf{x},\omega_i)$ is the insertion loss for the kth channel at the ith frequency, $F^2(\mathbf{x},\omega_i)$ is the return loss at the common port at the ith frequency, $S_{Uk}^1(\omega_i)(S_{Lk}^1(\omega_i))$ is the upper (lower) specification on insertion loss of the kth channel at the ith frequency, $S_U^2(\omega_i)$ $(S_L^2(\omega_i))$ is the upper (lower) specification on return loss at the ith frequency, and w_{Uk}^1, w_{Lk}^1, w_U^2, w_L^2 are the arbitrary user-chosen non-negative weighting factors.

A 12-channel, 12 GHz multiplexer without dummy channels has been optimized using spacings, input-output transformer ratios, cavity resonant frequencies as well as intercavity couplings as optimization variables. The filters are assumed lossy and dispersive; waveguide functions are assumed nonideal. The results of optimization are shown in Fig. 2. The problem has been solved using the MMLC package [11].

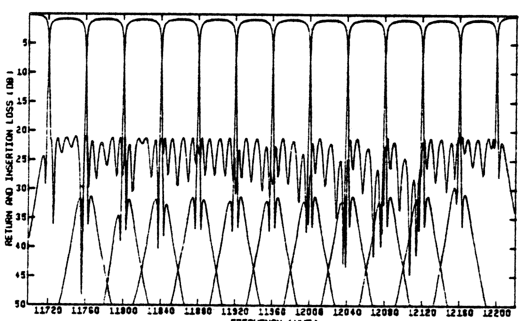

Fig. 2 Responses of a 12-channel multiplexer without dummy channels with optimized spacings, input-output transformer ratios, cavity resonances and coupling parameters.

The results presented in this paper have been obtained in close cooperation with members of ComDev Ltd. of Cambridge, Ontario, directly involved in multiplexer design and manufacture.

Low-Pass Filter

The LC low-pass filter shown in Fig. 3 is considered [1]. The problem is the optimal worst-case design embodying centering, tolerancing and tuning at the design stage. If the designer has no prior knowledge of the choice of the tuning components, we consider an objective function of the form

$$C \triangleq \sum_{i=1}^{3} \left[\frac{\phi_i^0}{\varepsilon_i} + c_i \frac{t_i}{\phi_i^0} \right], \quad (59)$$

where ϕ_i^0, ε_i and t_i represent nominal values, tolerances and tuning parameters of components L_1, C and L_2, respectively. The performance constraints may be written in the form

$$g_i = w(\omega_i)\,[S(\omega_i) - F(\phi, \omega_i)],\ i = 1,2,...,m_c, \quad (60)$$

where $w(\omega_i)$ denotes the weighting factor corresponding to frequency ω_i, $S(\omega_i)$ is the specification and $F(\phi, \omega_i)$ is the circuit response function evaluated at sample frequency ω_i. Table I summarizes the specifications for the filter. The critical vertices used can be obtained from published vertex selection schemes [23].

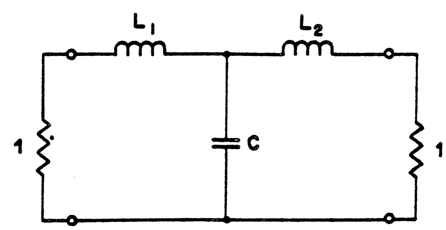

Fig. 3 The LC low-pass filter.

TABLE I
SPECIFICATIONS FOR LC LOW-PASS FILTER

Frequency Range (rad/s)	Sample Points (rad/s)	Insertion Loss Specification (dB)	Type	Weight w
0–1	0.45, 0.50, 0.55, 1.0	1.5	upper	+1
2.5	2.5	25	lower	–1

Table II summarizes the data for the filter including worst vertices, frequency points and weighting factors.

TABLE II
DATA FOR LOW-PASS FILTER

a \ i	1	2	3	4	5	6	7	8	9	10
r	6	6	6	8	1	3	3	3	3	3
	+1	+1	+1	+1	-1	-1	-1	-1	-1	-1
µ	-1	-1	-1	+1	-1	+1	+1	+1	+1	+1
	+1	+1	+1	+1	-1	-1	-1	-1	-1	-1
ω	0.45	0.50	0.55	1.0	2.5	0.45	0.50	0.55	1.0	2.5
S	1.5	1.5	1.5	1.5	25	1.5	1.5	1.5	1.5	25
w	1	1	1	1	-1	1	1	1	1	-1

There are 21 variables including nominal values, tolerances and tuning parameters as well as slack variables **ρ** which represent the settings of tuning components and 43 constraints including performance constraints and additional constraints on variables.

Table III summarizes the results for the LC low-pass filter problem. For this problem the choice of the cost coefficient c_i in (59) for tuning is very important. The most appropriate choice is the one for which both terms in the objective function (59) have the same order of magnitude. The advantage gained in the formulation using the ℓ_1 type of objective function is that the optimization will automatically choose the most appropriate component for tuning, which is the capacitor here. The results have been obtained using the MFNC package [14].

Best Mechanical Alignment Problem

An important practical extension of the problems discussed is the best alignment problem [24,25]. The problem arises in many practical situations when a relatively expensive manufactured product does not meet the design specifications and a decision is to be made for partial retreatment of the product. The problem is how to perform efficiently the part alignment process and, if reworking is needed, how to choose the best way to do it.

Suppose we have a set of points P in a two-dimensional space

$$P \triangleq \{p_1, p_2, ..., p_m\},\ m \geq 1, \quad (61)$$

TABLE III
OPTIMAL TUNING DESIGN OF THE LC LOW-PASS FILTER

Parameters	Solution
$L_1^0 = L_2^0$	2.06696
C^0	0.90758
$100\,\varepsilon_1/L_1^0 = 100\,\varepsilon_3/L_2^0$	18.01%
$100\,\varepsilon_2/C^0$	14.14%
$100\,t_1/L_1^0 = 100\,t_3/L_2^0$	0.00%
$100\,t_2/C^0$	16.43%
$\rho_1(6)$	-1.00000
$\rho_2(6)$	1.00000
$\rho_3(6)$	-1.00000
$\rho_1(8)$	-0.99935
$\rho_2(8)$	-1.00000
$\rho_3(8)$	-0.99935
$\rho_1(1)$	1.00000
$\rho_2(1)$	1.00000
$\rho_3(1)$	1.00000
$\rho_1(3)$	0.99885
$\rho_2(3)$	-0.06969
$\rho_3(3)$	0.99885
Cost Function	26.39285

and a system of coordinates $\overline{YOX}$ associated with this set.

Let

$$I \triangleq \{1,2,\ldots,m\} \tag{62}$$

be the index set for these points.

Suppose we have a set R of tolerance regions R_i, $i \in I \triangleq \{1,2,\ldots,m\}$, in a two-dimensional space,

$$R \triangleq \{R_1, R_2, \ldots, R_m\} \tag{63}$$

and a system of coordinates YOX associated with this set. We can define a one-to-one mapping g which assigns elements $R_i \in R$ to elements $p_i \in P$,

$$g: P \rightarrow R. \tag{64}$$

The regions $R_i \in R$, $i \in I$, may have different shapes (e.g., circular, rectangular), they may be defined using polar coordinates, rectangular coordinates or combined polar and rectangular coordinates.

The two systems of coordinates, YOX and $\overline{YOX}$ are related by the following transformation of coordinates

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} \cos\phi_3 & -\sin\phi_3 \\ \sin\phi_3 & \cos\phi_3 \end{bmatrix} \begin{bmatrix} \bar{x}_i \\ \bar{y}_i \end{bmatrix} + \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}, \tag{65}$$

where

$$\boldsymbol{\phi} \triangleq [\phi_1 \;\; \phi_2 \;\; \phi_3]^T \tag{66}$$

is the set of variables relating the two systems of coordinates.

The first step in the solution of the best alignment problem is to find $\boldsymbol{\phi}$ such that the maximum number of points $p_i \in P$, $i \in I$, are inside or on the boundary of the corresponding $R_i = g(p_i)$. However, the solution to the problem stated above may not be unique and may not be equal to the number of points m.

If it is not possible to find $\boldsymbol{\phi} = [\phi_1 \; \phi_2 \; \phi_3]^T$ such that all m points are inside or on the boundary of the corresponding tolerance region then it is necessary to delete one or more points in the set P to ensure that all other points satisfy this condition.

The best alignment problem can be formulated as

$$\underset{I_{del} \in 2^I}{\text{minimize}} \; n_{del} \triangleq \text{card}(I_{del}) \tag{67}$$

subject to constraint

$$\min_{\boldsymbol{\phi}} \max_{i} f_i(\boldsymbol{\phi}) \leq 0, \tag{68}$$

where I is the index set for points p_i which are to be aligned, I_{del} is the index set for points which should be deleted, 2^I is the family of all subsets of the set I and n_{del} is the cardinality of I_{del}.

The error function $f_i(\boldsymbol{\phi})$ is associated with the point p_i to indicate whether the point p_i is in ($f_i(\boldsymbol{\phi}) \leq 0$) or out ($f_i(\boldsymbol{\phi}) > 0$) of the tolerance region $R_i = g(p_i)$.

<u>Illustrative Example</u>: Suppose we have a set of points $P = \{p_1, p_2, p_3, p_4, p_5\}$ and a set of tolerance regions, $R = \{R_1, R_2, R_3, R_4, R_5\}$. Fig. 4 illustrates the situation before the alignment. Error functions at the starting point $\boldsymbol{\phi}_0 = [0.0 \; 0.0 \; 0.0]^T$ are

$$f_1 = 2.0710 \times 10^{-1},$$
$$f_2 = -5.0000 \times 10^{-1},$$
$$f_3 = 5.0000 \times 10^{-1},$$
$$f_4 = -5.0000 \times 10^{-1},$$
$$f_5 = 5.0000 \times 10^{-1}.$$

Fig. 5 shows the situation after running the alignment program. The best alignment was found at $\boldsymbol{\phi}_0 = [-2.316 \times 10^{-1} \; -2.792 \times 10^{-1} \; 4.758 \times 10^{-2}]^T$ with point 5 deleted. Remaining error functions at the solution are

$$f_1 = -1.5400 \times 10^{-1},$$
$$f_2 = -1.2060 \times 10^{-1},$$
$$f_3 = -1.2043 \times 10^{-2},$$
$$f_4 = -1.2043 \times 10^{-2}.$$

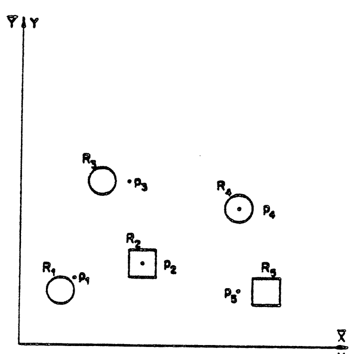

Fig. 4 Points and regions before alignment.

For the circular tolerance region the error function is the difference between the geometrical distance of a point from the center of the tolerance region and its radius. For the rectangular tolerance region the error function results from lower and upper bounds on coordinates of a point.

<u>Test Results on Practical Problems</u>: The algorithm described in [25] has been tested for seven sets of data supplied by the Woodward Governor Company. The data resulted from practical problems of part alignment in manufactured mechanical systems

and have been collected from inspecting actual parts. The points represent holes in one part which have to meet certain specifications when coupled together with another part. Test samples have different numbers of points, varying from 5 to 13 and specified tolerance regions of different shapes. The data as well as the results of running the alignment program for some samples are in [25].

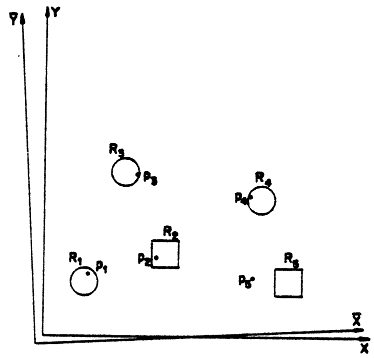

Fig. 5 Results of running the alignment program.

Parameter Identification Using the ℓ_1 Norm

In this application we deal with multi-coupled cavity narrow band-pass filters used in microwave communication systems (see Fig. 6).

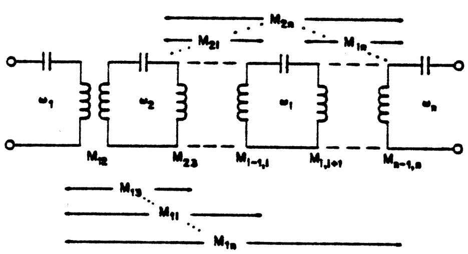

Fig. 6 Unterminated coupled-cavity filter illustrating the coupling coefficients.

A narrow-band lumped model of an unterminated multi-cavity filter has been given by Atia and Williams [26] as

$$\mathbf{ZI} = \mathbf{V}, \tag{69}$$

where

$$\mathbf{Z} = j(s\mathbf{1} + \mathbf{M}), \tag{70}$$

$$s = \frac{\omega_0}{\Delta\omega}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right), \tag{71}$$

$\mathbf{I}$ denotes an nxn identity matrix, $\mathbf{M}$ an nxn coupling matrix whose (i,j) element represents the normalized coupling between the ith and jth cavities, ω_0 is the center frequency and $\Delta\omega$ is the bandwidth parameter.

In practice it is often desired to determine the actual filter couplings based on response (return loss or insertion loss) measurements. The problem can be formulated as an optimization problem with the ℓ_1 objective function.

In this example we have used reflection coefficient as the filter response. The formulation is as follows.

$$\underset{\mathbf{x}}{\text{Minimize}} \sum_{j=1}^{m} |f(\mathbf{x}, \omega_j)|, \tag{72}$$

where

$$f(\mathbf{x},\omega_j) \triangleq w(\omega_j)(F^c(\mathbf{x},\omega_j) - F^m(\omega_j)), \tag{73}$$

$\mathbf{x}$ is the vector of filter couplings to be identified, F^c is the response calculated using the current parameter values and F^m is the measured response.

The filter response and its sensitivities are calculated using the formulas given in [22].

A 6th order filter centered at 12000 MHz with 40 MHz bandwidth is considered. Optimally designed filter parameters have been perturbed and the filter has been simulated. Reflection coefficient at 23 frequency points is used as the specification (measured response). The optimization problem (72) has been solved using optimal filter couplings as starting values. The results of parameter identification using the L1NONL package [20] are summarized in Table IV.

TABLE IV
RESULTS FOR THE 6TH ORDER FILTER EXAMPLE

Coupling	Percentage Deviation	
	Actual	Identified
M_{12}	2.0	2.0
M_{23}	-1.0	-1.0
M_{34}	5.0	5.0
M_{16}	5.0	5.0
M_{25}	-4.0	-4.0
M_{45}	-1.0	-1.0
M_{56}	2.0	2.0
Number of Function Evaluations		24
Execution Time (secs) on Cyber 170/815		6.2

CONCLUSIONS

We have described concepts and formulations developed for design centering, tolerancing and tuning of engineering systems. Formulations are fully supported by implementations of new and highly efficient algorithms for linearly constrained minimax and ℓ_1 optimization and for optimization with general constraints. We do not presume to be able to solve all problems associated with any overall engineering system. Applications of the approach proposed will be immediately apparent in many cases. Often it will also occur that familiarity with the concepts and techniques presented will itself clarify certain problem aspects which have been obscured or unrecognized. Examples have been described using electrical and mechanical systems. We feel that the formulations and techniques presented are powerful tools for solving difficult engineering design problems.

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