LEAST pTH AND MINIMAX OBJECTIVES FOR AUTOMATED NETWORK DESIGN†

Indexing terms: Lumped-parameter networks, Linear-network synthesis, Computer-aided circuit design, Frequency-domain analysis

It is shown how the adjoint-network approach to automated design due to Director and Rohrer can be implemented in gradient calculations for least pth and minimax response objective functions. Their formulas, for example, for frequency-domain design of networks consisting of lumped, linear and time invariant elements may be used without change, provided that the adjoint network is appropriately excited.

The concept of the adjoint network and its relevance to automated network design have been recently discussed by Director and Rohrer.[1, 2] They demonstrated the possibility of evaluating the gradient vector for a least-squares type of response objective function with respect to all existing and also nonexisting elements from only two analyses, one of the given network and one of its topologically equivalent adjoint network.

A least-squares performance criterion is only one of several possible performance criteria that could provide acceptable responses.[3] In the frequency domain, in particular, least pth[3, 4] and minimax[3, 5, 6] objectives are often more desirable than least-squares objectives from the point of view of obtaining smaller maximum deviations of network responses from desired responses. Minimax approximation, for example, is an approximation in the Chebyshev sense.

Hence, the question immediately arises: can the adjoint-network approach be implemented in gradient calculations for least pth and minimax performance criteria? This letter shows that the answer is 'yes'. Moreover, because of the linear relationship between the adjoint-network variables and the network-sensitivity components, the least-squares gradient formulas may be used without change, provided that the adjoint network is excited in an appropriate fashion.

Since the frequency- and time-domain cases are fairly similar, only the former will be considered here. For the time domain, the excitations would be analogous, and the adjoint network would have to be analysed again in backward time.

Least p th objective: A generalised-performance error criterion in the frequency domain can be defined as

$$E = \sum_{i=1}^{n_V} \int_{\Omega} \frac{1}{p} W_i(\omega)\,|I_i(j\omega) - \hat{I}_i(j\omega)|^p\, d\omega + \sum_{i=n_V+1}^{n_V+n_I} \int_{\Omega} \frac{1}{p} W_i(\omega)\,|V_i(j\omega) - \hat{V}_i(j\omega)|^p\, d\omega \quad (1)$$

where

Ω defines the frequency range of interest

I_i, V_i = actual complex port responses

$\hat{I}_i, \hat{V}_i$ = desired complex port responses

W_i = nonnegative real weight

p = positive integer

n_V = number of independent voltage sources

n_I = number of independent current sources

The network is excited as indicated in Fig. 1A, where some sources may have the value zero. The elements are assumed to be lumped, linear and time invariant, as allowed by Director and Rohrer.[2]

The partial derivative of E with respect to a network parameter x is

$$\frac{\partial E}{\partial x} = \sum_{i=1}^{n_V} \int_{\Omega} \mathrm{Re}\left[W_i(\omega)\,|I_i(j\omega) - \hat{I}_i(j\omega)|^{p-2}\,\{I_i^*(j\omega) - \hat{I}_i^*(j\omega)\}\,\frac{\partial I_i(j\omega)}{\partial x}\right] d\omega + \sum_{i=n_V+1}^{n_V+n_I} \int_{\Omega} \mathrm{Re}\left[W_i(\omega)\,|V_i(j\omega) - \hat{V}_i(j\omega)|^{p-2}\,\{V_i^*(j\omega) - \hat{V}_i^*(j\omega)\}\,\frac{\partial V_i(j\omega)}{\partial x}\right] d\omega \quad (2)$$

where * denotes the complex conjugate.

Tellegen's theorem can now be invoked in the manner of Director and Rohrer.[2] It follows from the relation between the network sensitivities, and those parts of the Tellegen sum involving the port variables, that the gradient vector ∇E as given by Director and Rohrer is applicable here, provided that the adjoint network is excited as shown in Fig. 1B by independent voltage sources

$$\Psi_i(j\omega) = -W_i(\omega)\,|I_i(j\omega) - \hat{I}_i(j\omega)|^{p-2}\,\{I_i^*(j\omega) - \hat{I}_i^*(j\omega)\} \qquad i = 1, 2, \ldots, n_V \quad (3a)$$

at the first n_V ports, and by independent current sources

$$\Phi_i(j\omega) = W_i(\omega)\,|V_i(j\omega) - \hat{V}_i(j\omega)|^{p-2}\,\{V_i^*(j\omega) - \hat{V}_i^*(j\omega)\} \qquad i = n_V+1, \ldots, n_V+n_I \quad (3b)$$

at the remaining n_I ports.

Minimax objectives: One possible way of formulating a minimax objective function is

$$\left.\begin{aligned} E &= \max_{i_V, i_I, \Omega} \{W_{i_V}(\omega)\,|I_{i_V}(j\omega) - \hat{I}_{i_V}(j\omega)|,\ W_{i_I}(\omega)\,|V_{i_I}(j\omega) - \hat{V}_{i_I}(j\omega)|\} \\ E &= \max_{i_V, i_I, \Omega} \{e_{i_V}(\omega), e_{i_I}(\omega)\} \begin{cases} i_V = 1, 2, \ldots, n_V \\ i_I = n_V+1, \ldots, n_V+n_I \end{cases} \end{aligned}\right\} \quad (4)$$

This function is characterised in general by discontinuous derivatives.[3] But suppose, for example, that the maximum occurs uniquely at the kth voltage-excited port, at $\omega = \omega_0$. Then

$$E = W_k(\omega_0)\,|I_k(j\omega_0) - \hat{I}_k(j\omega_0)| = e_k(\omega_0) \quad . \quad . \quad . \quad (5)$$

so that, under circumstances usually fulfilled in practice,

$$\frac{\partial E}{\partial x} = \mathrm{Re}\left\{W_k(\omega_0)\,\frac{I_k^*(j\omega_0) - \hat{I}_k^*(j\omega_0)}{|I_k(j\omega_0) - \hat{I}_k(j\omega_0)|}\,\frac{\partial I_k(j\omega_0)}{\partial x}\right\} \quad (6)$$

Following arguments similar to those used earlier, it can be seen that *(a)* the adjoint network need be analysed only at $\omega = \omega_0$, and *(b)* at this frequency all the port sources of the adjoint network are set to zero except the source at the kth port, where an independent voltage source

$$\Psi_k(j\omega_0) = -W_k(\omega_0)\,\frac{I_k^*(j\omega_0) - \hat{I}_k^*(j\omega_0)}{|I_k(j\omega_0) - \hat{I}_k(j\omega_0)|} \quad . \quad . \quad . \quad (7)$$

is applied. The gradient components given by Director and Rohrer can again be employed; the adjoint network need be analysed at a single frequency only, namely $\omega = \omega_0$, integration over Ω not being required.‡

The implementation of an objective function of the form of eqn. 4 does, however, pose several difficulties as discussed by Bandler.[3] One snag is the scarcity of minimisation algorithms for dealing, in general, with such problems. An alternative formulation can be to minimise a nonnegative independent variable E, subject to

$$\left.\begin{aligned} E &\geqslant g_{i_V}(\omega_d) \qquad i_V = 1, 2, \ldots, n_V \\ E &\geqslant g_{i_I}(\omega_d) \qquad i_I = n_V+1, \ldots, n_V+n_I \end{aligned}\right\} \quad . \quad . \quad (8)$$

for all $\omega_d \in \Omega_d$, a given discrete set of frequencies, and where

$$\left.\begin{aligned} g_{i_V}(\omega_d) &= \tfrac{1}{2}W_{i_V}(\omega_d)\,|I_{i_V}(j\omega_d) - \hat{I}_{i_V}(j\omega_d)|^2 \\ g_{i_I}(\omega_d) &= \tfrac{1}{2}W_{i_I}(\omega_d)\,|V_{i_I}(j\omega_d) - \hat{V}_{i_I}(j\omega_d)|^2 \end{aligned}\right\} \quad . \quad . \quad (9)$$

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‡ The preceding ideas can, of course, be applied in a similar way to a current-excited port

Thus

$$\left.\begin{aligned}\frac{\partial g_{i_V}(\omega_d)}{\partial x} &= \mathrm{Re}\left[W_{i_V}(\omega_d)\{I^*_{i_V}(j\omega_d)-\hat{I}^*_{i_V}(j\omega_d)\}\frac{\partial I_{i_V}(j\omega_d)}{\partial x}\right]\\ \frac{\partial g_{i_I}(\omega_d)}{\partial x} &= \mathrm{Re}\left[W_{i_I}(\omega_d)\{V^*_{i_V}(j\omega_d)-\hat{V}^*_{i_V}(j\omega_d)\}\frac{\partial V_{i_I}(j\omega_d)}{\partial x}\right]\end{aligned}\right\}\quad \ldots \tag{10}$$

The adjoint-network excitations are as given by eqn. 3 with $p = 2$ and $\omega = \omega_d$. Appropriate minimisation techniques are available.[3, 5, 6]

The computational effort for obtaining gradients for the least pth and minimax objectives does not appear to be much greater than for the least-squares case. Thus the decision as to which formulation should be used will depend on the type of solution required and the availability of suitable minimisation methods.

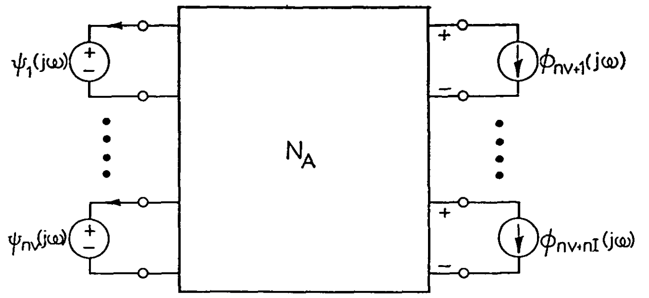

Fig. 1A *Multiport network consisting of lumped linear time-invariant elements*

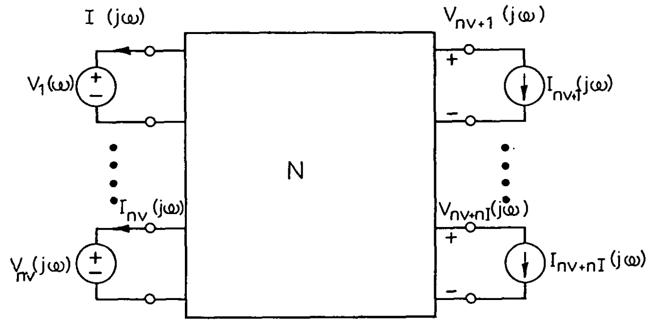

Fig. 1B *Adjoint network with corresponding port excitations*

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CURRENT-MODE CIRCUITS FOR THE UNARY FUNCTIONS OF A TERNARY VARIABLE

Indexing terms: Logic circuits, Logic design, Combinatorial switching

The unary functions of a ternary variable other than those of constant value are classified into three basic types. Current-mode circuits are given for the realisation of each function type. The circuits have high speed and large fan-out capabilities and are especially suited to fabrication by i.c. methods.

A recent letter[1] describing current-mode circuits for the realisation of ternary-combinational-logic expressions included design information for the two-valued unary operators x^0, x^2, x^{01}, x^{12} and x^1 and the ternary inverse function. The same circuit techniques, which are well suited for integrated-circuit manufacture, can be extended to the design of the remaining unary functions that cannot be implemented by a d.c. level.

Table 1 lists the unary functions of a ternary variable other than those of constant value and classifies these in groups according to the function type; the first group comprising two-valued ascending or descending functions, the second group two-valued functions with an intermediate maximum or

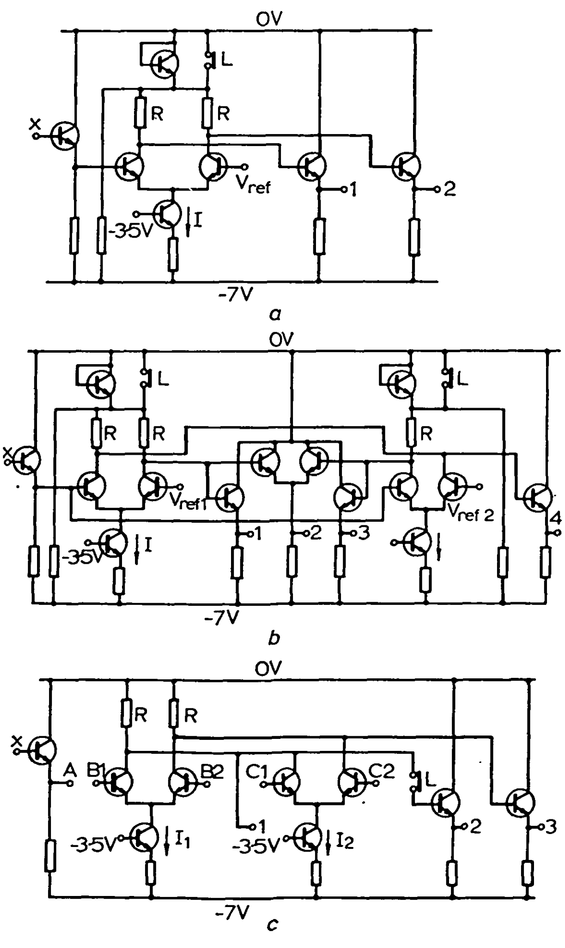

Fig. 1 *Current-mode circuits to give the unary functions of Table 1*
a From group 1
b From group 2
c From group 3