

EXACT SENSITIVITY ANALYSIS FOR OPTIMIZATION OF MULTI-COUPLED CAVITY FILTERS

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SUMMARY

This paper describes an efficient approach to the simulation and exact sensitivity evaluation of multi-coupled cavity filters. The approach uses sensitivity formulae for such responses as input or output reflection coefficient, return loss, insertion loss, transducer loss, gain slope and group delay, which are derived for a two-port equivalent of a general network described by its symmetrical impedance matrix. The formulae are specialized to the case of multi-coupled cavity filters, using a filter model which takes into account many non-ideal factors such as losses, frequency dependent coupling parameters and stray couplings. The formulation also treats synchronously or asynchronously tuned structures in a unified manner. Explicit tables of first- and second-order sensitivities w.r.t. all variables of interest, including frequency, are presented. Three problems of significant practical value in manufacturing of multi-cavity filters are solved with the direct application of our formulae. A 10th-order filter is considered for all three cases. The first case is simultaneous optimization of the amplitude and delay responses to obtain a self-equalized filter. The second case is accurate prediction of the responses for a lossy filter by simulating a lossless filter. The third case involves parameter identification of the filter from simulated measurements on its responses.

1. INTRODUCTION

The application of multi-coupled cavity microwave filters in modern communication systems has received increasing attention. The theory originated by Atia and Williams¹ has inspired many advances in this area. These advances have been responsible for many of the improvements in satellite multiplexing networks, as has been extensively discussed in the literature.¹⁻⁵

The growing variety and complexity of the design and manufacture of these filters necessitate the employment of modern computer-aided design techniques. For example, the traditional approach to an analytical solution may become inappropriate when asynchronous tuning realizing asymmetric characteristics or non-minimum-phase designs necessary to meet tight amplitude and delay specifications are of interest. Furthermore, CAD techniques can be used to predict the effects of non-ideal factors such as modelling or manufacturing imperfections which are not to be ignored in microwave devices. Unfortunately, and as not infrequently encountered, the practicality of the CAD approach could be fatally degraded by an inefficient simulation method and/or the lack of exact sensitivity information.

This paper describes a systematic and efficient approach to the simulation and exact sensitivity evaluation of narrow-band multi-coupled cavity microwave filters. The approach is based on the development of a set of response and sensitivity formulae for a two-port model of a general network described by its symmetrical impedance matrix. The responses of interest are input or output reflection coefficient, return loss, insertion loss, transducer loss, gain slope and group delay. The formulae are then specialized to the case of multi-cavity filters.

The presentation is organized in the following way. First, a brief description of the notation used throughout the paper is given in section 2. Then, in section 3, a comprehensive set of formulae for the simulation and sensitivity evaluation of various amplitude and phase responses of interest are derived which apply to a general class of networks characterized by their symmetrical impedance matrices. Derivations are detailed and the systems of equations to be solved for a complete analysis are defined in a natural way as the derivation proceeds. In section 4, after the description of a model for multi-coupled

cavity filters which takes into account such non-ideal effects as losses, frequency dependent coupling parameters and stray couplings, the general formulae for a two-port are specialized to the case of the filters. Synchronously and asynchronously tuned structures are treated in a unified manner. Practical formulae which can be readily implemented are tabulated. Computational considerations for various models of the filter are also discussed.

In section 5, our sensitivity formulae are applied directly to solve three problems of current interest in the manufacturing of multi-cavity filters. A 10th-order filter is considered for all three cases. The first problem involves the simultaneous optimization of amplitude and delay responses, i.e. design of self-equalized filters. To take advantage of state-of-the-art gradient-based optimization techniques, it is required to evaluate the responses as well as their sensitivities for this problem. This is a direct application of our formulae for the first-order sensitivity w.r.t. design variables, frequency and the second-order sensitivities. Although CAD based on frequency response is practical only if discrete frequency points are selected, sensitivities w.r.t. frequency are also used in conjunction with a cubic interpolation technique to optimally select sample frequency points which effectively transform the discrete frequency response design to a continuous one. The second problem is the prediction of responses for a filter which takes into account a non-ideal but realistic effect, dissipation in this case, by simulation of the ideal filter. This is a case for which sensitivities w.r.t. non-existing parameters, i.e. the parameters that for an ideal design are not present, have to be used. The third problem involves parameter identification of the filter from simulated measurements on its responses. Once again to apply gradient-based optimization techniques, it is required to evaluate the responses used for identification and their sensitivities w.r.t. the parameters to be identified.

2. NOTATION

Unless otherwise specified, the notation used in this paper is as follows. Boldface lower-cases \mathbf{p} , \mathbf{q} , etc., denote column vectors. Boldface capitals, \mathbf{M} , \mathbf{Z} , etc., denote matrices except \mathbf{I} and \mathbf{V} , which are reserved to denote current and voltage vectors, respectively. A letter with a double subscript denotes an element of the corresponding matrix, e.g. M_{lk} denotes the element at the l th row and k th column of matrix \mathbf{M} . A letter with a single subscript denotes a component of the corresponding vector, e.g. p_n denotes the n th component of vector \mathbf{p} . A letter with a single or double Greek subscript represents a first- or second-order derivative, respectively, e.g. \mathbf{Z}_ϕ denotes the first-order derivative of \mathbf{Z} w.r.t. design variable ϕ and $(I_1)_{\phi\omega}$ denotes the second-order derivative of I_1 w.r.t. ϕ and frequency ω . $\mathbf{1}$ denotes an identity matrix of appropriate dimension. \mathbf{e}_k denotes a vector whose components are zero except the k th component which is 1. $\bar{\mathbf{I}}$ denotes a rotation matrix of zero entries except the anti-diagonal elements which are 1.

3. A GENERALIZED TWO-PORT MODEL

Narrow-band multi-coupled cavity filters belong to a more general class of networks characterized by their symmetrical impedance matrices. If only input and output responses are of interest, an elegant approach to the analysis of such networks is to construct their two-port equivalents. In this section, using compact matrix notation, we develop a comprehensive set of formulae for simulation, first- and second-order sensitivities of a general two-port equivalent circuit.

Unterminated two-port

Consider a circuit described by

$$j\mathbf{Z}\mathbf{I}' = \mathbf{V}' \quad (1)$$

where $\mathbf{V}' \triangleq [V'_1 \ 0 \ \dots \ 0 \ V'_n]^T$ and $\mathbf{I}' \triangleq [I'_1 \ I'_2 \ \dots \ I'_n]^T$ are the voltage excitations and the currents, respectively. We use $j\mathbf{Z}$ to denote the $n \times n$ symmetrical impedance matrix, where \mathbf{Z} is a real matrix for a lossless circuit. Figure 1 shows a block representation of the circuit and its two-port equivalent.

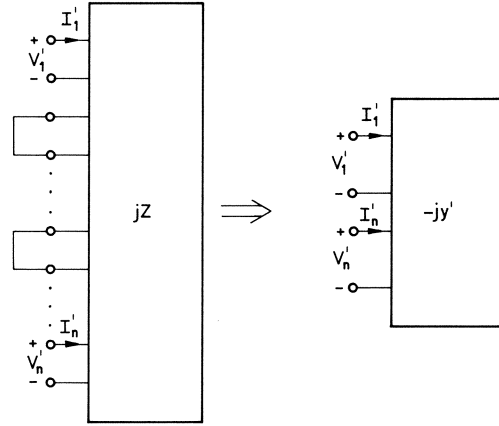


Figure 1. Block representation and two-port equivalent of a circuit characterized by its impedance matrix

Denote the two-port currents and voltages by $\mathbf{I}'_p = [I'_1 \ I'_n]^T$, $\mathbf{V}'_p = [V'_1 \ V'_n]^T$. By defining

$$\mathbf{U} \triangleq [\mathbf{e}_1 \ \mathbf{e}_n] \quad (2)$$

where \mathbf{e}_1 and \mathbf{e}_n are unit vectors consistent with the definition in section 2, we can express the relationship between the two-port and mesh currents and voltages as

$$\mathbf{I}'_p = \mathbf{U}^T \mathbf{I}' \quad (3)$$

and

$$\mathbf{V}' = \mathbf{U} \mathbf{V}'_p \quad (4)$$

From (1), (3) and (4) we can solve for

$$\mathbf{I}'_p = -j \mathbf{U}^T \mathbf{Z}^{-1} \mathbf{U} \mathbf{V}'_p = -j \mathbf{y}' \mathbf{V}'_p \quad (5)$$

where

$$\mathbf{y}' \triangleq \mathbf{U}^T \mathbf{Z}^{-1} \mathbf{U} = \begin{bmatrix} \mathbf{e}_1^T \mathbf{Z}^{-1} \mathbf{e}_1 & \mathbf{e}_1^T \mathbf{Z}^{-1} \mathbf{e}_n \\ \mathbf{e}_n^T \mathbf{Z}^{-1} \mathbf{e}_1 & \mathbf{e}_n^T \mathbf{Z}^{-1} \mathbf{e}_n \end{bmatrix} \quad (6)$$

In equation (5), $-j \mathbf{y}'$ is the conventionally defined short-circuit admittance matrix. Denote the two vectors $\mathbf{Z}^{-1} \mathbf{e}_1$ and $\mathbf{Z}^{-1} \mathbf{e}_n$, which are required to evaluate \mathbf{y}' , by \mathbf{p} and \mathbf{q} . We can obtain \mathbf{p} and \mathbf{q} by solving the systems of equations

$$\mathbf{Z} \mathbf{p} = \mathbf{e}_1 \quad (7)$$

and

$$\mathbf{Z} \mathbf{q} = \mathbf{e}_n \quad (8)$$

respectively. The elements of \mathbf{y}' are then given by

$$\mathbf{y}' = \begin{bmatrix} \mathbf{e}_1^T \mathbf{p} & \mathbf{e}_1^T \mathbf{q} \\ \mathbf{e}_n^T \mathbf{p} & \mathbf{e}_n^T \mathbf{q} \end{bmatrix} = \begin{bmatrix} p_1 & q_1 \\ p_n & q_n \end{bmatrix} = \begin{bmatrix} p_1 & p_n \\ p_n & q_n \end{bmatrix} \quad (9)$$

where $p_n = q_1$ because the matrix \mathbf{Z} is symmetrical.

We can now show that the solutions of (7) and (8) also provide sufficient information for the first-order sensitivity calculations. Differentiating (6) w.r.t. a variable ϕ in \mathbf{Z} , we have

$$\mathbf{y}'_\phi = -\mathbf{U}^T \mathbf{Z}^{-1} \mathbf{Z}_\phi \mathbf{Z}^{-1} \mathbf{U} = [-\mathbf{p} \ \mathbf{q}]^T \mathbf{Z}_\phi [\mathbf{p} \ \mathbf{q}] \quad (10)$$

where \mathbf{y}'_ϕ and \mathbf{Z}_ϕ denote the first-order derivatives of \mathbf{y}' and \mathbf{Z} w.r.t. ϕ and we have used the identity

$$\mathbf{Z}^{-1}\mathbf{U} = [\mathbf{Z}^{-1}\mathbf{e}_1 \quad \mathbf{Z}^{-1}\mathbf{e}_n] = [\mathbf{p} \quad \mathbf{q}] \quad (11)$$

For computation of gain slope and group delay sensitivities, second-order derivatives, namely $\mathbf{y}'_{\phi\omega}$, are needed. First, we obtain \mathbf{y}'_ω from (10) by letting $\phi = \omega$, as

$$\mathbf{y}'_\omega = -\mathbf{U}^T \mathbf{Z}^{-1} \mathbf{Z}_\omega \mathbf{Z}^{-1} \mathbf{U} \quad (12)$$

Then, differentiation of (12) w.r.t. ϕ gives

$$\begin{aligned} \mathbf{y}'_{\phi\omega} &= \mathbf{U}^T \mathbf{Z}^{-1} (\mathbf{Z}_\omega \mathbf{Z}^{-1} \mathbf{Z}_\phi - \mathbf{Z}_{\phi\omega} + \mathbf{Z}_\phi \mathbf{Z}^{-1} \mathbf{Z}_\omega) \mathbf{Z}^{-1} \mathbf{U} \\ &= [\mathbf{p} \quad \mathbf{q}]^T (\mathbf{Z}_\omega \mathbf{Z}^{-1} \mathbf{Z}_\phi - \mathbf{Z}_{\phi\omega} + \mathbf{Z}_\phi \mathbf{Z}^{-1} \mathbf{Z}_\omega) [\mathbf{p} \quad \mathbf{q}] \end{aligned} \quad (13)$$

where $\mathbf{y}'_{\phi\omega}$ and $\mathbf{Z}_{\phi\omega}$ denote the second-order derivatives of \mathbf{y}' and \mathbf{Z} . Denote $\mathbf{Z}^{-1} \mathbf{Z}_\omega [\mathbf{p} \quad \mathbf{q}]$ in equation (13) by $[\bar{\mathbf{p}} \quad \bar{\mathbf{q}}]$. We can obtain $\bar{\mathbf{p}}$ and $\bar{\mathbf{q}}$ by solving the systems of equations

$$\mathbf{Z}\bar{\mathbf{p}} = \mathbf{Z}_\omega \mathbf{p} \quad (14)$$

and

$$\mathbf{Z}\bar{\mathbf{q}} = \mathbf{Z}_\omega \mathbf{q} \quad (15)$$

respectively. Equation (13) can then be rewritten as

$$\mathbf{y}'_{\phi\omega} = [\bar{\mathbf{p}} \quad \bar{\mathbf{q}}]^T \mathbf{Z}_\phi [\mathbf{p} \quad \mathbf{q}] - [\mathbf{p} \quad \mathbf{q}]^T \mathbf{Z}_{\phi\omega} [\mathbf{p} \quad \mathbf{q}] + [\mathbf{p} \quad \mathbf{q}]^T \mathbf{Z}_\phi [\bar{\mathbf{p}} \quad \bar{\mathbf{q}}] \quad (16)$$

From a computational point of view, we notice that the systems of equations (7), (8), (14) and (15) have the same coefficient matrix \mathbf{Z} . Therefore, their solutions require only one LU factorization of \mathbf{Z} .

Terminated two-port

The two-port terminated, through input and output transformers, by an arbitrary load Z_L and a voltage source $E = 1$ V with an impedance Z_S is shown in Figure 2. In Figure 2, I_1 , I_n , V_1 and V_n are the currents and voltages that correspond to a two-port which includes the transformers. We define $\mathbf{I}_p \triangleq [I_1 \ I_n]^T$, $\mathbf{V}_p \triangleq [V_1 \ V_n]^T$ and

$$\mathbf{N} \triangleq \begin{bmatrix} n_1 & 0 \\ 0 & n_2 \end{bmatrix} \quad (17)$$

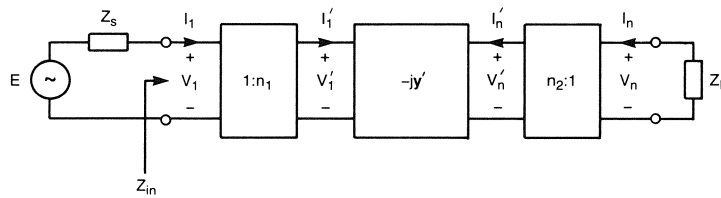


Figure 2. Representation of the overall network showing the two-port and the terminations

where n_1 and n_2 are the input and output transformer ratios, respectively. Consequently, \mathbf{I}_p and \mathbf{V}_p are related to \mathbf{I}'_p and \mathbf{V}'_p by

$$\mathbf{I}_p = \mathbf{N} \mathbf{I}'_p \quad (18)$$

and

$$\mathbf{V}'_p = \mathbf{N} \mathbf{V}_p \quad (19)$$

Also, as given in equation (5), \mathbf{I}'_p and \mathbf{V}'_p are related by $\mathbf{I}'_p = -j\mathbf{y}'\mathbf{V}'_p$. It follows that

$$\mathbf{I}_p = -j\mathbf{N}\mathbf{y}'\mathbf{N}\mathbf{V}_p = -j\mathbf{y}\mathbf{V}_p \quad (20)$$

where

$$\mathbf{y} \triangleq \mathbf{N}\mathbf{y}'\mathbf{N} \quad (21)$$

For the circuit under consideration, the terminating conditions are

$$V_1 = 1 - Z_S I_1 \quad (22)$$

and

$$V_n = -Z_L I_n \quad (23)$$

Denoting

$$\mathbf{T} \triangleq \begin{bmatrix} Z_S & 0 \\ 0 & Z_L \end{bmatrix} \quad (24)$$

we write (22) and (23) in a compact form, as

$$\mathbf{V}_p = \mathbf{e}_1 - \mathbf{T}\mathbf{I}_p \quad (25)$$

Equation (20), subject to the terminating conditions (25), can be solved for \mathbf{I}_p as follows:

$$\mathbf{I}_p = -j(1 - j\mathbf{y}\mathbf{T})^{-1}\mathbf{y}\mathbf{e}_1 = -j\mathbf{H}\mathbf{y}\mathbf{e}_1 \quad (26)$$

where

$$\mathbf{H} \triangleq (1 - j\mathbf{y}\mathbf{T})^{-1} \quad (27)$$

Furthermore, the first-order sensitivities of \mathbf{I}_p can be derived as

$$\begin{aligned} (\mathbf{I}_p)_\phi &= -j[\mathbf{H}_\phi\mathbf{y} + \mathbf{H}\mathbf{y}_\phi]\mathbf{e}_1 \\ &= j[\mathbf{H}(\mathbf{H}^{-1})_\phi\mathbf{H}\mathbf{y} - \mathbf{H}\mathbf{y}_\phi]\mathbf{e}_1 \\ &= j\mathbf{H}[-j(\mathbf{y}_\phi\mathbf{T} + \mathbf{y}\mathbf{T}_\phi)\mathbf{H}\mathbf{y} - \mathbf{y}_\phi]\mathbf{e}_1 \\ &= j\mathbf{H}[-j\mathbf{y}\mathbf{T}_\phi\mathbf{H}\mathbf{y}\mathbf{e}_1 - \mathbf{y}_\phi(1 + j\mathbf{T}\mathbf{H}\mathbf{y})\mathbf{e}_1] \\ &= j\mathbf{H}[\mathbf{y}\mathbf{T}_\phi\mathbf{I}_p - \mathbf{y}_\phi(\mathbf{e}_1 - \mathbf{T}\mathbf{I}_p)] \\ &= j\mathbf{H}(\mathbf{y}\mathbf{T}_\phi\mathbf{I}_p - \mathbf{y}_\phi\mathbf{V}_p) \end{aligned} \quad (28)$$

By the definition of \mathbf{y} , given in equation (21), it is clear that \mathbf{y}_ϕ is given by $\mathbf{N}\mathbf{y}'_\phi\mathbf{N}$ where \mathbf{y}'_ϕ has already been evaluated (cf. equation (10)).

We have shown the derivation of $(\mathbf{I}_p)_\phi$ in an explicit manner. The derivation of the second-order derivatives is similar but lengthy. Therefore, we only state the result as follows:

$$\begin{aligned} (\mathbf{I}_p)_{\phi\omega} &= j\mathbf{H}\{\mathbf{y}_\omega[\mathbf{T}_\phi\mathbf{I}_p + \mathbf{T}(\mathbf{I}_p)_\phi] + \mathbf{y}_\phi[\mathbf{T}_\omega\mathbf{I}_p + \mathbf{T}(\mathbf{I}_p)_\omega] \\ &\quad + \mathbf{y}[\mathbf{T}_\phi(\mathbf{I}_p)_\omega + \mathbf{T}_\omega(\mathbf{I}_p)_\phi] + \mathbf{y}\mathbf{T}_{\phi\omega}\mathbf{I}_p - \mathbf{y}_{\phi\omega}\mathbf{V}_p\} \end{aligned} \quad (29)$$

Sometimes, as for the evaluation of the output reflection coefficient, it is also of interest to solve the network excited at the output port. The solution, denoted by $\hat{\mathbf{I}}_p$, can be obtained by simply replacing \mathbf{e}_1 by \mathbf{e}_n in (26), where $\mathbf{e}_n = [0 \ 1]^T$. The sensitivity expressions of $\hat{\mathbf{I}}_p$ are the same as those of \mathbf{I}_p except that \mathbf{e}_1 , \mathbf{I}_p and \mathbf{V}_p are replaced by \mathbf{e}_n , $\hat{\mathbf{I}}_p$ and $\hat{\mathbf{V}}_p$, as appropriate.

Various frequency responses

In practice, the performance of a two-port network is often evaluated via some conventionally defined frequency responses such as the input and output reflection coefficients, input and output return losses,

transducer loss, insertion loss, gain slope and group delay. These frequency responses and their sensitivities can be calculated using the formulae which were obtained for the two-port. Table I summarizes various frequency responses of interest and their sensitivities. The formulae for return loss, transducer loss, insertion loss, gain slope and group delay are clear from their conventional definitions, whereas the formulae for reflection coefficients are derived as follows.

Consider the input reflection coefficient, defined as

$$\rho_{in} \triangleq \frac{Z_{in} - Z_S^*}{Z_{in} + Z_S} \quad (30)$$

Table I. Sensitivity expressions for various frequency responses

Type	Response	Formula	Expression for sensitivity w.r.t. ϕ
Input reflection coefficient		$1 - 2R_S I_1$	$-2[R_S(I_1)_\phi + (R_S)_\phi I_1]$
Output reflection coefficient		$1 - 2R_L \hat{I}_n$	$-2[R_L(\hat{I}_n)_\phi + (R_L)_\phi \hat{I}_n]$
Input or output† return loss		$-20 \log_{10} \rho_{in,out} $	$c \operatorname{Re} \left[\frac{(\rho_{in,out})_\phi}{\rho_{in,out}} \right]$
Transducer loss		$-10 \log_{10} (4 I_n ^2 R_S R_L)$	$c \operatorname{Re} \left[\frac{(I_n)_\phi}{I_n} + \frac{(R_S)_\phi}{2R_S} + \frac{(R_L)_\phi}{2R_L} \right]$
Insertion loss		$-20 \log_{10} Z_T I_n $	$c \operatorname{Re} \left[\frac{(I_n)_\phi}{I_n} + \frac{(Z_T)_\phi}{Z_T} \right]$
Gain slope		$c \operatorname{Re} \left[\frac{(I_n)_\omega}{I_n} + \frac{(Z_T)_\omega}{Z_T} \right]$	$c \operatorname{Re} \left[\frac{(I_n)_{\phi\omega}}{I_n} - \frac{(I_n)_\phi (I_n)_\omega}{I_n^2} + \frac{(Z_T)_{\phi\omega}}{Z_T} - \frac{(Z_T)_\phi (Z_T)_\omega}{Z_T^2} \right]$
Group delay		$-\operatorname{Im} \left[\frac{(I_n)_\omega}{I_n} + \frac{(Z_L)_\omega}{Z_L} \right]$	$-\operatorname{Im} \left[\frac{(I_n)_{\phi\omega}}{I_n} - \frac{(I_n)_\phi (I_n)_\omega}{I_n^2} + \frac{(Z_L)_{\phi\omega}}{Z_L} - \frac{(Z_L)_\phi (Z_L)_\omega}{Z_L^2} \right]$

† $\rho_{in,out}$ denotes input (output) reflection coefficient.

Auxiliary notation: $R_S \triangleq \operatorname{Re}(Z_S)$, $R_L \triangleq \operatorname{Re}(Z_L)$, $Z_T \triangleq Z_S + Z_L$, $c \triangleq -20/\ln 10$.

where Z_{in} is the input impedance as shown in Figure 2. From (30) it follows that

$$\rho_{in} = 1 - \frac{2 \operatorname{Re}(Z_S)}{Z_{in} + Z_S} = 1 - 2I_1 \operatorname{Re}(Z_S) \quad (31)$$

The formula for the output reflection coefficient is derived in a similar way.

4. ANALYSIS OF MULTI-COUPLED CAVITY FILTERS

Basic model and sensitivities

The approach which was developed for analysing a generalized two-port can be used most beneficially for the case of multi-coupled cavity filters. The symmetrical impedance matrix for a narrow-band lumped model of an unterminated filter is given by

$$j\mathbf{Z} \triangleq j(s\mathbf{1} + \mathbf{M}) - r\mathbf{1} \quad (32)$$

where $\mathbf{1}$ denotes an $n \times n$ identity matrix and s is the normalized frequency variable given by

$$s \triangleq \frac{\omega_0}{\Delta\omega} \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right) \quad (33)$$

ω_0 and $\Delta\omega$ being the synchronously tuned cavity resonant frequency and the bandwidth parameter, respectively. We assume uniform dissipation for all cavities indicated by parameter r . In equation (32), \mathbf{M} is the coupling matrix whose (i, j) element represents the normalized coupling between the i th and j th cavities, as illustrated in Figure 3, and the diagonal entries M_{ii} represent the deviations from synchronous tuning. Element M_{ij} does not necessarily correspond to a desirable and designable coupling. It may as well represent a stray coupling which is excluded from the nominal electrical equivalent circuit. Dispersion effects on the filter can be modelled by a frequency dependent \mathbf{M} matrix.

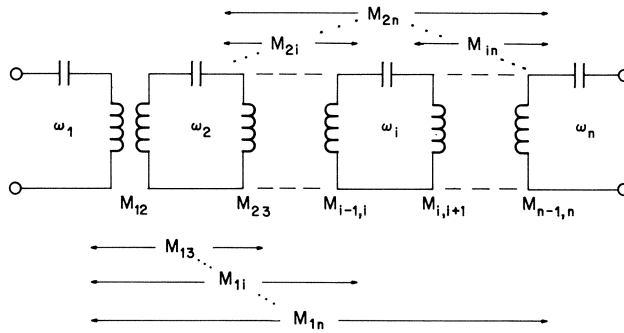


Figure 3. Underterminated coupled-cavity filter illustrating the coupling coefficients

The ideal model, namely the non-dispersive and lossless filter of Atia and Williams,¹ is recovered by considering a frequency independent \mathbf{M} matrix and letting r be zero.

Having modelled the multi-coupled cavity filter by its impedance matrix, we can calculate \mathbf{y}' and \mathbf{y} , their first- and second-order sensitivities and hence various frequency responses and sensitivities from the formulae developed for the two-port equivalent. More specifically, we can use equations (9), (10), (16) and (21) for calculation of \mathbf{y}' , its first- and second-order sensitivities and \mathbf{y} . The sensitivities of \mathbf{y} are obtained by pre- and post-multiplication by \mathbf{N} of the corresponding formulae for \mathbf{y}' . If the input or output transformer ratio is the variable of interest, i.e. $\phi \in \{n_1, n_2\}$, the sensitivities of \mathbf{y} are obtained in a different but straightforward way. In this case, \mathbf{y}_ϕ and $\mathbf{y}'_{\phi\omega}$ are derived from \mathbf{y}' and \mathbf{y}'_ω , appropriately pre- and post-multiplied by \mathbf{N}_ϕ and \mathbf{N} matrices.

Tables II–IV summarize the first- and second-order sensitivity expressions for variables of interest in multi-coupled cavity filters. The tables correspond to the case in which \mathbf{M} is frequency independent. The formulae derived for the generalized two-port can be used directly when dispersive effects are taken into account, namely, for the case of a frequency dependent \mathbf{M} matrix.

A special class of filters, namely the filters with dual-symmetrical impedance matrices are distinguished in Tables II–IV. In this paper a filter is said to be dual-symmetrical if its coupling matrix \mathbf{M} is symmetrical w.r.t. its anti-diagonal as well as its diagonal. For such a filter we have

$$M_{lk} = M_{ij} \quad (34)$$

where

$$i \triangleq n+1-k \quad \text{and} \quad j \triangleq n+1-l \quad (35)$$

or, using the matrix notation,

$$\bar{\mathbf{1}}\mathbf{M}\bar{\mathbf{1}} = \mathbf{M} \quad (36)$$

Table II. First-order sensitivity expressions for (a) symmetrical and (b) dual-symmetrical coupling matrices: basic solutions (a) $\mathbf{Zp} = \mathbf{e}_1$, $\mathbf{Zq} = \mathbf{e}_n$; (b) $\mathbf{Zp} = \mathbf{e}_1$, $\mathbf{q} = \bar{\mathbf{1}}\mathbf{p}$

Variable (ϕ)	First-order sensitivity expression (\mathbf{y}_ϕ)	
	Symmetrical case	Dual-symmetrical case
\mathbf{M}_{lk}^\dagger	$c_1 \begin{bmatrix} 2n_1^2 p_l p_k & n_1 n_2 (p_l q_k + p_k q_l) \\ n_1 n_2 (p_l q_k + p_k q_l) & 2n_2^2 q_l q_k \end{bmatrix}$	$2c_1 \begin{bmatrix} n_1^2 (p_l p_k + p_i p_j) & n_1 n_2 (p_l p_i + p_k p_j) \\ n_1 n_2 (p_l p_i + p_k p_j) & n_2^2 (p_l p_k + p_i p_j) \end{bmatrix}$
$\omega, \Delta\omega, \omega_0^\ddagger$	$-s_\phi \begin{bmatrix} n_1^2 \mathbf{p}^T \mathbf{p} & n_1 n_2 \mathbf{p}^T \mathbf{q} \\ n_1 n_2 \mathbf{p}^T \mathbf{q} & n_2^2 \mathbf{q}^T \mathbf{q} \end{bmatrix}$	$-s_\phi \begin{bmatrix} n_1^2 \mathbf{p}^T \mathbf{p} & n_1 n_2 \mathbf{p}^T \bar{\mathbf{1}}\mathbf{p} \\ n_1 n_2 \mathbf{p}^T \bar{\mathbf{1}}\mathbf{p} & n_2^2 \mathbf{p}^T \mathbf{p} \end{bmatrix}$
r	$j \begin{bmatrix} n_1^2 \mathbf{p}^T \mathbf{p} & n_1 n_2 \mathbf{p}^T \mathbf{q} \\ n_1 n_2 \mathbf{p}^T \mathbf{q} & n_2^2 \mathbf{q}^T \mathbf{q} \end{bmatrix}$	$j \begin{bmatrix} n_1^2 \mathbf{p}^T \mathbf{p} & n_1 n_2 \mathbf{p}^T \bar{\mathbf{1}}\mathbf{p} \\ n_1 n_2 \mathbf{p}^T \bar{\mathbf{1}}\mathbf{p} & n_2^2 \mathbf{p}^T \mathbf{p} \end{bmatrix}$
n_1	$\begin{bmatrix} 2n_1 p_1 & n_2 p_n \\ n_2 p_n & 0 \end{bmatrix}$	$\begin{bmatrix} 2n_1 p_1 & n_2 p_n \\ n_2 p_n & 0 \end{bmatrix}$
n_2	$\begin{bmatrix} 0 & n_1 p_n \\ n_1 p_n & 2n_2 q_n \end{bmatrix}$	$\begin{bmatrix} 0 & n_1 p_n \\ n_1 p_n & 2n_2 p_1 \end{bmatrix}$

$$\dagger c_1 \triangleq \begin{cases} -\frac{1}{2} & \text{if } l = k \text{ (for the dual-symmetrical case, also if } l + k = n + 1) \\ -1 & \text{otherwise} \end{cases}$$

$$\text{and } i \triangleq n + 1 - k, j \triangleq n + 1 - l$$

$$\ddagger s_\phi = \frac{\partial s}{\partial \phi}$$

Table III. Some second-order sensitivity expressions for a symmetrical coupling matrix: basic solutions $\mathbf{Zp} = \mathbf{e}_1$; $\mathbf{Zq} = \mathbf{e}_n$; $\mathbf{Z}\bar{\mathbf{p}} = \mathbf{p}$; $\mathbf{Z}\bar{\mathbf{q}} = \mathbf{q}$

Variable ϕ	Second-order sensitivity expression ($\mathbf{y}_{\phi\omega}$)
\mathbf{M}_{lk}^\dagger	$c_2 \begin{bmatrix} 2n_1^2 (p_l \bar{p}_k + p_k \bar{p}_l) & n_1 n_2 (p_l \bar{q}_k + p_k \bar{q}_l + \bar{p}_l q_k + \bar{p}_k q_l) \\ n_1 n_2 (p_l \bar{q}_k + p_k \bar{q}_l + \bar{p}_l q_k + \bar{p}_k q_l) & 2n_2^2 (q_l \bar{q}_k + q_k \bar{q}_l) \end{bmatrix}$
$\omega, \Delta\omega, \omega_0^\ddagger$	$2s_\phi s_\omega \begin{bmatrix} n_1^2 \mathbf{p}^T \bar{\mathbf{p}} & n_1 n_2 \mathbf{p}^T \bar{\mathbf{q}} \\ n_1 n_2 \mathbf{p}^T \bar{\mathbf{q}} & n_2^2 \mathbf{q}^T \bar{\mathbf{q}} \end{bmatrix} - s_{\phi\omega} \begin{bmatrix} n_1^2 \bar{p}_1 & n_1 n_2 \bar{p}_n \\ n_1 n_2 \bar{p}_n & n_2^2 \bar{q}_n \end{bmatrix}$
r	$-2j s_\omega \begin{bmatrix} n_1^2 \mathbf{p}^T \bar{\mathbf{p}} & n_1 n_2 \mathbf{p}^T \bar{\mathbf{q}} \\ n_1 n_2 \mathbf{p}^T \bar{\mathbf{q}} & n_2^2 \mathbf{q}^T \bar{\mathbf{q}} \end{bmatrix}$
n_1	$-s_\omega \begin{bmatrix} 2n_1 \bar{p}_1 & n_2 \bar{p}_n \\ n_2 \bar{p}_n & 0 \end{bmatrix}$
n_2	$-s_\omega \begin{bmatrix} 0 & n_1 \bar{p}_n \\ n_1 \bar{p}_n & 2n_2 \bar{q}_n \end{bmatrix}$

$$\dagger c_2 \triangleq \begin{cases} \frac{1}{2} s_\omega & \text{if } l = k \\ s_\omega & \text{otherwise} \end{cases}$$

$$\ddagger s_\phi = \frac{\partial s}{\partial \phi}, \quad s_\omega = \frac{\partial s}{\partial \omega}, \quad s_{\phi\omega} = \frac{\partial^2 s}{\partial \phi \partial \omega}$$

Table IV. Some second-order sensitivity expressions for a dual-symmetrical coupling matrix: basic solutions $\mathbf{Z}\mathbf{p} = \mathbf{e}_i$; $\mathbf{Z}\bar{\mathbf{p}} = \mathbf{p}$

Variable (ϕ)	Second-order sensitivity expression ($\mathbf{y}_{\phi\omega}$)
M_{lk}^\dagger	$\dot{\lambda}c_2 \begin{bmatrix} n_1^2(p_l\bar{p}_k + p_k\bar{p}_l + p_i\bar{p}_j + p_j\bar{p}_i) & n_1n_2(p_l\bar{p}_i + p_i\bar{p}_l + p_k\bar{p}_j + p_j\bar{p}_k) \\ n_1n_2(p_l\bar{p}_i + p_i\bar{p}_l + p_k\bar{p}_j + p_j\bar{p}_k) & n_2^2(p_l\bar{p}_k + p_k\bar{p}_l + p_i\bar{p}_j + p_j\bar{p}_i) \end{bmatrix}$
$\omega, \Delta\omega, \omega_0^\ddagger$	$2s_\phi s_\omega \begin{bmatrix} n_1^2\mathbf{p}^T\bar{\mathbf{p}} & n_1n_2\mathbf{p}^T\bar{\mathbf{I}}\bar{\mathbf{p}} \\ n_1n_2\mathbf{p}^T\bar{\mathbf{I}}\bar{\mathbf{p}} & n_2^2\mathbf{p}^T\bar{\mathbf{p}} \end{bmatrix} - s_{\phi\omega} \begin{bmatrix} n_1^2\bar{p}_1 & n_1n_2\bar{p}_n \\ n_1n_2\bar{p}_n & n_2^2\bar{p}_1 \end{bmatrix}$
r	$-2js_\omega \begin{bmatrix} n_1^2\mathbf{p}^T\bar{\mathbf{p}} & n_1n_2\mathbf{p}^T\bar{\mathbf{I}}\bar{\mathbf{p}} \\ n_1n_2\mathbf{p}^T\bar{\mathbf{I}}\bar{\mathbf{p}} & n_2^2\mathbf{p}^T\bar{\mathbf{p}} \end{bmatrix}$
n_1	$-s_\omega \begin{bmatrix} 2n_1\bar{p}_1 & n_2\bar{p}_n \\ n_2\bar{p}_n & 0 \end{bmatrix}$
n_2	$-s_\omega \begin{bmatrix} 0 & n_1\bar{p}_n \\ n_1\bar{p}_n & 2n_2\bar{p}_1 \end{bmatrix}$

$$\dagger c_2 \triangleq \begin{cases} \frac{1}{2}s_\omega & \text{if } l = k \text{ or } l+k = n+1 \\ s_\omega & \text{otherwise} \end{cases}$$

$$\text{and } i \triangleq n+1-k, j \triangleq n+1-l$$

$$\ddagger s_\phi = \frac{\partial s}{\partial \phi}, \quad s_\omega = \frac{\partial s}{\partial \omega}, \quad s_{\phi\omega} = \frac{\partial^2 s}{\partial \phi \partial \omega}$$

Computational considerations

For a generalized two-port, it was established that solving four systems of equations, namely $\mathbf{Z}\mathbf{p} = \mathbf{e}_1$, $\mathbf{Z}\mathbf{q} = \mathbf{e}_n$, $\mathbf{Z}\bar{\mathbf{p}} = \mathbf{Z}_\omega\mathbf{p}$ and $\mathbf{Z}\bar{\mathbf{q}} = \mathbf{Z}_\omega\mathbf{q}$, provides sufficient information for evaluation of input and output responses and their first- and second-order sensitivities. From a computational point of view, this means one LU factorization of the matrix \mathbf{Z} followed by four forward and backward substitutions. For a matrix \mathbf{Z} given by (32), we consider three special cases with each one leading to significant computational savings.

Case 1 corresponds to a lossless filter, for which r is zero in equation (32). This implies that \mathbf{Z} is a real matrix which itself leads to \mathbf{p} , \mathbf{q} , $\bar{\mathbf{p}}$ and $\bar{\mathbf{q}}$ being real. It is clear that solving real systems of equations and manipulating real arrays enjoy significant computational advantages over the complex calculations.

In case 2, \mathbf{M} is assumed to be frequency independent. In this case \mathbf{Z}_ω , which for a general dispersive filter is given by $s_\omega\mathbf{1} + \mathbf{M}_\omega$, reduces to $s_\omega\mathbf{1}$. This implies that matrix by vector multiplications of the form $\mathbf{Z}_\omega\mathbf{p}$ and $\mathbf{Z}_\omega\mathbf{q}$, reduce to scalar by vector multiplications.

In case 3, \mathbf{M} is dual symmetrical as described by (36). This leads to \mathbf{Z} also being dual symmetrical, or

$$\mathbf{Z} = \bar{\mathbf{I}}\mathbf{Z}\bar{\mathbf{I}} \quad (37)$$

Considering (37) and the four systems of equations required for the analysis, we have

$$\mathbf{Z}\bar{\mathbf{I}}\mathbf{p} = \bar{\mathbf{I}}\mathbf{Z}\mathbf{p} = \bar{\mathbf{I}}\mathbf{e}_1 = \mathbf{e}_n \quad (38)$$

which gives

$$\mathbf{q} = \bar{\mathbf{I}}\mathbf{p} \quad (39)$$

i.e.

$$q_i = p_{n+1-i}, \quad i = 1, 2, \dots, n \quad (40)$$

Similarly, it can be shown that

$$\bar{\mathbf{q}} = \bar{\mathbf{I}}\bar{\mathbf{p}} \quad (41)$$

The implication of (39) and (41) is that the two systems of equations involving \mathbf{q} and $\bar{\mathbf{q}}$ need not be solved since they only contain reordered elements of \mathbf{p} and $\bar{\mathbf{p}}$.

We conclude that the computational effort varies from solving two real systems for the case of a dual-symmetrical, lossless and non-dispersive filter, to solving four complex systems for the most general case.

Comparison with the traditional approach

Having discussed some computational aspects of our approach, it is also necessary to describe rigorously the advantages of the approach over the more traditional analysis methods for multi-coupled cavity filters.

Considering Figure 2 and Table I, it is clear that all frequency responses of interest are evaluated via the calculation of I_1 and I_n . In a direct and traditional approach, I_1 and I_n can be calculated by solving the system of equations

$$\frac{1}{n_1} \mathbf{Z} \mathbf{I}' = \mathbf{e}_1 \quad (42)$$

where

$$\mathbf{Z} = j(s\mathbf{1} + \mathbf{M}) + r\mathbf{1} + n_1^2 \mathbf{Z}_S \mathbf{e}_1 \mathbf{e}_1^T + n_2^2 \mathbf{Z}_L \mathbf{e}_n \mathbf{e}_n^T \quad (43)$$

with \mathbf{Z} representing an impedance matrix which takes into account terminations \mathbf{Z}_S and \mathbf{Z}_L by appropriately adjusting the impedance matrix defined by (32). I_1 and I_n are then given by

$$I_1 = n_1 I'_1 \quad (44)$$

and

$$I_n = n_2 I'_n \quad (45)$$

The first-order sensitivities of I_1 and I_n with respect to any of the variables, e.g. elements of the coupling matrix, input-output transformer ratios, frequency or dissipation parameter, are simply evaluated by the well-known finite difference method. Each variable candidate is perturbed by a small amount once in the positive direction and once in the negative direction and two additional solutions of (42) are computed. Neglecting the calculations in (44) and (45) and considering n_ω frequency points and n_ϕ variables, a total of $n_\omega(2n_\phi + 1)LU$ factorizations plus forward and backward substitutions are required to complete the evaluations of I_1 , I_n and their first-order sensitivities. The total number of complex multiplications and divisions is estimated as

$$N_p = n_\omega(2n_\phi + 1) \left(\frac{n^3}{3} + n^2 - \frac{n}{3} \right) \quad (46)$$

A typical formula for n_ϕ is

$$n_\phi = n + (n - 1) + m + 4 \quad (47)$$

with n diagonal elements of the coupling matrix, i.e. deviations from synchronous tuning for each cavity, $n - 1$ superdiagonal couplings and m cross-couplings taken as variables. The other four variables are input and output transformer ratios, dissipation parameter and frequency. For a 6th-order filter with one cross-coupling, we have $N_p = 3498n_\omega$.

Without going into details, an estimated number of multiplications and divisions for evaluation of I_1 , I_n and their first-order sensitivities, by using our approach and having similar variables as the previous case, is

$$N_e = n_\omega \left[\left(\frac{n^3}{3} + n^2 - \frac{n}{3} \right) + n^2 + 4(n - 1 + m) + 3n + 8n_\phi \right] \quad (48)$$

(See (7), (8) and (28), and Table II.) For $n = 6$ and $m = 1$, we have $N_e = 312n_o$. Although a few overhead calculations have been neglected in deriving (48), it can be safely said that using our approach achieves an approximate 10-fold reduction in the number of operations over the direct approach.

Evaluation of second-order sensitivities by perturbations is not only cumbersome but also not sufficiently accurate in some cases. Our exact formulae, which basically add two more forward and backward substitutions to previous calculations become very attractive.

5. APPLICATIONS

Three examples of significant practical value are selected to illustrate the direct application of the approach presented. A 10th-order multi-coupled cavity filter with a centre frequency of 4 GHz and a bandwidth of 40 MHz is considered. The coupling matrix of the filter is dual-symmetrical, as described in section 4. The first example describes a non-minimum-phase self-equalized design achieved by simultaneous optimization of the amplitude and group delay responses. For the second example, the sensitivities w.r.t. cavity dissipation are used to predict the amplitude response of a lossy filter. The parameter identification of the filter from simulated measurements is described in the third example. These problems reflect typical situations in engineering practice where the tremendous advantage of modern CAD techniques can be exploited.

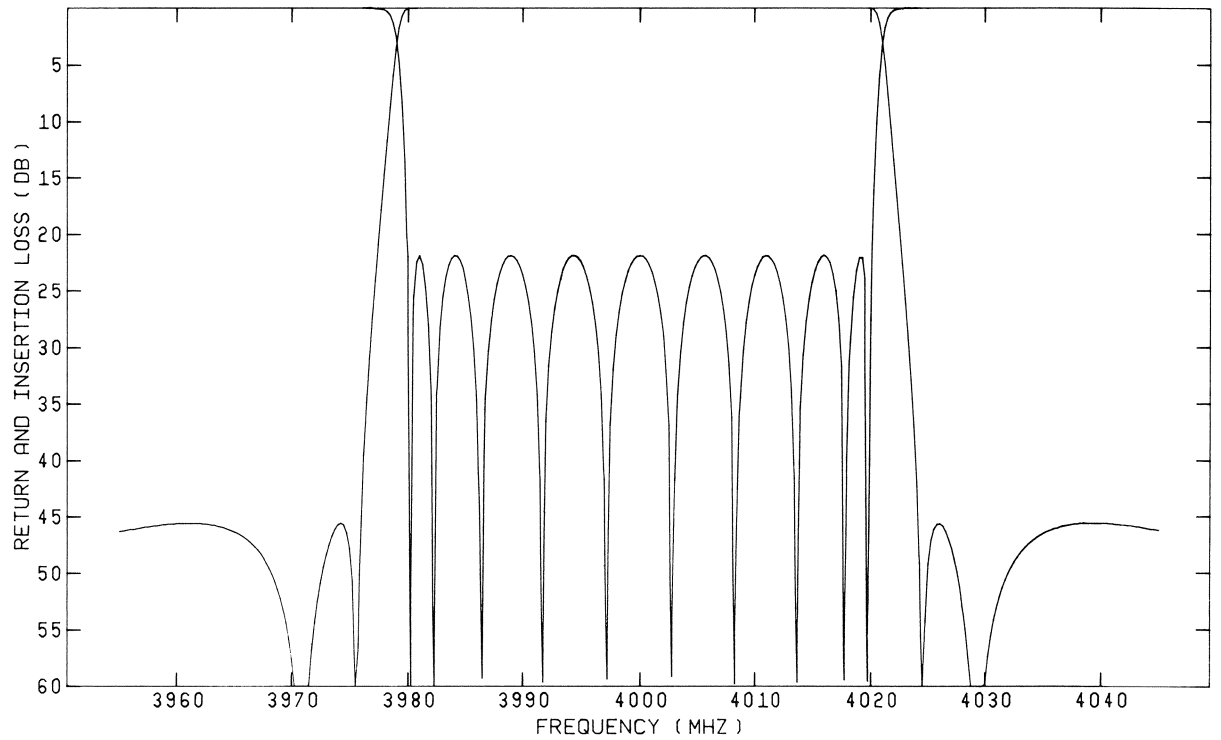
A quasi-elliptic self-equalized filter

Non-minimum-phase function filters can be designed to achieve optimal trade-offs between the attenuation and group delay characteristics, as required for high fidelity signal transmission. A 10th-order quasi-elliptic, self-equalized filter has been obtained from simultaneous optimization of the amplitude and delay responses. A powerful gradient-based minimax optimization method⁶ is employed. The objective functions to be optimized are formulated from the filter responses including the reflection coefficient for both the passband and the stopband and the relative group delay for the passband. Figure 4 shows the amplitude and group delay responses of the filter designed. The optimization directly solves for the non-zero couplings and transformer ratios as

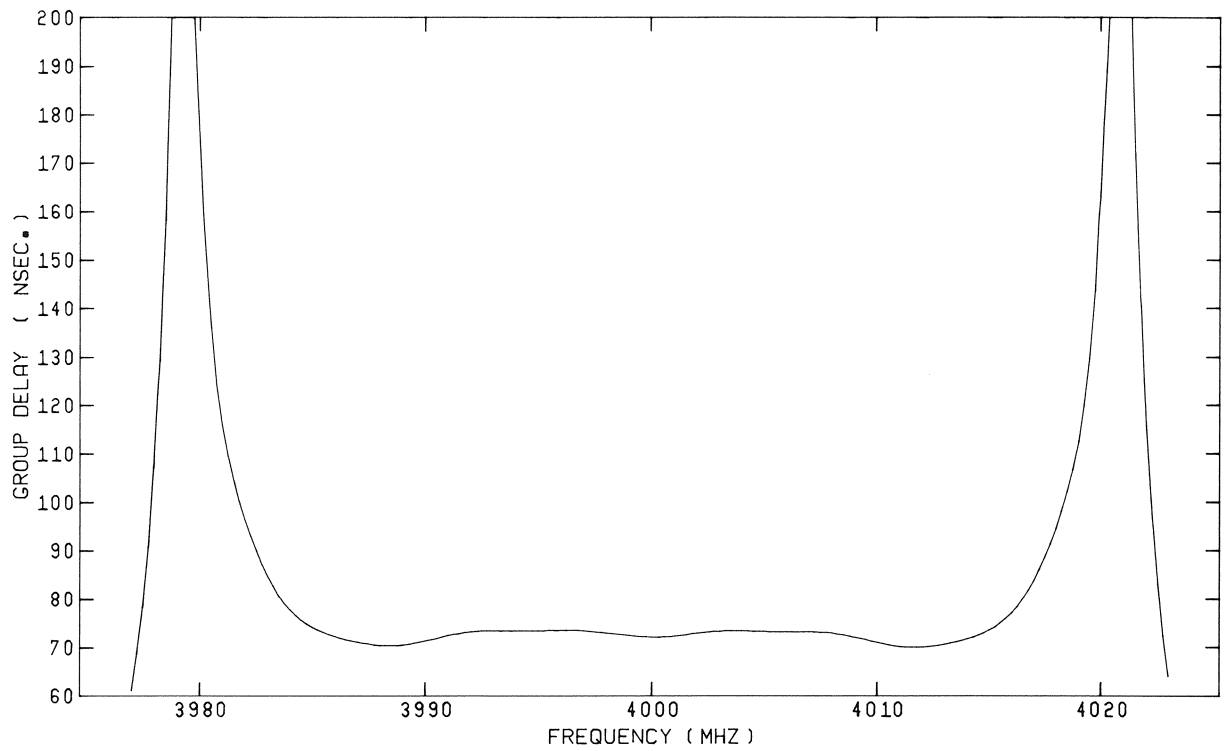
$$\begin{aligned}
 M_{12} &= M_{21} = M_{9,10} = M_{10,9} = 0.84424 \\
 M_{23} &= M_{32} = M_{89} = M_{98} = 0.59318 \\
 M_{34} &= M_{43} = M_{78} = M_{87} = 0.54438 \\
 M_{45} &= M_{54} = M_{67} = M_{76} = 0.53059 \\
 M_{56} &= M_{65} = 0.46916 \\
 M_{1,10} &= M_{10,1} = 0.01597 \\
 M_{29} &= M_{92} = -0.02673 \\
 M_{38} &= M_{83} = -0.05570 \\
 M_{47} &= M_{74} = 0.13067 \\
 n_1^2 &= n_2^2 = 1.04566
 \end{aligned}$$

The exact gradients required for optimization are provided using our first- and second-order sensitivity formulae. Some typical results of the sensitivities are shown in Table V.

In practical design optimization of networks in the frequency domain, the responses are discretized by selecting sample frequency points. This is an important but difficult task, since in many situations a poor selection may degenerate the efficiency and accuracy of the algorithm. We have adapted cubic interpolation formulae which use the exact derivatives w.r.t. frequency to detect and locate the ripples of the amplitude



(a)



(b)

Figure 4. Responses of the 10th-order quasi-elliptic self-equalized filter showing optimized amplitude and group delay: (a) return loss and insertion loss response; (b) group delay response

Table V. Some typical results of sensitivities

Frequency (MHz)	Response	Sensitivity w.r.t.				ω
		M_{12}	$M_{1,10}$	M_{56}	M_{28}^*	
3965	Insertion loss (dB)	22.1	-1365.5	-0.9	-313.9	0.49
3990	Return loss (dB)	189.7	-126.0	-278.5	-201.9	3.41
3995	Group delay (ns)	-20.1	46.5	1.3	-180.3	0.09

* M_{28} represents a stray coupling whose nominal value for a synchronously tuned design is zero. One of the advantages of the approach presented is that the sensitivities w.r.t. certain non-existing elements can be evaluated. This permits the prediction of the effect on responses due to small deviations of such elements.

response, whereby the sample frequency points can be automatically and optimally selected. As can be seen in Figure 4, an equal-ripple amplitude solution, optimal in the continuous sense, is virtually ensured.

Prediction of the effect of cavity dissipation

Ideal lossless models are often used to obtain nominal designs. In reality, the actual devices are subject to certain imperfections such as dissipation. The performance of a lossy filter can, of course, be re-evaluated by exact simulation. This, however, requires complex matrix analyses and, if different values of the quality factor are considered, such complex analyses have to be repeated for each Q . A much more efficient method of predicting the non-ideal response is using the sensitivities to obtain a first-order estimation. We have used this method to compute the response of the 10th-order filter given in the first example with $Q = 10,000$. Figure 5 shows the predicted passband insertion loss, which is indistinguishable from the exact simulation of the lossy filter (the numerical difference is less than 0.001 dB).

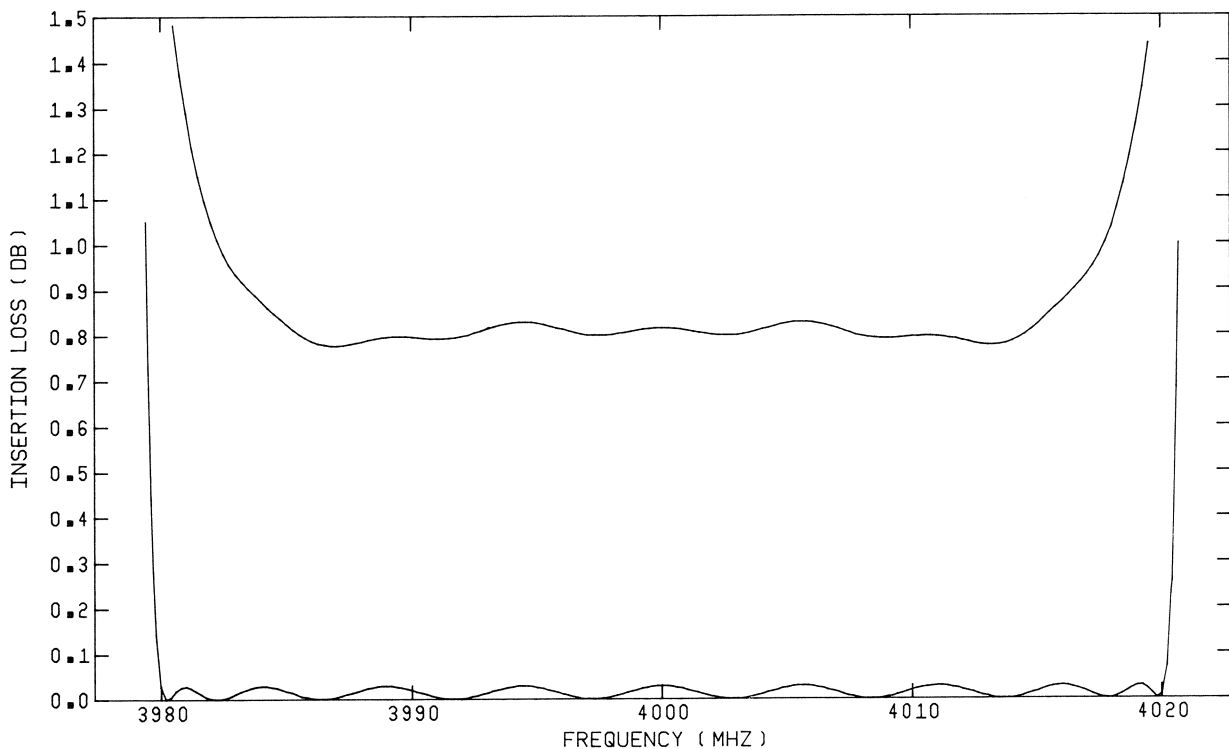


Figure 5. Passband insertion loss of the 10th-order filter example for $Q = 10,000$ and $Q = \infty$

It has been proved⁷ that the sensitivity of the amplitude w.r.t. dissipation is proportional to the group delay. Hence a filter with flat delay is also less sensitive to the dissipation. Figure 5 shows that with $Q = 10,000$ the insertion loss variation is less than 0.1 dB over 80 per cent of the passband.

Parameter identification using simulated measurements

Identification of network parameters from external measurements provides the necessary guidance for post-production tuning. The objective of such identification is to obtain by optimization an electrical equivalent circuit capable of reproducing frequency responses which correlate accurately with the measurements. An identification algorithm will necessarily involve extensive network analyses. Our simulation and sensitivity analysis approach, with its computational efficiency and flexibility, is most beneficial in the development of a practical identification algorithm. Such an algorithm has been implemented and tested on the following problem. It is desired to identify the parameters of the filter that deviate from their nominal values and are given by

$$M_{12} = M_{21} = M_{9,10} = M_{10,9} = 0.8$$

$$M_{23} = M_{32} = M_{89} = M_{98} = 0.6$$

$$M_{34} = M_{43} = M_{78} = M_{87} = 0.5$$

$$M_{45} = M_{54} = M_{67} = M_{76} = 0.5$$

$$M_{56} = M_{65} = 0.5$$

$$M_{1,10} = M_{10,1} = -0.1$$

$$M_{29} = M_{92} = 0.1$$

$$M_{38} = M_{83} = -0.2$$

$$M_{47} = M_{74} = 0.1$$

$$n_1^2 = n_2^2 = 1.0$$

using the amplitude responses shown in Figure 6 as simulated measurements.

Employing gradient-based optimization techniques and using starting values of initial guesses for variables that vary from the required solution by up to 50 per cent, all parameters are accurately identified.

6. CONCLUSION

An efficient and flexible approach to the simulation and exact sensitivity analysis of multi-coupled cavity filters has been presented. The basic theory of this approach has been derived, using an elegant and compact notation, for a two-port equivalent. The results actually apply to a class of general networks. By specializing this theory to the case of multi-coupled cavity filters, explicit formulae and tables have been given in a self-contained form. Since dissipation, dispersion and stray couplings have been accommodated by our filter model, the application is not restricted to an idealized situation. Also, synchronously or asynchronously tuned structures are treated in a unified manner. Simultaneously, special features of some particular cases have been exploited to minimize the computational effort. Illustrative examples of practical engineering problems solved by the actual implementation of our approach have also been provided. With its computational efficiency and structural flexibility, the approach presented provides, for the first time, a basis for the development of more advanced CAD software for automatic design, modelling and tuning of multi-coupled cavity filters and multiplexing networks.⁸ Such a prospect makes this work extremely attractive.

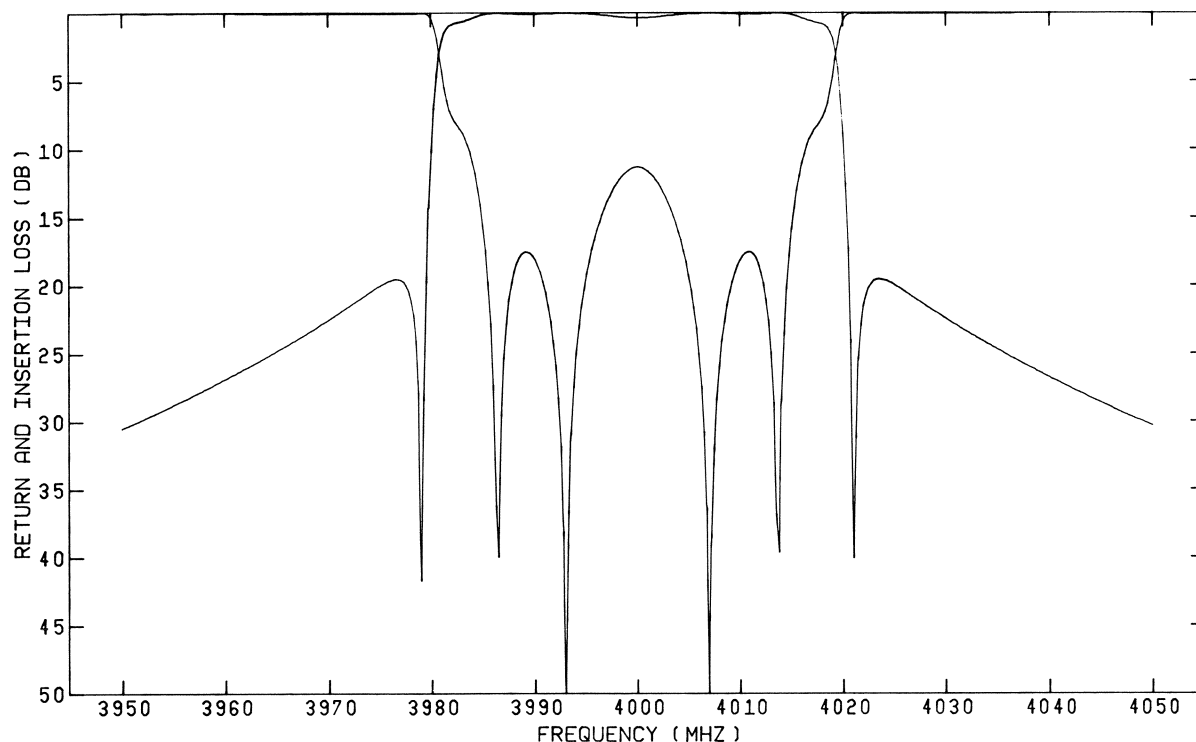


Figure 6. Return loss and insertion loss responses of a 10th-order detuned filter. The parameters capable of reproducing such responses are identified

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