

## MICROWAVE DEVICE MODELLING USING EFFICIENT $\ell_1$ OPTIMIZATION: A NOVEL APPROACH

J.W. Bandler\*, S.H. Chen and S. Daijavad

Simulation Optimization Systems Research Laboratory  
and Department of Electrical and Computer Engineering  
McMaster University, Hamilton, Canada L8S 4L7

### ABSTRACT

A powerful modelling technique which exploits the unique properties of the  $\ell_1$  norm is presented. Self-consistent models for passive and active devices are achieved by an approach that automatically checks the validity of model parameters obtained from optimization. Practical use of an efficient  $\ell_1$  algorithm in complicated problems for which gradient evaluation may not be feasible, is discussed. Examples in modelling of multi-coupled cavity filters and GaAs FET's are presented.

### INTRODUCTION

The problem of approximating a measured response by a network or system response has been formulated as an optimization problem w.r.t. the equivalent circuit parameters of a proposed model. The traditional approach in modelling is almost entirely directed at achieving the best possible match between measured and calculated responses. This approach has serious shortcomings in two frequently encountered cases. The first case is when the equivalent circuit parameters are not unique w.r.t. the responses selected and the second is when nonideal effects are not modelled adequately, the latter causing an imperfect match, even if small measurement errors are allowed for. In both cases, a family of solutions for circuit model parameters may exist which produce a reasonable and similar match between measured and calculated responses.

In this paper, we present a new formulation for modelling that automatically checks the validity of the circuit parameters, with a simultaneous attempt in matching measured and calculated responses. If successful, the method provides confidence in the validity of the model parameters, otherwise it proves their incorrectness. The use of the  $\ell_1$  norm, with its unique properties, is an integral part of the approach. We discuss the use of an efficient  $\ell_1$  algorithm [1-3] both in problems for which response gradients can be evaluated, and in complicated problems for which gradient evaluation is not feasible. The use of a gradient-based  $\ell_1$  algorithm and utilizing a variation of Broyden's formula to update gradients internally [3], makes it possible to employ a state-of-the-art optimization algorithm with any simulation package capable simply of providing responses. Therefore, widely used microwave design programs, e.g., SUPER-COMPACT [4] and TOUCHSTONE [5] which do not calculate exact gradients, could employ such an algorithm with an appropriate interface. As a result, it is

conceivable that the modelling technique described could find its way into microwave engineering practice in the near future.

### THE TRADITIONAL APPROACH

The traditional approximation problem is stated as follows

$$\underset{\mathbf{x}}{\text{minimize}} \|\mathbf{f}\| \quad (1)$$

where a typical component of vector  $\mathbf{f}$ , namely  $f_i$  evaluated at the frequency point  $\omega_i$ , is given by

$$f_i \triangleq w_i (F_i^c(\mathbf{x}) - F_i^m), \quad i=1,2,\dots,k. \quad (2)$$

$F_i^m$  is a measured response at  $\omega_i$  and  $F_i^c$  is the response of an appropriate network which depends nonlinearly on a vector of model parameters  $\mathbf{x} \triangleq [x_1 \ x_2 \ \dots \ x_n]^T$  and  $w_i$  denotes a nonnegative weighting factor.  $\|\cdot\|$  denotes an appropriate norm.

It is usually assumed that the expected values of the components of  $\mathbf{f}$  are zero. However, this cannot be realized in practice due to the presence of measurement errors in observing  $F_i^m$  and, more importantly, as a result of the imperfections and nonideal effects which may not have been accounted for in the topology of the equivalent circuit. Since the components of  $\mathbf{f}$  generally cannot go to zero simultaneously, different norms of  $\mathbf{f}$  may give different results for  $\mathbf{x}$ .

### NEW APPROACH USING MULTIPLE SETS OF MEASUREMENTS

The model parameters  $\mathbf{x}$  are generally controlled by some physical parameters  $\boldsymbol{\phi} \triangleq [\phi_1 \ \phi_2 \ \dots \ \phi_\ell]^T$ . For instance, in active device modelling intrinsic network parameters are controlled by bias voltages or currents. The actual functional relationship between  $\boldsymbol{\phi}$  and  $\mathbf{x}$  is usually not known, however, we often know which element or elements of  $\mathbf{x}$  are affected by an adjustment on an element of  $\boldsymbol{\phi}$ . We exploit this knowledge to propose the following formulation.

Suppose that after taking measurements on a microwave device at a number of frequency points, we make an easy-to-achieve adjustment on an element of  $\boldsymbol{\phi}$  such that one or a few components of  $\mathbf{x}$  are changed in a dominant fashion and the rest remain constant or change slightly. Consider the  $\ell_1$  optimization problem

$$\underset{\mathbf{x}}{\text{minimize}} \sum_{t=1}^2 \sum_{i=1}^{k_t} |f_i^t| + \sum_{j=1}^n \beta_j |x_j^1 - x_j^2| \quad (3)$$

where

$$f_i^t \triangleq w_i^t [F_i^c(\mathbf{x}^t) - (F_i^m)^t], \quad (4)$$

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada under Grant G1135.

\* J.W. Bandler is also with Optimization Systems Associates, 163 Watson's Lane, Dundas, Ontario, Canada L9H 6L1

with superscript  $t$  identifying the original network model ( $t=1$ ) or the model after adjustment on  $\Phi$  ( $t=2$ ).  $\beta_j$  represents an appropriate weighting factor and  $k_t$  is an index whose value depends on  $t$ , i.e., a different number of frequencies may be used for the original and the perturbed model.  $\mathbf{x}$  is a vector which contains circuit parameters of both the original and perturbed networks, i.e.,

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}^1 \\ \mathbf{x}^2 \end{bmatrix}. \quad (5)$$

The above formulation has the following properties:

- 1) Considering only the first part of the objective function, the formulation is equivalent to performing two optimizations, i.e., matching the calculated response of the original circuit model with its corresponding measurements and repeating the procedure for the perturbed circuit.
- 2) By adding the second part to the objective function, we take advantage of the knowledge that only one or a few components of  $\mathbf{x}$  should change dominantly by perturbing a component of  $\Phi$ . Therefore, we penalize the objective function for any change in  $\mathbf{x}$ . However, since the  $\ell_1$  norm is used, one or a few large changes in  $\mathbf{x}$  are still allowed.

The confidence in the validity of the equivalent circuit parameters increases if a) an optimization using the objective function of (3) results in a reasonable match between calculated and measured responses for both circuits 1 and 2 (original and perturbed) and b) the examination of the solution vector  $\mathbf{x}$  reveals changes from  $\mathbf{x}^1$  to  $\mathbf{x}^2$  which are consistent with the adjustment on  $\Phi$ , i.e., only the expected components have changed significantly. We can build upon our confidence even more by generalizing the technique to more adjustments on  $\Phi$ , i.e., formulating the optimization problem as

$$\underset{\mathbf{x}}{\text{minimize}} \quad \sum_{t=1}^{n_c} \sum_{i=1}^{k_t} \left| r_i^t \right| + \sum_{t=2}^{n_c} \sum_{j=1}^n \beta_j^t \left| x_j^1 - x_j^t \right|, \quad (6)$$

where  $n_c$  circuits and their corresponding sets of responses, measurements and parameters are considered and the first circuit is the reference model before any adjustment on  $\Phi$ .

By observing inconsistencies in changes of  $\mathbf{x}$  with the actual change in  $\Phi$ , the new technique exposes the existence of nonideal effects not taken into account in the model. Having confidence in parameters as well as observing a good match between measured and modelled responses means that the parameters and the model are valid even if different responses or different frequency ranges are used.

## PRACTICAL APPLICATION OF THE $\ell_1$ ALGORITHM

The  $\ell_1$  optimization problem is formulated in (6). The success of the new technique described relies upon the use of an efficient and robust  $\ell_1$  algorithm. Recently, a superlinearly convergent algorithm for nonlinear  $\ell_1$  optimization has been described [1]. The algorithm, based on the original work of Hald and Madsen [2], is a combination of a first-order method that approximates the solution by successive linear programming and a quasi-Newton method using approximate second-order information to solve the system of nonlinear equations resulting from the first-order necessary conditions for an optimum.

The most efficient use of the  $\ell_1$  algorithm requires the user to supply function and gradient values of the individual functions in (6), i.e., network responses as well as their gradients are needed. Starting with the impedance or nodal admittance description of a network model for which only input

and output port responses are of interest, we have derived analytical formulas for evaluation of first-order sensitivities of two port S-parameters w.r.t. any circuit parameter appearing in the impedance or admittance matrix. The formulas have been used in the GaAs FET modelling example which follows.

In many practical problems, e.g., in the presence of nonlinear devices or complicated field problems, the evaluation of gradients is not feasible. In such cases, it is possible to estimate the gradients using the numerical difference method. However, this is computationally slow and consequently expensive. To take advantage of a fast gradient-based approach without requiring user-supplied gradients or using the numerical difference method, the original  $\ell_1$  algorithm has been modified [3]. Different and flexible versions of the modified algorithm exist. A typical version estimates the gradients using the numerical difference method only once and updates the gradients with minimum extra effort by applying a variation of Broyden's formula as the optimization proceeds. All approximations are performed internally, therefore, the optimization could be linked to any analysis program which provides only the responses.

## EXAMPLES

### A. Modelling of Multi-Coupled Cavity Filters

A 6th order multi-coupled cavity filter centered at 11785.5 MHz with a 56.2 MHz bandwidth is considered. Measurements on input and output return loss, insertion loss and group delay of an optimally tuned filter and the same filter after a deliberate adjustment on the screw which dominantly controls coupling  $M_{12}$ , were provided by ComDev Ltd., Cambridge, Canada [6]. Although the passband return loss changes significantly, we anticipate that such a physical adjustment affects only model parameters  $M_{12}$ ,  $M_{11}$  and  $M_{22}$  (the last two correspond to cavity resonant frequencies) in a dominant fashion, possibly with slight changes in other parameters.

Using the new technique described in this paper, we simultaneously processed measurements on passband return loss (input reflection coefficient with a weighting of 1), and stopband insertion loss (with a weighting of 0.05) of both filters, i.e., the original and perturbed models. The  $\ell_1$  algorithm with exact gradients was used. The evaluation of sensitivities is discussed in detail by Bandler et al. [7]. The model parameters identified for two filters are summarized in Table I. Figs. 1 and 2 illustrate the measured and modelled responses of the original filter. Fig. 3 shows the measured and modelled input return loss for the filter after adjustment. An examination of results in Table I and Figs. 1-3 shows that not only an excellent match between measured and modelled responses has been achieved, but also the changes in parameters are completely consistent with the actual physical adjustment. Therefore, by means of only one optimization, we have built confidence in the validity of equivalent circuit parameters. The problem involved 84 nonlinear functions ( $42 \times 2$  responses for original and perturbed filters) and 12 linear functions (change in parameters of two circuit equivalents) and 24 variables. The solution was achieved in 72 seconds of CPU time on the VAX 11/780 system.

### B. GaAs FET Modelling

Using S-parameter data for the device B1824-20C from 4 to 18 GHz, Curtice and Camisa have achieved a very good model for the FET chip [8]. They have used the traditional least squares optimization of responses utilizing SUPER-COMPACT. Their success is due to the fact that they have reduced the number of possible variables from 16 to 8 by using dc and zero-bias measurements. We used their equivalent circuit at normal

TABLE I  
RESULTS FOR THE 6TH ORDER FILTER EXAMPLE

Coupling	Original Filter	Perturbed Filter	Change in Parameter
M <sub>11</sub>	-0.0473	-0.1472	-0.0999*
M <sub>22</sub>	-0.0204	-0.0696	-0.0492*
M <sub>33</sub>	-0.0305	-0.0230	0.0075
M <sub>44</sub>	0.0005	0.0066	0.0061
M <sub>55</sub>	-0.0026	0.0014	0.0040
M <sub>66</sub>	0.0177	-0.0047	-0.0224
M <sub>12</sub>	0.8489	0.7119	-0.1370*
M <sub>23</sub>	0.6064	0.5969	-0.0095
M <sub>34</sub>	0.5106	0.5101	-0.0005
M <sub>45</sub>	0.7709	0.7709	0.0000
M <sub>56</sub>	0.7898	0.7806	-0.0092
M <sub>36</sub>	-0.2783	-0.2850	-0.0067

\* significant change in parameter value

operating bias (including the carrier), as illustrated in Fig. 4, and created artificial measurements using TOUCHSTONE. Two sets of S-parameter measurements were created; one set using the parameters reported by Curtice and Camisa (operating bias  $V_{ds} = 8.0$  V,  $V_{gs} = -2.0$  V and  $I_{ds} = 128.0$  mA) and the other by changing the values of  $C_1$ ,  $C_2$ ,  $L_g$  and  $L_d$  to simulate the effect of taking different reference planes for the carriers. Both sets of data are shown in Fig. 5, where the S-parameters of the two circuits are plotted on a Smith Chart. Using the technique described in this paper, we processed the measurements on the two circuits simultaneously by minimizing the function defined in (3). The objective of this experiment is to show that even if the equivalent circuit parameters were not known, as is the case using real measurements, the consistency of the results would be proved only if the intrinsic parameters of the FET remain unchanged between the two circuits. This was indeed the case for the experiment performed. Although the maximum number of possible variables, namely 32 (16 for each circuit), were allowed for in the optimization, the intrinsic parameters were found to be the same between the two circuits and, as expected,  $C_1$ ,  $C_2$ ,  $L_g$  and  $L_d$  changed from circuit 1 to 2. Table II summarizes the parameter values obtained. The problem involved 128 nonlinear functions (real and imaginary parts of 4 S-parameters, at 8 frequencies, for two circuits), 16 linear functions and 32 variables. The CPU time on the VAX 11/780 system was 79 seconds.

### CONCLUSIONS

We have described a new technique for modelling of microwave devices which exploits multi-circuit measurements. The use of the  $\ell_1$  norm is an integral part of the approach. We discussed the use of an efficient  $\ell_1$  algorithm in problems for which gradient evaluation may not be feasible, or in conjunction with widely used analysis programs.

The results for modelling of narrowband multi-coupled cavity filter and wideband GaAs FET examples are promising. The authors believe that the use of multiple sets of measurements and a formulation which ties modelling to the actual physical adjustments on the device, will enhance further developments in modelling and tuning of microwave devices.

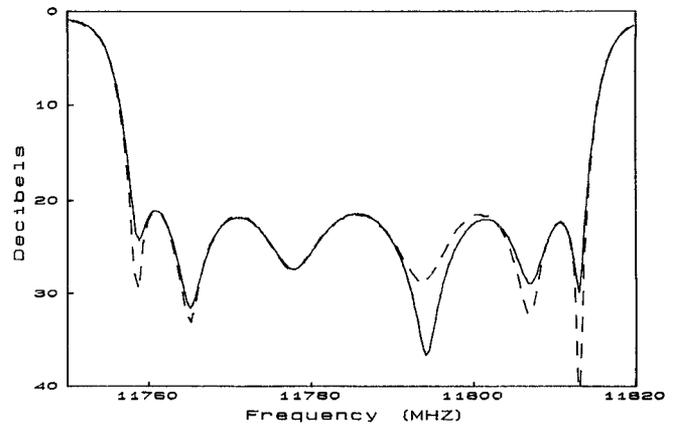


Fig 1 Input return loss for the filter before adjusting the screw. Solid line represents the modelled response and dashed line shows measurement data.

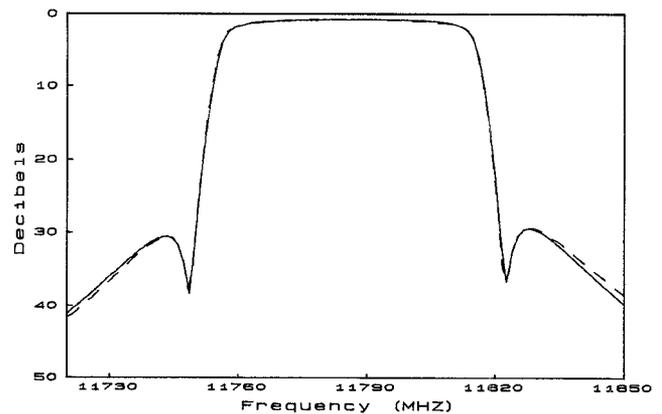


Fig 2 Insertion loss for the filter before adjusting the screw. Solid line represents the modelled response and dashed line shows measurement data.

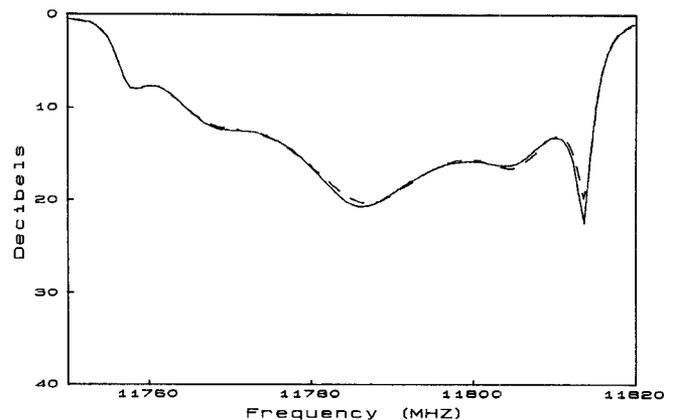


Fig. 3 Input return loss for the filter after adjusting the screw. Solid line represents the modelled response and dashed line shows measurement data.

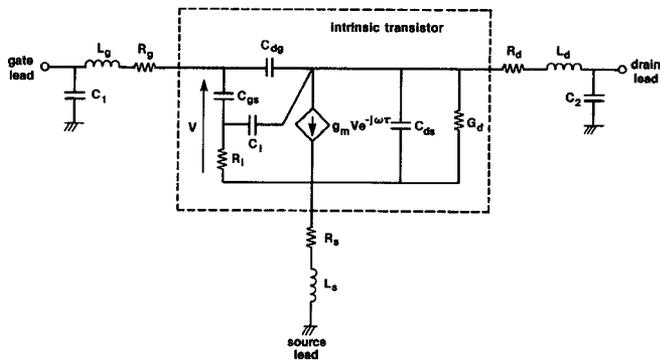


Fig 4 Equivalent circuit of carrier-mounted FET.

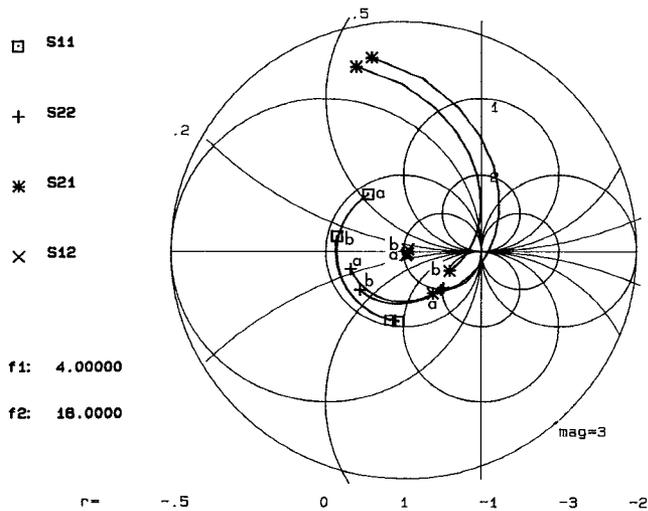


Fig. 5 Smith Chart display of  $S_{11}$ ,  $S_{22}$ ,  $S_{12}$  and  $S_{21}$ , for the carrier-mounted FET, before and after adjustments on parameters. Points a and b mark the high frequency end of original and perturbed network responses, respectively.

#### REFERENCES

[1] J.W. Bandler, W. Kellermann and K. Madsen, "A superlinearly convergent algorithm for nonlinear  $\ell_1$  optimization with circuit applications", *Proc. IEEE Int. Symp. Circuits and Systems* (Kyoto, Japan, 1985), pp. 977-980.  
 [2] J. Hald and K. Madsen, "Combined LP and quasi-Newton methods for nonlinear  $\ell_1$  optimization", *SIAM J. Numerical Analysis*, vol. 22, 1985, pp 68-80.

TABLE II  
RESULTS FOR THE GaAs FET EXAMPLE

Parameter	Original Circuit	Perturbed Circuit
$C_1$ (pF)	0.0440	0.0200*
$C_2$ (pF)	0.0389	0.0200*
$C_{dg}$ (pF)	0.0416	0.0416
$C_{gs}$ (pF)	0.6869	0.6869
$C_{ds}$ (pF)	0.1900	0.1900
$C_i$ (pF)	0.0100	0.0100
$R_g$ ( $\Omega$ )	0.5490	0.5490
$R_d$ ( $\Omega$ )	1.3670	1.3670
$R_s$ ( $\Omega$ )	1.0480	1.0480
$R_i$ ( $\Omega$ )	1.0842	1.0842
$G_d^{-1}$ (k $\Omega$ )	0.3761	0.3763
$L_g$ (nH)	0.3158	0.1500*
$L_d$ (nH)	0.2515	0.1499*
$L_s$ (nH)	0.0105	0.0105
$g_m$ (S)	0.0423	0.0423
$\tau$ (ps)	7.4035	7.4035

\* significant change in parameter value

[3] J.W. Bandler, and S.H. Chen, S. Daijavad and K. Madsen, "Efficient gradient approximations for nonlinear optimization of circuits and systems", *Proc. IEEE Int. Symp. Circuits and Systems* (San Jose, CA), 1986.  
 [4] COMSAT General Integrated Systems, Inc., SUPER-COMPACT Users Manual, 1982.  
 [5] TOUCHSTONE, EEsof, Inc., Westlake Village, CA, 1985.  
 [6] ComDev Ltd., 155 Sheldon Drive, Cambridge, Ontario, Canada N1R 7H6, private communications, 1985.  
 [7] J. W. Bandler, S.H. Chen and S. Daijavad, "Exact sensitivity analysis for optimization of multi-coupled cavity filters", *Int. J. Circuit Theory and Applic.* vol. 14, 1986.  
 [8] W.R. Curtice and R.L. Camisa, "Self-consistent GaAs FET models for amplifier design and device diagnostics", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, 1984, pp 1573-1578.