COMPUTATION OF SENSITIVITIES

FOR OPTIMAL DESIGN OF MICROWAVE NETWORKS

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Introduction

In recent contributions [1, 2], Director and Rohrer discussed the concept of the adjoint network and its relevance to automated design of networks in the frequency and time domains. Employing Tellegen's theorem [3, 4] they demonstrated how the gradient vector for a least squares type of response objective function with respect to all existing (and nonexisting, if desired) elements could be evaluated from only two complete analyses, one of the given network and one of its topologically equivalent adjoint network. In the frequency domain [2] they considered both reciprocal and nonreciprocal lumped, linear and time invariant elements. More recently [5], it was shown how their approach could be implemented for least pth and minimax response objective functions[6].

The purpose of this paper is to show how the adjoint network approach may be used to advantage in gradient calculations for a broad class of multiport commensurate and noncommensurate structures of interest to microwave engineers. The results can then be incorporated into an automatic optimization algorithm in which such functions as gain, insertion loss, reflection coefficient or any other desired response function can be optimized to meet least pth or minimax performance specifications. The essence of this approach is that all required partial derivatives of the objective function may be obtained from the results of at most two complete analyses of the network at each frequency regardless of the number of variable parameters and without actually perturbing them. The computational inefficiency and uncertainty inherent in the numerical estimation of partial derivatives by perturbation could be circumvented and the use of efficient gradient methods of minimization [6] could more profitably be exploited in producing optimal designs.

Numerical Example

Referring to Figure 1 suppose we have to minimize

$$U = \sum_{\Omega_d} \frac{1}{p} \left| L(\omega_d) - \hat{L}(\omega_d) \right|^p \qquad (1)$$

where L is the insertion loss between R_g and R_L

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$\hat{L}$ is the desired insertion loss between R_g and R_L

Ω_d is a set of discrete frequencies ω_d

p is a positive integer

that is to say to approximate a specified insertion loss function in a least pth sense over a set of frequencies in the range of interest, where

$$L(\omega_d) = -20 \log_{10} \left| \frac{I_L(j\omega_d)}{V_g(j\omega_d)} \right| (R_g + R_L) \qquad (2)$$

Consider the noncommensurate network shown in Figure 2 having 13 variables. An objective function of the form of (1) was chosen with $\hat{L} = 0$, $p = 10$ and Ω_d consisting of .5, .6, .7, .8, .9 and 1.0 GHz. For the element values shown in Figure 2 $U = 3.04383 \times 10^8$. Table I shows the components of the gradient vector ∇U estimated from 1% and .001% incremental changes in the parameters compared with those obtained using the adjoint network approach with two network analyses.

It is readily shown that for design on the reflection coefficient basis only one analysis is required. In the case of lossless two-ports (such as the present example) it may thus be preferable to design on this basis rather than insertion loss.

Conclusions

Growing new elements in the manner of Director and Rohrer[2] can also be envisaged. One has to be more careful at microwave frequencies, however, concerning the location and in choosing the nature of the element, i.e., distributed or lumped. It may be more convenient in some instances to obtain the partial derivatives with respect to geometrical dimensions rather than characteristic impedance or admittance. Since Tellegen's theorem can also be written in terms of wave variables we can carry out the preceding analysis in terms of the scattering matrix.

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References

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TABLE I

Element			Gradient Components		
Type	Parameter	Value	1% increment	.001% increment	Adjoint Network
parallel capacitor	C	2pF	1.416×10^7	1.3756×10^7	1.3755×10^7
parallel s.c. line	Y	.0125 mho	-7.324×10^9	-7.3952×10^9	-7.3952×10^9
	ℓ	6 cm	1.884×10^7	1.8510×10^7	1.8509×10^7
transmission line	Z	25Ω	-1.166×10^8	-1.2276×10^8	-1.2276×10^8
	ℓ	8 cm	-1.275×10^7	-1.3300×10^7	-1.3300×10^7
parallel inductor	Γ	.1 $(nH)^{-1}$	-3.178×10^9	-3.2166×10^9	-3.2167×10^9
parallel capacitor	C	3pF	6.563×10^7	6.5130×10^7	6.5130×10^7
series o.c. line	Z	40Ω	3.439×10^7	3.3764×10^7	3.3763×10^7
	ℓ	5 cm	-4.408×10^8	-4.5832×10^8	-4.5834×10^8
parallel capacitor	C	4pF	3.296×10^8	3.2384×10^8	3.2384×10^8
transmission line	Z	50Ω	-4.191×10^6	-4.2482×10^6	-4.2483×10^6
	ℓ	1 cm	1.393×10^8	1.3932×10^8	1.3932×10^8
series inductor	L	3nH	3.442×10^6	3.3200×10^6	3.3199×10^6

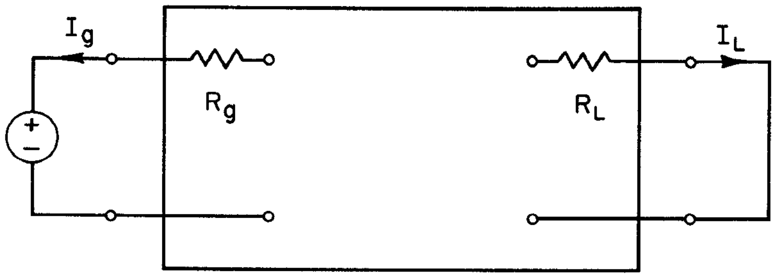

Figure 1 Network for insertion loss design.

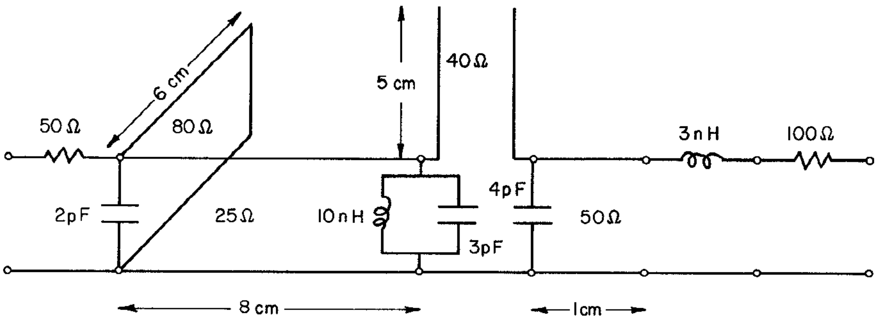

Figure 2 Noncommensurate network having 13 variables terminated in 50Ω and 100Ω.