MM-1

GRADIENT QUADRATIC APPROXIMATION SCHEME FOR YIELD-DRIVEN DESIGN

J.W. Bandler*, R.M. Biernacki*, S.H. Chen*, J. Song, S. Ye and Q.J. Zhang**

Simulation Optimization Systems Research Laboratory
and Department of Electrical and Computer Engineering
McMaster University, Hamilton, Canada L8S 4L7

ABSTRACT

A new approach to modeling of circuit responses and gradients is proposed. We exploit multidimensional quadratic approximation and take full advantage of available gradient information. Efficiency and accuracy are demonstrated by gradient-based yield optimization of a filter and an MMIC amplifier.

INTRODUCTION

Yield-driven design has become widely accepted as a necessary design tool to decrease manufacturing cost [1,2]. However, the existing design approaches are computationally intensive and, therefore, for large-scale problems the effort required can be prohibitive.

In this paper, we propose to utilize an efficient quadratic approximation scheme [3,4] to replace the expensive repeated circuit simulations and gradient evaluations, in order to speed up the optimization process. The novelty of this utilization is that not only circuit performance functions, but also their gradients are approximated. In a gradient-based optimization procedure, such as the one-sided ℓ_1 centering approach [5], gradient information is critical in determining the direction for optimization iterations to follow. Higher gradient accuracy will improve the overall performance of the optimization process.

EFFICIENT QUADRATIC APPROXIMATION

The quadratic model to be used to approximate a response or a gradient function f(x), $x = [x_1\ x_2\ \ldots\ x_n]^T$, is an interpolating polynomial of the form

$$q(x) = a_0 + \sum_{i=1}^{n} a_i(x_i - r_i) + \sum_{\substack{j,i=1\\ j\geq i}}^{n} a_{ij}(x_i - r_i)(x_j - r_j), \qquad (1)$$

where

$$\mathbf{r} = [r_1\ r_2\ \ldots\ r_n]^T \qquad (2)$$

is a known reference point. Using actual circuit simulation, the function f(x) is evaluated at points x^i, i = 1, 2, . . ., m, where m > n+1. These points are called the base points. Using $f(x^i)$, we set up the system of linear equations

$$\begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \end{bmatrix}, \qquad (3)$$

where

$$\mathbf{a} = [a_0\ a_1\ a_2\ \ldots\ a_n]^T \qquad (4)$$

and

$$\mathbf{v} = [a_{11}\ a_{22}\ \ldots\ a_{nn}\ a_{12}\ a_{13}\ \ldots\ a_{n-1,n}]^T, \qquad (5)$$

respectively. The vectors $\mathbf{f}_1$ and $\mathbf{f}_2$ contain the function values $f(x^i)$, and the matrices $\mathbf{Q}_{ij}$, i, j = 1, 2, are determined from the coordinates of the base points. If m < (n+1)(n+2)/2 (which is normally the case since we want to perform as few actual circuit simulations as possible) the above system is under-determined. As pointed out in [3], when the least-squares constraint is applied to $\mathbf{v}$, a unique solution to (3) can be found and that solution is called the maximally flat quadratic interpolation.

Following the approach proposed in [4], we use only m, where $n+1 < m \leq 2n+1$, base points. The reference point r is selected as the first base point x^1. The next n base points are selected by perturbing one variable at a time around r, i.e.,

$$x^{i+1} = \mathbf{r} + [0\ \ldots\ 0\ \beta_i\ 0\ \ldots\ 0]^T,\ i = 1, 2, ..., n, \qquad (6)$$

where β_i is a predetermined perturbation. The remaining m-(n+1) points follow to provide the second-order information on the function. They are

$$x^{n+1+i} = \mathbf{r} + [0\ \ldots\ 0\ \gamma_i\ 0\ \ldots\ 0]^T,\ i = 1, 2, ..., m-(n+1), \qquad (7)$$

where γ_i is another perturbation of r_i, which must not equal β_i. This particular arrangement of base points leads to simple, closed-form formulas for determining the coefficients in (4) and (5). Efficiency of the approach is unsurpassed and the computational effort increases only linearly with the number of variables n. Additionally, the resulting maximally flat quadratic interpolation has the property that the coefficients a_{ij} for $i \neq j$ are zero.

QUADRATIC APPROXIMATION TO RESPONSES AND GRADIENTS

In [3] and [4], only circuit responses are modeled by quadratic functions. The gradients of the responses are either not used or their approximate values are calculated by differentiating the quadratic approximate responses. To further improve the performance of the gradient-based yield-driven optimization, more accurate gradients are preferable.

Consider a response function with n variables. The gradient of the response is a vector of functions of the same n variables, each of the functions being the partial derivative of the response w.r.t. one *designable* variable. In yield optimization we typically

MM

* J.W. Bandler, R.M. Biernacki and S.H. Chen are also with Optimization Systems Associates Inc., P.O. Box 8083, Dundas, Ontario, Canada L9H 5E7.

** Q.J. Zhang is now with the Department of Electronics, Carleton University, Ottawa, Canada K1S 5B6.

deal with three types of variables, namely, n_{DS} designable variables x_{DS} with statistics, n_D designable variables x_D without statistics, and n_S non-designable variables x_S which are subject to statistical variations. Suppose that there are k responses, R_i, i = 1, 2, ..., k. The gradients of the responses with respect to the designable variables are

$$\nabla R_i = [(\partial R_i/\partial x^0_{DS})^T \ (\partial R_i/\partial x^0_D)^T]^T, \ i = 1, 2, ..., k, \quad (8)$$

where x^0 stands for the nominal values and the dimension of the gradient vector is $(n_{DS} + n_D)$.

For yield-driven design, circuit responses and their gradients have to be evaluated at a number of statistical outcomes. Each statistical outcome is generated in a $(n_{DS} + n_S)$-dimensional space according to a known statistical distribution and can be expressed as

$$[x_{DS}^T \ x_S^T]^T = [x^0_{DS}{}^T \ x^0_S{}^T]^T + [\Delta x_{DS}^T \ \Delta x_S^T]^T, \quad (9)$$

where Δx_{DS}^T and Δx_S^T are outcome specific deviates from the nominal values. Because of a large number of statistical outcomes needed for a meaningful yield estimate the main saving of the computational effort is achieved by building the models in the $(n_{DS} + n_S)$-dimensional space of the statistical variables (9). In other words, we consider (9) as the variables in the quadratic model, that is, the vector x in (1) is

$$x = [x_{DS}^T \ x_S^T]^T. \quad (10)$$

Locality of statistical spreads assures a good level of model accuracy. The models are built for the current (optimization specific) nominal point and utilized for as many statistical outcomes as desired. In addition to the response functions, each entry to the gradient vectors can be approximated by a separate quadratic function in a similar manner as the response functions are. Thus, for k responses there is a total of $k(1 + n_{DS} + n_D)$ functions to be approximated, i.e.,

$$[R_1 \ \nabla R_1^T \ R_2 \ \nabla R_2^T \ ... \ R_k \ \nabla R_k^T]^T. \quad (11)$$

It should be pointed out that, if the adjoint technique is used, the gradient can be available at a low additional cost to the circuit simulation, and can be returned from the simulator regardless of whether it is utilized or not. For example, the *FAST* technique [6] requires very little additional computational effort to evaluate sensitivities in the harmonic balance environment [7]. Therefore, the proposed method can not only utilize information that would otherwise be lost, but also allows for reduction of the model dimensionality by n_D, as is clearly seen from (10).

The resulting quadratic model for the gradient is more accurate than the one that could be obtained by differentiating the quadratic model of the response, because the partial second-order information is incorporated in the model.

EXAMPLES

The proposed quadratic approximation technique has been implemented [8]. $2(n_{DS} + n_S) + 1$ base points, defined by (6) and (7), are used. An interface has been developed for the response and gradient approximation module which is very flexible in dealing with different types of variables involved in yield-driven design. The following examples illustrate efficiency and accuracy of the proposed method.

A. 13-Element Low-Pass Filter

The low-pass filter shown in Fig. 1 [9] is considered. The circuit must meet the specifications: insertion loss less than 0.4dB at the angular frequencies

{0.25, 0.27, 0.29, 0.31, 0.33, 0.67, 0.69, 0.71, 0.73, 0.75, 0.90, 0.905, 0.91, 0.92, 0.93, 0.978, 0.981, 0.984, 0.986, 0.988, 1},

and greater than 49dB at

{1.04569, 1.056, 1.059, 1.063, 1.067, 1.071, 1.115}.

There are 13 design variables. A normal distribution with 0.5% standard deviation is assumed for all variables. The starting point is the optimal minimax solution, which has an estimated yield of 33.4%. To illustrate the efficiency of the new quadratic approximation approach, we solve the problem using both approximate simulations from the quadratic model and exact simulations. This problem involves 28 frequency points and 100 statistical outcomes, resulting in 2800 error functions. The final yields for both approaches are 75.6 and 80.7%. Computational details are given in Table I. CPU times for the two designs were 7 and 30 minutes, respectively.

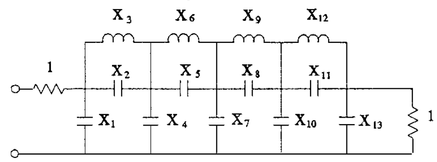


Fig. 1. Circuit schematic of the LC 13-element filter [9].

B. Two-Stage GaAs MMIC Feedback Amplifier

We consider a two-stage 2-6GHz GaAs MMIC feedback amplifier [2]. The equivalent circuit model for the FET and the circuit are shown in Fig. 2. The specifications are a small-signal gain of 8dB±1dB, VSWR at the input port of less than 2, and VSWR at the output port less than 2.2. A total of 9 sampling frequency points equally spaced with the step of 0.5GHz are used.

It is intended to manufacture high-volume, high-yield, and, consequently, low-cost microwave op amps. The size of the IC has a strong effect on the cost. Therefore, we consider the mean values of most capacitors as fixed to keep the size of the chip reasonable. The mean value of the gate width is fixed because of the assumed FET process, but a 3% standard deviation is allowed. Since the RF responses are not very sensitive to changes in the bias resistors, no tolerances are assigned to the resistors. Two feedback resistors and a forward capacitor are chosen as design variables. A standard deviation of 2% is assumed for the design variables. For nominal values and standard deviations of other elements, see Table II.

The first step in the entire optimization procedure is to find a minimax solution as the starting point for yield-driven design. The minimax solution is found and listed in Table II. The yield estimate at this point is 32.1%. Two yield-driven optimization processes are carried out with and without the new quadratic approximation. Two solutions and the final yields are given in Table III. The actual yields based on a Monte Carlo analysis of 1000 outcomes are 77.8% and 77.3%, respectively. The corresponding CPU times are 9 and 39 minutes.

TABLE I

YIELD OPTIMIZATION OF THE LC 13-ELEMENT FILTER WITH AND WITHOUT QUADRATIC APPROXIMATIONS

Parameter	Initial	Solution†	Solution††
x_1	0.2088	0.2145	0.2205
x_2	0.03594	0.03642	0.03929
x_3	0.1822	0.1800	0.1775
x_4	0.2340	0.2347	0.2266
x_5	0.2424	0.2426	0.2556
x_6	0.08776	0.08702	0.08426
x_7	0.1333	0.1290	0.1234
x_8	0.3549	0.3535	0.3551
x_9	0.06477	0.06496	0.06481
x_{10}	0.1674	0.1625	0.1561
x_{11}	0.1422	0.1435	0.1498
x_{12}	0.1140	0.1120	0.1098
x_{13}	0.1433	0.1414	0.1303
Yield Estimate	33.4%	75.6%	80.7%
CPU*		7min.	30min.

† The solution after one phase of yield optimization with quadratic approximation.
†† The solution after one phase of yield optimization with exact simulations and numerical gradients.
* On the Sun SPARCstation 1.

Comments: Normal distribution of $\sigma = 0.5\%$ is assumed for all parameters. 100 outcomes are used in the optimization. 1000 outcomes are used in the yield estimation.

Parameters are scaled down by the factor 2π, e.g., the actual element value of x_1 is $2\pi \times 0.2088$.

TABLE II

PARAMETER VALUES AND TOLERANCES FOR THE MMIC AMPLIFIER

Element Parameter	Mean Value	Standard Deviation
Z(μm)	300	3%
$R_4(\Omega)$	400	0%
C_5(pF)	4	2%
$R_6(\Omega)$	20	2%
C_7(pF)	10	2%
$R_8(\Omega)$	145	2%
$R_9(\Omega)$	2200	0%
C_{10}(pF)	4	2%
$R_{11}(\Omega)$	6000	0%
$R_{12}(\Omega)$	500	2%
C_{13}(pF)	10	2%

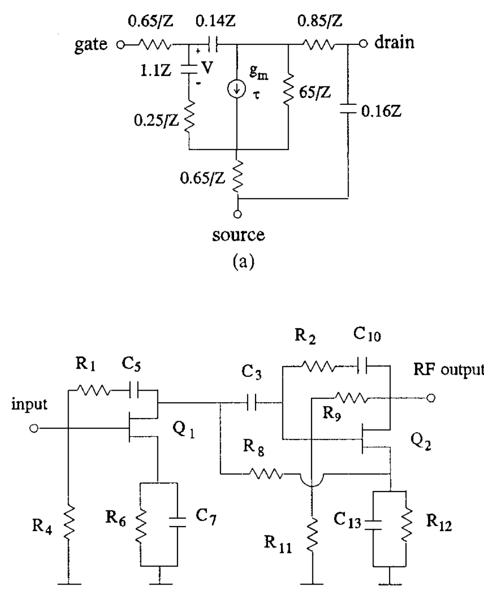
TABLE III

YIELD OPTIMIZATION OF THE MMIC AMPLIFIER WITH AND WITHOUT QUADRATIC APPROXIMATIONS

Parameter	Initial	Solution†	Solution††
R_1	201.02	207.63	207.73
R_2	504.82	627.94	630.53
C_3	5.3501	2.7742	2.7563
Yield Estimate	32.1%	77.8%	77.3%
CPU*		9min.	39min.

† The solution after one phase of yield optimization with quadratic approximations.
†† The solution after one phase of yield optimization with exact simulations and numerical gradients.
* On the Sun SPARCstation 1.

Comments: 100 outcomes are used in the optimization. 1000 outcomes are used in the yield estimation.



(b)

Fig. 2. (a) Normalized GaAs MESFET model [2]. Z is the gate width in millimeters. $g_m = 0.17Z$, and $\tau = 2.5$ps. All resistors are in ohms. All capacitors are in picofarads. (b) A two-stage amplifier [2].

CONCLUSIONS

We have presented a new scheme for quadratic modeling where both circuit response functions and their gradients are simultaneously approximated. It is especially suitable for gradient based yield-driven design, making the model more accurate and robust. A standard test problem and an MMIC amplifier design illustrate the merits of our method.

REFERENCES

[1] J.W. Bandler and S.H. Chen, "Circuit optimization: the state of the art", *IEEE Trans. Microwave Theory Tech.*, vol. 36, 1988, pp. 424-443.

[2] G.D. Vendelin, A.M. Pavio and U.L. Rohde, *Microwave Circuit Design Using Linear and Nonlinear Techniques*. New York: Wiley, 1990.

[3] R.M. Biernacki and M.A. Styblinski, "Statistical circuit design with a dynamic constraint approximation scheme", *Proc. IEEE Int. Symp. Circuits Syst.* (San Jose, CA), 1986, pp. 976-979.

[4] R.M. Biernacki, J.W. Bandler, J. Song and Q.J. Zhang, "Efficient quadratic approximation for statistical design," *IEEE Trans. Circuits Syst.*, vol. 36, 1989, pp. 1449-1454.

[5] J.W. Bandler, S.H. Chen and K. Madsen, "An algorithm for one-sided ℓ_1 optimization with application to circuit design centering", *Proc. IEEE Int. Symp. Circuits Syst.* (Espoo, Finland), 1988, pp. 1795-1798.

[6] J.W. Bandler, Q.J. Zhang, J. Song and Biernacki, "FAST gradient based yield optimization of nonlinear circuits", *IEEE Trans. Microwave Theory Tech.*, vol. 38, 1990, pp. 1701-1710.

[7] K.S. Kundert and A Sangiovanni-Vincentelli, "Simulation of nonlinear circuits in the frequency domain," *IEEE Trans. Circuits Syst.*, vol. CAD-5, 1986, pp. 521-535.

[8] *OSA90*™, Optimization Systems Associates Inc., Dundas, Ontario, Canada L9H 5E7, 1990.

[9] E. Wehrhahn and R. Spence, "The performance of some design centering methods", *Proc. IEEE Int. Symp. Circuits Syst.* (Montreal, Canada), 1984, pp. 1424-1438.