HARMONIC BALANCE SIMULATION AND OPTIMIZATION OF NONLINEAR CIRCUITS

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Abstract - This paper addresses the recent advances in harmonic balance simulation and optimization of nonlinear circuits operating in the steady state. The basic concepts and formulations for single tone and multitone simulation are reviewed. A unified theory for nonlinear adjoint sensitivity analysis is summarized. The novel *FAST* technique linking efficient simulation to gradient-based optimization is described. The application of the harmonic balance method is expanded from circuit analysis to modeling, nominal design and yield optimization.

I. INTRODUCTION

Harmonic balance is an attractive method for steady-state simulation of nonlinear circuits because of its efficiency and flexibility [1-3]. By directly calculating the steady-state solution, it avoids the transient analysis performed by traditional time-domain simulators. When the circuit contains widely separated time constants, the transient analysis can be lengthy and expensive. With multitone input signals, it can be difficult to predict the length of the transient state. Harmonic balance is a mixed domain method: the linear elements are evaluated in the frequency domain and the nonlinear devices are modeled in the time domain. Distributed elements and time delays, which are difficult to handle by time-domain simulators, can be easily accommodated by the harmonic balance method.

Harmonic balance simulation has applications to a wide range of nonlinear circuits. For some circuits, the harmonics generated by the nonlinearity are detrimental to performance (e.g., causing distortion in power amplifiers). Using harmonic balance, we can analyze and even minimize such undesirable effects. Yet, harmonic balance simulation is particularly valuable when nonlinearity is exploited as an essential part of the circuit design (e.g., for mixers, oscillators, frequency multipliers and dividers).

This paper reviews recent advances in applying the harmonic balance concept to nonlinear circuit simulation, sensitivity analysis and optimization. A unified theory for nonlinear adjoint sensitivity analysis is summarized. The novel *Feasible Adjoint Sensitivity Technique* (*FAST*) which provides an efficient link between harmonic balance simulation and gradient-based optimization is described. The recent advances have expanded the scope of circuit applications from nonlinear analysis to modeling, nominal design and statistical yield optimization.

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II. HARMONIC BALANCE EQUATIONS

In harmonic balance simulation, the circuit is partitioned into a linear subcircuit and a nonlinear subcircuit connected through a number of ports. A suitable set of state variables is chosen, such as the port voltages represented by frequency-domain complex phasors at all the harmonic frequencies of interest. The linear subcircuit is evaluated in the frequency domain, given the excitations and state variables. The nonlinear subcircuit is simulated in the time-domain, and the responses are converted into the frequency domain by the Fourier transform.

The set of harmonic balance equations can be defined as

$$\mathbf{F}(\mathbf{V}) = \mathbf{I}_{NL}(\mathbf{V}) + \mathbf{I}_{L}(\mathbf{V}) = \mathbf{0} \qquad (1)$$

where $\mathbf{V}$ represents the state variables, $\mathbf{I}_{NL}$ and $\mathbf{I}_L$ are the responses of the nonlinear and linear subcircuits, respectively.

The nonlinear harmonic balance equations can be solved by Newton iterations or by optimization. The choice of starting point is important or even crucial for convergence. The continuation method is often used to overcome convergence difficulties. For instance, to improve convergence at high input power levels, we can first solve the harmonic balance equations at a low input power level and gradually increase the input power to the desired levels. A starting point may also be obtained from small-signal simulation of the linearized circuit around the dc operating point. It has been shown [4] that at sufficiently low input power levels, dc/small-signal simulation and harmonic balance simulation lead to consistent results, as illustrated in Fig. 1.

A different approach suggested by Rizzoli *et al.* [2] employs a quasi-Newton iteration to obtain starting points.

Harmonic balance simulation is certainly not limited to circuits with single-frequency sinusoidal excitations. The input signal may contain nonzero higher harmonic components. For instance, an input of triangular waveforms may be represented by its truncated frequency spectrum. For frequency dividers, the input is treated as the second harmonic of the desired output frequency.

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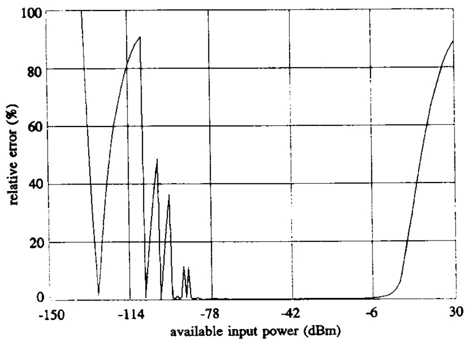


Fig. 1. Relative errors between the voltage gain calculated by harmonic balance and $|S_{21}|$ by small-signal analysis with respect to different input power levels. The errors at high input power levels are due to circuit nonlinearity. Numerical errors become dominant at extremely low input power levels.

Harmonic balance simulation under multitone excitations is useful for intermodulation and mixer analyses. The number of spectral frequencies which need to be considered grows rapidly with the number of tones. The selection of time samples (for the simulation of the nonlinear subcircuit) is another challenging aspect of multitone simulation. A number of competing approaches have been proposed: random sampling, the APFT algorithm by Kundert and Sangiovanni-Vincentelli [5], multidimensional FFT [6], and a recent quasi-orthogonal matrix sampling algorithm by Ngoya *et al.* [7]. Mixers, as a special case of multitone circuits, can also be analyzed by the frequency conversion method [8].

III. HARMONIC BALANCE SENSITIVITIES

In order to incorporate harmonic balance simulation into gradient-based optimization, we need to estimate the sensitivities of circuit responses. The simplest way is by perturbations, but it would require that the nonlinear harmonic balance equations be repeatedly solved after each perturbation.

Exact harmonic balance sensitivity expressions have been derived by Bandler *et al.* [9]. Suppose we are interested in the sensitivity of

$$V_{out} = c^T V \quad (2)$$

where c is a linear transfer vector linking the output voltage with the state variables. An adjoint system is then defined as

$$J^T \hat{V} = c \quad (3)$$

The adjoint system is a set of linear equations whose coefficient matrix J is the Jacobian matrix at the solution of the harmonic balance equations (1). Once (1) is solved by Newton iterations, little additional effort is required to solve the adjoint system. Detailed adjoint sensitivity expressions are given in [9].

The exact adjoint analysis is theoretically the most precise and efficient method for calculating sensitivities. To implement it in practice, however, can be a very complicated task, especially for general-purpose applications. A novel approach called *Feasible Adjoint Sensitivity Technique* (*FAST*) was proposed by Bandler *et al.* [10,11]. The *FAST* technique exploits the adjoint principles to derive high-level sensitivity expressions in which some of the terms are calculated by perturbations at the elementary level. It retains most of the elegance and accuracy of exact adjoint analysis while avoiding the implementational complexity.

For brevity, we consider ϕ as one of the designable variables. If we perturb ϕ by $\Delta\phi$, the corresponding change in V_{out} can be approximated by

$$\Delta V_{out} \approx \Delta c^T V + c^T \Delta V \quad (4)$$

To obtain the incremental change in the state variables ΔV, we utilize the adjoint solution given by (3):

$$\Delta V_{out} \approx \Delta c^T V - \hat{V}^T \Delta F \quad (5)$$

where ΔF is the residual function of the harmonic balance equations (1) at $\phi + \Delta\phi$ (F is theoretically zero at the harmonic balance solution). The evaluation of ΔF at $\phi + \Delta\phi$ requires much less effort than to repeat the harmonic balance solution. The incremental change in the linear transfer vector, namely Δc, can be obtained by either an adjoint analysis of the linear subcircuit or perturbation. In fact, if ϕ is a parameter in the nonlinear subcircuit, Δc is not needed:

$$\Delta V_{out} \approx -\hat{V}^T \Delta F \quad (6)$$

The *FAST* technique is substantially more efficient than the perturbation method, and it is also much easier to implement than the exact nonlinear adjoint formulas in [9]. Another alternative is the approach of optimization with integrated gradient approximations [12].

IV. NONLINEAR MODELING USING HARMONICS

In conventional frequency-domain linear analysis, nonlinear devices are represented by linearized equivalent circuit models around the dc operating point. Typically, the models are extracted from small-signal and/or dc measurements. Such models can be inadequate or even unsuitable for large-signal simulation. Using harmonic balance, large-signal nonlinear models can be extracted directly from power spectrum and/or waveform measurements [13]. Fig. 2 shows the match between FET power spectrum measurements and the simulated responses of a large-signal nonlinear model obtained using our modeling system HarPE [14].

V. UNIFIED SMALL- AND LARGE-SIGNAL DESIGN

Another significant approach is to combine dc, small-signal and large-signal performance specifications into one unified design optimization problem. This is possible if the same nonlinear circuit model is used in dc, small-signal and large-signal analyses, providing analytically consistent results [4]. The ability to optimize different types of responses simultaneously instead of separately brings important benefits to a CAD system, especially when some of the variables affect both the small- and large-signal performance.

This approach can be illustrated through a broad-band amplifier [4]. The dynamic range of the amplifier is taken into consideration in addition to the small-signal gain. The small-signal gain of the amplifier is optimized simultaneously for a range of input power levels. The higher harmonic responses (undesirable) are also minimized, using our CAD

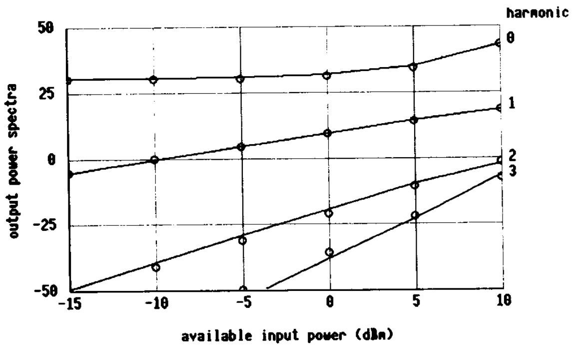
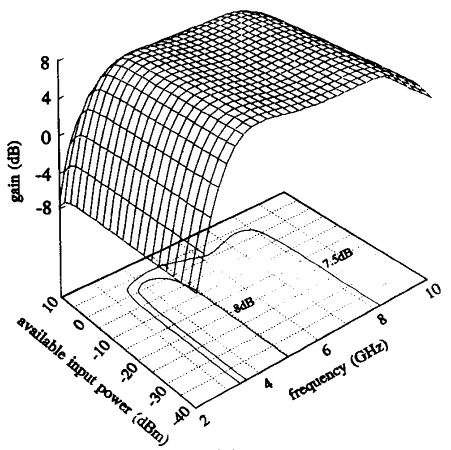
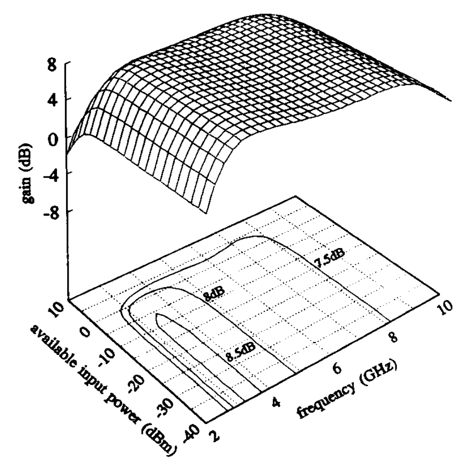


Fig. 2. Measured (circles) and simulated (solid lines) output power spectra of a MESFET.



(a)



(b)

Fig. 3. The gain surface and selected contour projections for the amplifier (a) by conventional small-signal design and (b) by simultaneous small- and large-signal design. The specification is 8 ± 0.5dB.

system OSA90/hope [15]. As shown in Fig. 3, the simultaneous optimization of small- and large-signal responses leads to a flatter gain surface and larger area in which the gain specification is met, as compared with a conventional small-signal design.

VI. NONLINEAR YIELD OPTIMIZATION

Harmonic balance can also be applied to statistical optimization of nonlinear circuits. In order to reduce the computational effort involved in the Monte Carlo simulation of a large number of random outcomes (i.e., circuits with statistically perturbed parameter values), a recent approach applies quadratic approximation not only for the circuit responses but also for their derivatives [16]. Since the determination of the quadratic model coefficients is independent of the number of outcomes, we can improve the sampling accuracy using a large number of outcomes without excessive computational effort.

Nonlinear yield optimization can be demonstrated through a frequency doubler [11]. The doubler consists of a common-source FET with a lumped input matching network and a microstrip output matching and filter section. The statistics of the nonlinear FET model parameters are described by a multidimensional normal distribution, including correlations. The statistics of the linear parameters are represented by uniform distributions. The optimization variables are parameters of the matching circuit. Responses to be optimized include conversion gain and spectral purity. Using the one-sided ℓ_1 centering algorithm [17] implemented in OSA90/hope [15], the yield of the doubler, as estimated from 500 random outcomes, was increased from 31% to 74%. The histograms of the spectrual purity before and after yield optimization are shown in Fig. 4.

VII. DATAPIPE ARCHITECTURE

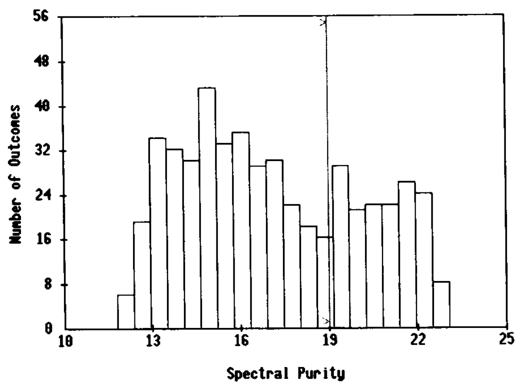
As optimization is applied to an expanding variety of circuit applications, software modules of diverse origins such as specialized simulators must be accommodated in a general CAD system.

We developed a software architecture called Datapipe which utilizes UNIX interprocess pipes for high speed data communication between independent programs. Utilizing the Datapipe architecture, an optimizer in the parent program will be able accept response functions and gradients from external simulators running as child programs. For example, we are able to interface our optimizers and statistical modules with SPICE-PAC [18].

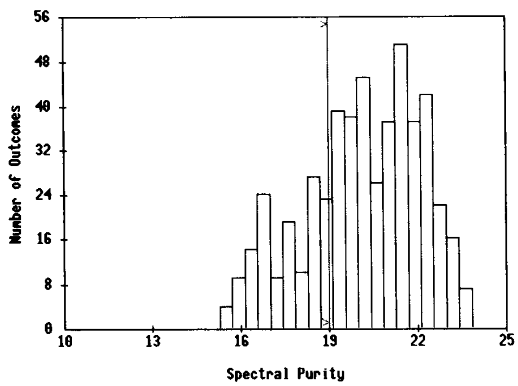
One of the most active research areas at the present is the development of nonlinear device models for harmonic balance and/or time-domain simulation. The Datapipe concept can facilitate the testing of such models without interfering with an existing CAD system.

VIII. CONCLUSIONS

The basic formulation and advantages of harmonic balance simulation have been described. Recent advances in circuit applications, including device modeling, nominal design and statistical optimization, have been reviewed and illustrated through examples. Nonlinear adjoint sensitivity analysis and especially the *FAST* technique have been discussed. The harmonic balance concept combined with adequate nonlinear circuit models can provide a unifying basis for consistent dc, small-signal and large-signal simulation and optimization.



(a)



(b)

Fig. 3. Histogram of spectral purity of the frequency doubler based on 500 outcomes (a) before yield optimization and (b) after yield optimization. The specification is shown by a vertical line.

REFERENCES

[1] K.S. Kundert and A. Sangiovanni-Vincentelli, "Simulation of nonlinear circuits in the frequency domain", *IEEE Trans. Computer-Aided Design*, vol. CAD-5, pp. 521-535, 1986.

[2] V. Rizzoli and A. Neri, "State of the art and present trends in nonlinear microwave CAD techniques", *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 343-365, 1988.

[3] R. Gilmore, "Nonlinear circuit design using the modified harmonic balance algorithm", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1294-1307, 1986.

[4] J.W. Bandler, R.M. Biernacki, S.H. Chen, J. Song, S. Ye and Q.J. Zhang, "Analytically unified DC/small-signal /large-signal circuit design", *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 1076-1082, 1991.

[5] K.S. Kundert and A. Sangiovanni-Vincentelli, "Applying harmonic balance to almost-periodic circuits", *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 366-378, 1988.

[6] V. Rizzoli, C. Cecchetti, A. Lipparini and F. Mastri, "General-purpose harmonic balance analysis of nonlinear microwave circuits under multitone excitation", *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 1650-1660, 1988.

[7] E. Ngoya, J. Rousset, M. Gayral, R. Quere and J. Obregon, "Efficient algorithms for spectral calculations in nonlinear microwave circuits simulators", *IEEE Trans. Circuits and Systems*, vol. 37, pp. 1339-1355, 1990.

[8] S.A. Maas, *Nonlinear Microwave Circuits*, Norwood, MA: Artech House, 1988.

[9] J.W. Bandler, Q.J. Zhang and R.M. Biernacki, "A unified theory for frequency-domain simulation and sensitivity analysis of linear and nonlinear circuits", *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 1661-1669, 1988.

[10] J.W. Bandler, Q.J. Zhang and R.M. Biernacki, "Practical, high-speed gradient computation for harmonic balance simulators", *IEEE MTT-S Int. Microwave Symp. Digest* (Long Beach, CA), pp. 363-367, 1989.

[11] J.W. Bandler, Q.J. Zhang, J. Song and R.M. Biernacki, "FAST gradient based yield optimization of nonlinear circuits", *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1701-1709, 1990.

[12] J.W. Bandler, S.H. Chen, S. Daijavad and K. Madsen, "Efficient optimization with integrated gradient approximations", *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 444-454, 1988.

[13] J.W. Bandler, Q.J. Zhang, S. Ye and S.H. Chen, "Efficient large-signal FET parameter extraction using harmonics", *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 2099-2108, 1989.

[14] *HarPE™ User's Manual*, Optimization Systems Associates Inc., P.O. Box 8083, Dundas, Canada, L9H 5E7, 1991.

[15] *OSA90/hope™ User's Manual*, Optimization Systems Associates Inc., P.O. Box 8083, Dundas, Canada, L9H 5E7, 1991.

[16] J.W. Bandler, R.M. Biernacki, S.H. Chen, J. Song, S. Ye and Q.J. Zhang, "Gradient quadratic approximation scheme for yield-driven design", *IEEE MTT-S Int. Microwave Symp. Digest* (Boston, MA), pp. 1197-1200, 1991.

[17] J.W. Bandler and S.H. Chen, "Circuit optimization: the state of the art", *IEEE Trans. Microwave Theory Tech.*, vol. 36, pp. 424-443, 1988.

[18] W.M. Zuberek, "SPICE-PAC 2G6c - User's Guide", Technical Report #8902, Department of Computer Science, Memorial University of Newfoundland, St. John's, Canada A1C 5S7, 1989.