MULTILEVEL MULTIDIMENSIONAL QUADRATIC MODELING FOR YIELD-DRIVEN ELECTROMAGNETIC OPTIMIZATION

J.W. Bandler*, R.M. Biernacki*, S.H. Chen*, S. Ye* and P.A. Grobelny

Simulation Optimization Systems Research Laboratory
and Department of Electrical and Computer Engineering
McMaster University, Hamilton, Canada L8S 4L7

ABSTRACT

Powerful multilevel multidimensional quadratic modeling has been developed for efficient yield-driven design. This approach makes it possible, for the first time, to perform direct yield optimization of circuits with components simulated by an electromagnetic simulator. Efficiency and accuracy of our technique are demonstrated by yield optimization of a small-signal amplifier.

INTRODUCTION

A new multilevel multidimensional modeling technique is presented for effective and efficient yield-driven design. This approach makes it possible, for the first time, to perform yield optimization of circuits with microstrip structures simulated by an electromagnetic (EM) simulator.

Yield optimization is now recognized as effective, not only for massively manufactured circuits but also to ensure first-pass success in any design where the prototype development is lengthy and expensive. Complexity of calculations involved in yield optimization requires special numerical techniques, e.g., [1-4]. In this paper we extend our previously published [2,4], highly efficient quadratic approximation technique to multilevel modeling. It is particularly suitable for circuits containing complex subcircuits or components whose simulation requires significant computational effort.

With the increasing availability of EM simulators [5-7] it is very tempting to include them into performance-driven and even yield-driven circuit optimization. However, direct utilization of EM simulation for yield optimization might seem to be computationally prohibitive. By constructing local quadratic models for each component simulated by an EM simulator we effectively overcome the computational burden of repeated EM simulations, which would otherwise be invoked for many statistical circuit outcomes throughout all yield optimization iterations.

This work was supported in part by Optimization Systems Associates Inc. and in part by the Natural Sciences and Engineering Research Council of Canada under Grants OGP0007239, OGP0042444 and STR0117819 and through an Industrial Research Fellowship to S. Ye.

*J.W. Bandler, R.M. Biernacki and S.H. Chen are also with and S. Ye is with Optimization Systems Associates Inc., P.O. Box 8083, Dundas, Ontario, Canada L9H 5E7.

When the proposed multilevel quadratic modeling technique is used together with expensive, but more accurate simulations at the component level, the results are more reliable than those obtained from traditional empirical component simulations.

Efficiency and accuracy of our technique are demonstrated by yield optimization of a small-signal amplifier. Optimization was performed within the OSA90/hope™ [8] simulation and optimization environment with Empipe™ [9] driving *em*™ [7].

YIELD OPTIMIZATION

Formally, the problem of yield optimization can be formulated as

$$\underset{\phi^0}{maximize}\ \{\ Y(\phi^0) = \int_{R^n} I_a(\phi) f_\phi(\phi^0,\phi) d\phi\ \} \tag{1}$$

where ϕ^0 and ϕ are vectors in R^n and represent the nominal circuit parameters and the actual circuit outcome parameters, respectively, $Y(\phi^0)$ is the design yield and $f_\phi(\phi^0, \phi)$ is the probability density function of ϕ around ϕ^0. $I_a(\phi) = 1$ if $\phi \in A$ and $I_a(\phi) = 0$, otherwise, where A is the acceptability region in which all design specifications are met. In practice, the integral in (1) is approximated using K Monte Carlo circuit outcomes ϕ^i and yield is estimated by

$$Y(\phi^0) \simeq \frac{1}{K}\left[\sum_{i=1}^{K} I_a(\phi^i)\right] \tag{2}$$

The outcomes ϕ^i are generated by a random number generator according to $f_\phi(\phi^0, \phi)$. To estimate yield we create a set of multi-circuit error functions $e(\phi^1)$, $e(\phi^2)$, ..., $e(\phi^K)$. The error functions $e(\phi^i)$ are derived from the circuit responses R_j and lower specifications (S_l) and upper specifications (S_u) as

$$e_j(\phi^i) = R_j(\phi^i) - S_{uj} \quad \text{or} \quad e_j(\phi^i) = S_{lj} - R_j(\phi^i) \tag{3}$$

Notice that a positive (negative) error function indicates that the corresponding specification is violated (satisfied).

For yield optimization we use the one-sided ℓ_1 objective function [1,10]

$$U(\phi^0) = \sum_{i \in J} \alpha_i v(\phi^i) \tag{4}$$

where

$$J = \{i \mid v(\phi^i) > 0\} \tag{5}$$

OF2

α_i are suitably chosen positive multipliers and $v(\phi^i)$ is the generalized ℓ_1 function having the property that it is positive if at least one specification is violated, i.e., $\phi^i \notin A$, and it is negative if all specifications are satisfied. Consequently, $U(\phi^0)$ becomes an approximation to the percentage of outcomes violating design specifications and minimization of $U(\phi^0)$ leads to yield improvement.

EFFICIENT QUADRATIC MODELING [2,3,4]

The *Q-model* to approximate a generic response $f(x)$, i.e., any response or gradient function at any level (see Fig.1) for which we want to build and utilize the model, is a multidimensional quadratic polynomial of the form

$$q(x) = a_0 + \sum_{i=1}^{n} a_i(x_i - r_i) + \sum_{\substack{i=1 \\ j \geq i}}^{n} a_{ij}(x_i - r_i)(x_j - r_j) \qquad (6)$$

where $x = [x_1\ x_2\ \ldots\ x_n]^T$ is the vector of generic parameters in terms of which the response is defined, and $r = [r_1\ r_2\ \ldots\ r_n]^T$ is a chosen reference point in the parameter space.

To build the model we use $n+1 < m \leq 2n+1$ base points at which the function $f(x)$ is evaluated. The reference point r is selected as the first base point x^1. The remaining $m-1$ base points are selected by perturbing one variable at a time around r with a predetermined perturbation β_i.

$$x^{i+1} = r + [0\ \ldots\ 0\ \beta_i\ 0\ \ldots\ 0]^T, \quad i = 1, 2, \ldots, n \qquad (7)$$

$$x^{n+1+i} = r + [0\ \ldots\ 0\ -\beta_i\ 0\ \ldots\ 0]^T, \quad i = 1, 2, \ldots, m-(n+1) \qquad (8)$$

If a variable is perturbed twice the second perturbation is located symmetrically w.r.t. r. We have applied the Maximally Flat Quadratic Interpolation (MFQI) technique [3] to such a set of base points (see [2] for details). MFQI builds the Q-model by minimizing in the least-squares sense all the second-order coefficients in (6). It is intuitively equivalent to constructing an approximation which has the smallest deviation from the linear approximation. Applying MFQI to the base points defined by r, (7) and (8) yields the following formula for $q(x)$.

$$\begin{aligned} q(x) = f(r) + \sum_{i=1}^{m-(n+1)} &\Big\{[(f(x^{i+1}) - f(x^{n+1+i}) + (f(x^{i+1}) + f(x^{n+1+i}) \\ &- 2f(r))(x_i - r_i)/\beta_i](x_i - r_i)/(2\beta_i)\Big\} \\ &+ \sum_{i=m-n}^{n} \Big\{[f(x^{i+1}) - f(r)](x_i - r_i)/\beta_i\Big\} \end{aligned} \qquad (9)$$

To apply a gradient-based optimizer we need to provide the gradient of functions $q(x)$ which are actually used by the optimizer. Differentiating (9) w.r.t. x_i results in

$$\begin{aligned} \partial q(x)/\partial x_i = [(f(x^{i+1}) - f(x^{n+1+i}))/2 + (f(x^{i+1}) + f(x^{n+1+i}) \\ -2f(r))(x_i - r_i)/\beta_i]/\beta_i, \quad i = 1, \ldots, m-(n+1) \end{aligned} \qquad (10a)$$

and

$$\partial q(x)/\partial x_i = [f(x^{i+1}) - f(r)]/\beta_i, \quad i = m-n, \ldots, n, \quad m \neq 2n+1 \qquad (10b)$$

The simplicity of (9) and (10) results in unsurpassed efficiency of the approach. Note that the computational effort increases only linearly with the number of variables n. Also, a variable number of base points m offers a trade-off between accuracy and cost of circuit analysis.

MULTILEVEL MODELING

Multilevel simulation and modeling is depicted in Fig. 1. The circuit under consideration is divided into subcircuits, possibly in a hierarchical manner. At the lowest level we have circuit components which are the smallest entities that can be handled by the available simulators, e.g., a lumped capacitor or a microstrip structure. For EM simulators the definition of a component is not as straightforward and typically the components are defined by the user to encompass parts of the structure that can be isolated from other parts.

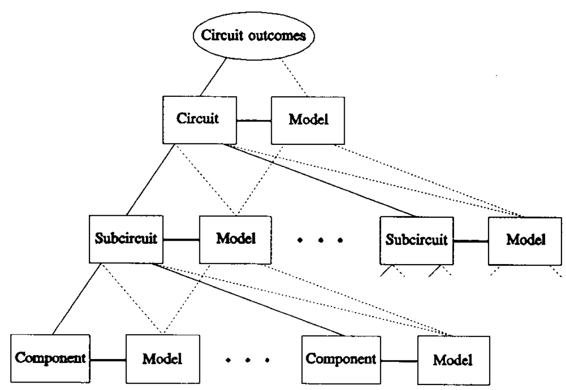


Fig. 1 Schematic diagram illustrating multilevel modeling for yield-driven optimization. Solid and dotted lines distinguish between exact and approximate responses.

A Q-model can be established and subsequently utilized at any level for some or all subcircuits and components. The models are built from the results of exact simulations of the corresponding component, subcircuit, or the overall circuit. Once the Q-model is established, it is used in place of the corresponding simulator. For particular problems many Q-models may exist changing the path of calculations as indicated by different links in Fig. 1.

A number of experiments were conducted on a 3-section 3:1 microstrip impedance transformer to investigate advantages of multilevel quadratic modeling. First, from results of ***em*** [7] simulations we established the component level Q-models for each section of the transformer. Utilizing these Q-models we performed yield optimization using (1) single-level (component) modeling, and (2) two-level (component and circuit response) modeling. We also used ***em*** [7] to create just one Q-model for the entire microstrip transformer structure. Similarly, we performed yield optimization employing both single- and two-level modeling. The solutions in all cases were almost identical. However, the CPU time was significantly reduced when two-level modeling was used.

The circuit, subcircuit, or component parameters can be, in general, categorized as: designable x_D, statistical x_S, or discrete x_G. All other parameters are fixed. In (6)-(10), the vector x of model variables may contain different combinations of x_D, x_S and x_G, depending on the capabilities of the corresponding simulator. For example, as proposed in [4] the Q-models at the circuit level can be built for both response and gradient functions in terms of x_S only.

The importance of bringing the discrete parameters x_G into the Q-model is illustrated in Fig. 2. The discrete parameters are those for which simulation can only be performed at discrete values located on the grid. For example, this is applicable in numerical EM simulation. Normally, the reference vector r is taken as the nominal point x^1. This is likely to be off-the-grid. Similarly, the other base points x^{i+1} and x^{n+1+i} are likely to be off-the-grid. Local interpolation involving several simulations on the grid in the vicinity of each of the base points must then be performed. In order to avoid these excessive simulations those base points are modified to snap to the grid. Significant computational savings can be achieved.

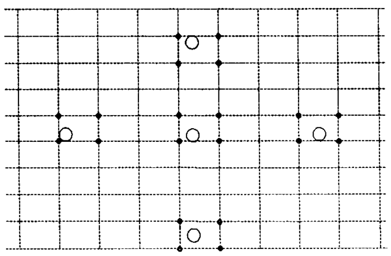


Fig. 2 Illustration of base points and discrete points. The large circles represent possible location of base points w.r.t. a grid. The solid dots indicate discrete simulation points on the grid. If the base points are snapped to the grid, the number of simulations can be significantly reduced.

YIELD OPTIMIZATION OF A SMALL-SIGNAL AMPLIFIER

The specification for a typical single-stage 6-18 GHz small-signal amplifier shown in Fig. 3, is

$$7 \text{ dB} \leq |S_{21}| \leq 8 \text{ dB, from 6 GHz to 18 GHz}$$

The error functions for yield optimization are calculated at frequencies from 6 GHz to 18 GHz with a 1 GHz step. The gate and drain circuit microstrip T-junctions and the feedback microstrip line are built on a 10 mil thick substrate with relative dielectric constant 9.9.

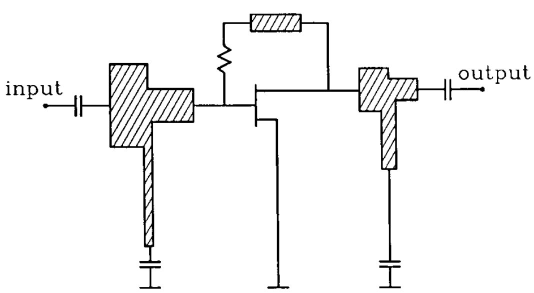
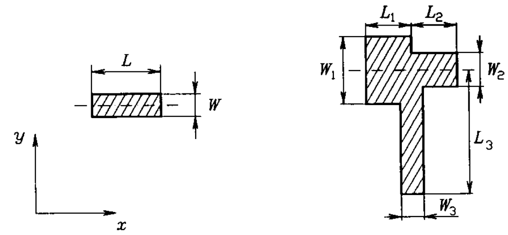


Fig. 3 Circuit diagram of the 6-18 GHz small-signal amplifier. We use ***em*** [7] to model the two T-junctions and the microstrip line.

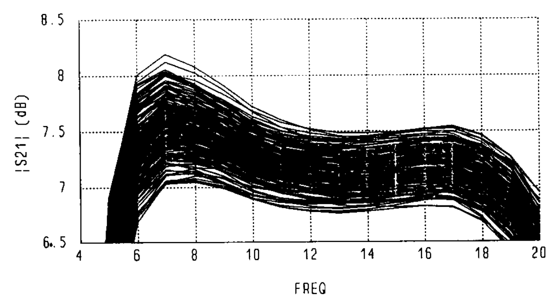
First, we performed nominal minimax optimization using analytical/empirical microstrip component models. W_{g1}, L_{g1}, W_{g2}, L_{g2} of the gate circuit T-junction and W_{d1}, L_{d1}, W_{d2}, L_{d2} of the drain circuit T-junction were selected as optimization variables. W_{g3}, L_{g3}, W_{d3} and L_{d3} of the T-junctions, W and L of the feedback microstrip line, as well as the FET parameters were not optimized. Fig. 4 shows the parameters of the T-junctions and the microstrip line.

We assumed 0.5 mil tolerance and uniform distribution for all geometrical parameters of the microstrip components. The statistics of the small-signal FET model were extracted from measurement data [11]. We built the component level Q-models from ***em*** [7] simulations of all microstrip components. Monte Carlo simulation with 250 outcomes performed at the nominal solution reported 55% yield. Using the component level Q-models we then performed yield optimization of the amplifier. Yield estimated by 250 Monte Carlo simulations was increased to 82%. Monte Carlo sweeps before and after yield optimization are shown in Fig. 5. The parameter values of the microstrip components before and after optimization are given in Table I.

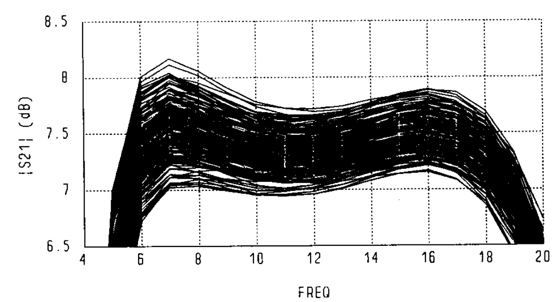


(a) (b)

Fig. 4 Parameters of (a) the feedback microstrip line and (b) the microstrip T-junctions.



(a)



(b)

Fig. 5 $|S_{21}|$ of the small-signal amplifier for 250 statistical outcomes with microstrip components simulated by the ***em*** [7] simulator: (a) at the nominal minimax solution, and (b) after yield optimization.

TABLE I
MICROSTRIP PARAMETERS OF THE AMPLIFIER

Parameters	Nominal design	Centered design
W_{g1}	17.45	19
L_{g1}	35.54	34.53
W_{g2}	9.01	8.611
L_{g2}	30.97	32
W_{g3}	3^*	3^*
L_{g3}	107^*	107^*
W_{d1}	8.562	7
L_{d1}	4.668	6
W_{d2}	3.926	3.628
L_{d2}	9.902	11
W_{d3}	3.5^*	3.5^*
L_{d3}	50^*	50^*
W	2^*	2^*
L	10^*	10^*
Yield (250 outcomes)	55%	82%

* Parameters not optimized.

Dimensions of the parameters are in mils. 50 outcomes were used for yield optimization. 0.5 mil tolerance and uniform distribution were assumed for all the parameters.

CONCLUSIONS

We have presented a new multilevel quadratic modeling technique suitable for effective and efficient yield-driven design optimization. This approach is particularly useful for circuits containing complex subcircuits or components whose simulation requires significant computational effort. The efficiency of this technique allowed us to perform yield-driven design of circuits containing microstrip structures accurately simulated by ***em*** [7]. Our approach, illustrated by yield optimization of a small-signal amplifier, significantly extends the microwave CAD applicability of yield optimization techniques.

ACKNOWLEDGEMENT

The authors thank Dr. J.C. Rautio of Sonnet Software, Inc., Liverpool, NY for his initiatives and making ***em*** available for this work.

REFERENCES

[1] J.W. Bandler and S.H. Chen, "Circuit optimization: the state of the art," *IEEE Trans. Microwave Theory Tech.*, vol. 36, 1988, pp. 424-443.

[2] R.M. Biernacki, J.W. Bandler, J. Song and Q.J. Zhang, "Efficient quadratic approximation for statistical design," *IEEE Trans. Circuit Syst.*, vol. 36, 1989, pp. 1449-1454.

[3] R.M. Biernacki and M.A. Styblinski, "Efficient performance function interpolation scheme and its application to statistical circuit design," *IEEE Int. J. Circuit Theory and Appl.*, vol. 19, 1991, pp. 403-422.

[4] J.W. Bandler, R.M. Biernacki, S.H. Chen, J. Song, S. Ye and Q.J. Zhang, "Gradient quadratic approximation scheme for yield-driven design," *IEEE MTT-S Int. Microwave Symp. Dig.* (Boston, MA), 1991, pp. 1197-1200.

[5] T. Itoh, Ed., *Numerical Techniques for Microwave and Millimeter-Wave Passive Structures.* New York: Wiley, 1989.

[6] R.H. Jansen and P. Pogatzki, "A hierarchically structured, comprehensive CAD system for field theory-based linear and nonlinear MIC/MMIC design," *1992 2nd Int. Workshop of the German IEEE MTT/AP Joint Chapter on Integrated Nonlinear Microwave and Millimeterwave Circuits Dig.* (Duisburg, Germany), 1992, pp. 333-341.

[7] *Em User's Manual* and *Xgeom User's Manual*, Sonnet Software, Inc., 135 Old Cove Road, Suite 203, Liverpool, NY 13090-3774, May 1992.

[8] *OSA90/hope™ Version 2.0 User's Manual*, Optimization Systems Associates Inc., P.O. Box 8083, Dundas, Ontario, Canada L9H 5E7, 1992.

[9] *Empipe™ Version 1.0 Technical Brief*, Optimization Systems Associates Inc., P.O. Box 8083, Dundas, Ontario, Canada L9H 5E7, 1992.

[10] J.W. Bandler, S.H. Chen and K. Madsen, "An algorithm for one-sided ℓ_1 optimization with application to circuit design centering," *Proc. IEEE Int. Symp. Circuits Syst.* (Espoo Finland), 1988, pp. 1795-1798.

[11] Measurement data provided by Plessey Research Caswell Ltd., Caswell, Towcester, Northamptonshire, England NN12 8EQ, 1990.