The Huber Concept in Device Modeling, Circuit Diagnosis and Design Centering

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ABSTRACT

We present exciting applications of the Huber concept in circuit modeling and optimization. By combining the desirable properties of the ℓ_1 and ℓ_2 norms, the Huber function is robust against gross errors and smooth w.r.t. small variations in the data. We extend the Huber concept by introducing a one-sided Huber function tailored to design optimization with upper and lower specifications. We demonstrate the advantages of Huber optimization in the presence of faults, large and small measurement errors, bad starting points and statistical uncertainties. Circuit applications include parameter identification, design optimization, statistical modeling, analog fault location and yield optimization.

INTRODUCTION

Realistic circuit optimization must take into account model/measurement/statistical errors, variations and uncertainties. Least-squares (ℓ_2) solutions are notoriously susceptible to the influence of gross errors: just a few "wild" data points can alter the results significantly. The ℓ_1 method is robust against gross errors [1,2]. However, it inappropriately treats small variations in the data. In other words, neither the ℓ_1 nor ℓ_2 alone is capable of providing solutions which are robust against large errors *and* flexible w.r.t. small variations in the data.

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The Huber function [3-5] is a hybrid of the ℓ_1 and ℓ_2 norms. The large errors are treated in the ℓ_1 sense and the small errors are measured in terms of least squares. Consequently, the Huber solution can provide a smooth model from data which contains many small variations and such a model is also robust against gross errors.

We extend the Huber concept by introducing a "one-sided" Huber function for design optimization with upper and/or lower specifications. The minimax objective is often chosen to achieve an equal-ripple design. However, from a "bad" starting point, a minimax optimizer can be trapped by the initial large errors. We have demonstrated [5] that the one-sided Huber function can be employed in a "preprocessing" optimization to overcome such a bad starting point. In this paper, we compare minimax optimization of multicavity filters with and without one-sided Huber preprocessing from randomly generated starting points.

Also, for the first time, we present a one-sided Huber approach to yield optimization of linear and nonlinear circuits.

Our approach is implemented in the CAD system OSA90/hope™ [6] which was used to produce the examples in this presentation.

THEORY

The Huber function is defined as [3-5]

$$\rho_k(f) = \begin{cases} f^2/2 & if\ |f| \le k \\ k|f| - k^2/2 & if\ |f| > k \end{cases} \qquad (1)$$

where k is a positive constant threshold value and f represents an error function.

The sum of ρ_k is a hybrid of the ℓ_2 (when $|f| \leq k$) and the ℓ_1 (when $|f| > k$) norms. The definition of ρ_k ensures a smooth transition between ℓ_2 and ℓ_1 at $|f| = k$. The threshold k separates "large" and "small" errors. With a sufficiently large k, ρ_k becomes least squares. As k approaches zero, ρ_k approaches the ℓ_1 norm. By changing k, we can alter the proportion of error functions to be treated in the ℓ_1 or ℓ_2 sense.

We define the "one-sided" Huber function as

$$\rho_k^+(f) = \begin{cases} 0 & if\ f \leq 0 \\ \rho_k(f) & if\ f > 0 \end{cases} \tag{2}$$

This definition is tailored to one-sided (upper/lower) specifications. A negative value of f indicates that the corresponding specification is satisfied and is therefore truncated.

MULTICAVITY FILTER PARAMETER IDENTIFICATION

We consider a 6th-order multicavity filter [2]. The input reflection coefficient is used as simulated measurement. Two large errors are deliberately introduced into this data. The task is to identify the parameters from the contaminated data [2]. Our results obtained using the Huber function (1) are shown in Table I for selected couplings. The percentage entries represent the relative differences between the identified parameter values and the actual parameter values.

In Case A, the two large errors are the only errors contained in the data. The results in Table I show that the ℓ_2 solution is hopelessly corrupted by the gross errors, whereas the ℓ_1 and Huber solutions are equally robust.

In Case B, the data is truncated to the first two significant digits to emulate the limited accuracy of measurement equipment. The truncation errors are small relative to the two gross errors. We choose a threshold value commensurate with the magnitude of the truncation errors so that they are treated in the ℓ_2 sense by the Huber norm. Consequently, the Huber solution is less affected than the corresponding ℓ_1 solution (see Table I).

In Case C, we introduced into the data small errors randomly generated from the uniform distribution [-0.01 0.01]. Again, the Huber solution is better in comparison with the ℓ_1 solution, as shown in Table I.

TABLE I
PARAMETER IDENTIFICATION FOR MULTICAVITY FILTER

Couplings		M_{12}	M_{45}	M_{16}
Actual Values		0.859956	0.526602	0.087293
Starting Point		0.819006	0.511264	0.093863
Case A:	ℓ_2	-11%	7.3%	278%
	ℓ_1	0.05%	-0.06%	-0.01%
	Huber	0.02%	0.01%	-1.2%
Case B:	ℓ_1	0.51%	-2.9%	-14%
	Huber	0.15%	-0.01%	-8.3%
Case C:	ℓ_1	1.8%	-4.1%	-43%
	Huber	0.41%	0.04%	-27%

ANALOG FAULT LOCATION

Consider a resistive mesh network which has been used to demonstrate the ℓ_1 approach to analog fault location [2]. We have reported successful application of the Huber function to this problem [5]. In this paper, we present new results which take into account data truncation errors to represent the limited accuracy of measurement equipment.

Selected parameter values of the mesh network are listed in Table II. Two faults were assumed, namely G_2 and G_{18}. We generated simulated node voltage measurements at the accessible nodes. The voltages were then truncated to the first two significant digits.

TABLE II
FAULT LOCATION OF THE RESISTIVE MESH CIRCUIT

Element	Nominal Value	Actual Value	Percentage Deviation		
			Actual	ℓ_1	Huber
G_2	1.0	0.50	-50.0	-47.55	-54.40
G_3	1.0	1.05	5.0	-25.45	-3.68
G_{16}	1.0	0.95	-5.0	-20.24	-3.53
G_{17}	1.0	1.05	5.0	0.00	-0.81
G_{18}	1.0	0.50	-50.0	-8.90	-49.97
G_{19}	1.0	0.95	-5.0	-25.32	-4.74
G_{20}	1.0	0.95	-5.0	-20.73	-5.98

The nominal parameter values are used as the starting point for optimization. The results listed in Table II show that the ℓ_1 optimization failed to isolate the faults. The ℓ_1 optimization attempts to suppress as many parameter deviations as possible to exactly zero, which may lead to an incorrect solution, as demonstrated in this case.

ONE-SIDED HUBER PREPROCESSING OF ARBITRARY STARTING POINTS

We have exploited the potential of using one-sided Huber preprocessing to overcome bad starting points in large-scale multiplexer optimization [5]. In this paper, we expand our investigation by testing several starting points for optimization.

For the same 6th-order multicavity filter, 30 starting points were generated using uniform distribution centered at a "good" starting point with ±30% spread of the parameter values. The input return loss of the filter at these starting points is shown in Fig. 1(a). Clearly, some of the starting points are very bad.

From each starting point, we performed: (1) direct minimax optimization and (2) one-sided Huber optimization (preprocessing) followed by minimax optimization. The optimized responses are shown in Figs. 2(b) and 2(c), respectively. Although one-sided Huber preprocessing did not guarantee convergence to the optimal solution from all the starting points, it produced more focused results.

FET STATISTICAL MODELING

In our approach to statistical device modeling [7] we first extract model parameters for individual devices from device measurements and then postprocess the sample of model parameters to estimate the statistics (means, standard deviations and correlations). At the postprocessing stage we normally apply least-squares estimators using the error functions $f_j(\overline{\phi}) = \overline{\phi} - \phi^j$ to estimate mean values, or $f_j(V_\phi) = V_\phi - (\phi^j - \overline{\phi})^2$ to estimate standard deviations. ϕ^j is the extracted value of a parameter of the jth device, $j = 1, 2, ..., N$ and N is the total number of devices. V_ϕ denotes the estimated variance from which we can calculate the standard deviation σ_ϕ.

The Huber function (1) can be used as an automatic robust statistical estimator in place of least-squares estimators. If the sample of device measurements contains some wild points (e.g., due to faulty devices) they will severely degrade the least-squares estimates.

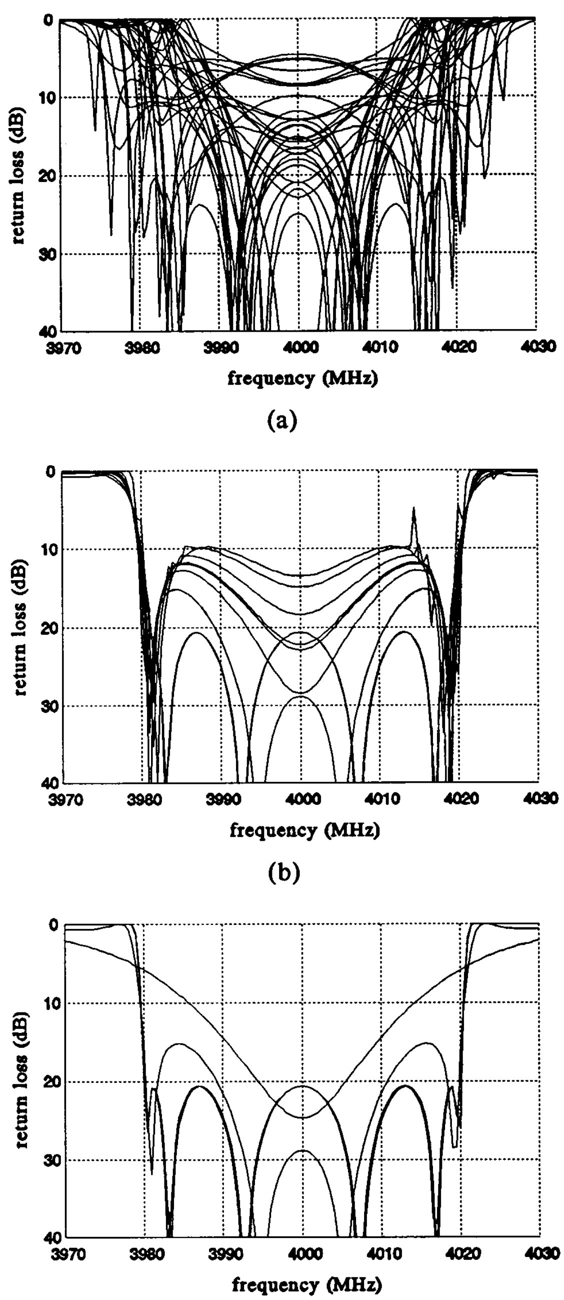

Figure 1

In our earlier work [7] using the ℓ_2 estimator, the wild points had to be manually excluded. Applying Huber estimators to the same data [5] we obtained similar results but without excluding any points.

ONE-SIDED HUBER FORMULATION FOR YIELD OPTIMIZATION

In Monte Carlo analysis, we consider a number of statistical outcomes of circuit parameters denoted by ϕ^i. Following the generalized ℓ_p centering approach [1], for each outcome we create a generalized ℓ_p function $\nu(\phi^i)$ which has a positive value if the outcome violates the design specifications or a zero or negative value if the specifications are satisfied.

In our earlier work [1,8], we have formulated yield optimization as a one-sided ℓ_1 problem. Here, we formulate yield optimization as a one-sided Huber problem in which the objective function is defined as

$$U(\phi^0) = \sum_{i=1}^{N} \rho_k^+(\alpha_i \, \nu(\phi^i)) \qquad (3)$$

where ϕ^0 represents the nominal circuit parameters to be centered, α_i is a positive multiplier associated with the ith outcome, N the total number of outcomes and ρ_k^+ the one-sided Huber function defined in (2).

We considered a linear LC filter [9]. The one-sided ℓ_1 method needed 160 CPU seconds (11 iterations) on a Sun SPARCstation 10 while the one-sided Huber yield optimization with k=0.2 finished in 123 CPU seconds (9 iterations). Both produced 75% yield.

We also considered a nonlinear frequency doubler [10]. We assumed uniform tolerances for the linear matching subcircuits and normal distributions (with correlations) for the intrinsic FET parameters. At the nominal design (before yield optimization) yield was 28%. The centered designs were obtained after 17 iterations and 337 CPU seconds using one-sided ℓ_1 centering, and after 29 iterations and 574 CPU seconds using one-sided Huber technique, both on a Sun SPARCstation 10. The optimized yield values were 76% and 77%, respectively. Thus, the new one-sided Huber approach proved to be a competitive alternative to the one-sided ℓ_1 centering approach.

CONCLUSIONS

We have presented exciting developments in applying a novel Huber approach to parameter identification, preprocessing of arbitrary starting points, statistical modeling, analog fault location and design centering. Compared with ℓ_1, ℓ_2 and minimax techniques, the Huber approach has demonstrated robustness and consistency in the presence of large and small errors, deterministic and statistical variations, which are critical considerations for practical CAD in an engineering environment.

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