Device statistical modelling and verification

Optimization Systems Associates of Ontario, Canada has specialized in microwave CAD using statistical modelling and yield-driven design. In this paper John Bandler, Radek Biernacki, Qian Cai and Shao Hua Chen describe modelling of active devices based on "indirect" or "direct" determination of the statistical distribution of the model.

In integrated circuit manufacturing, fabricated circuits and devices exhibit parameter values deviating randomly from their nominal (or designed) values. These random variations result in statistical spreads of circuit responses and directly affect production yield and cost. Since all the active and passive components are fabricated on a common semi-insulating substrate, post-production tuning is restricted and device replacement is not feasible. Therefore, yield and cost analysis and optimization are becoming widely accepted as indispensable ingredients of circuit CAD methodology.

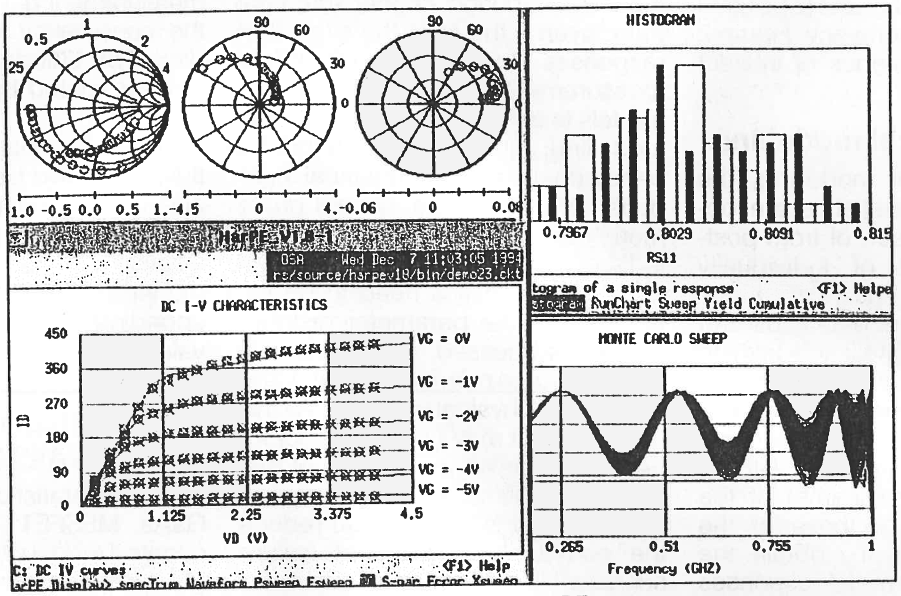


Figure 1: Various device responses featured by HarPE.

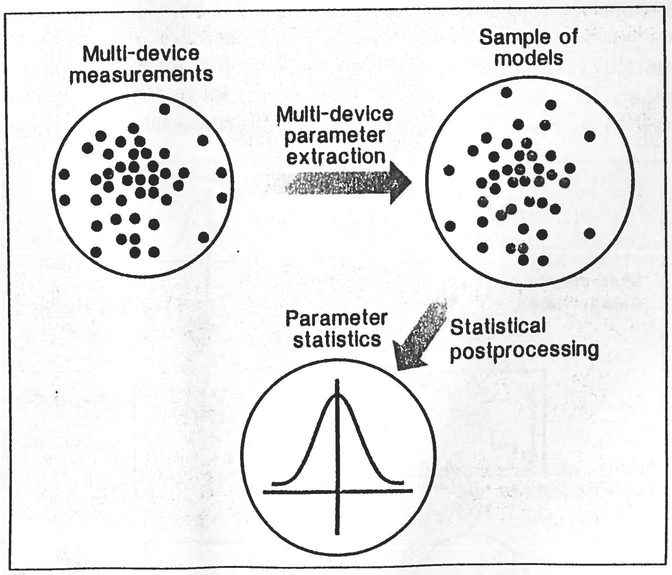


Figure 2: Illustration of indirect statistical modelling.

Statistical modelling is a prerequisite for yield and cost analysis, and consequently optimization. In statistical modelling we determine a device model whose parameters are described as random variables. The distribution of the model responses due to random variations of the model parameters must reflect the actual distribution of device responses. The latter is characterized by multi-device measurement data (measurements taken on a number of supposedly identical devices). Therefore, statistical modelling is a process of matching the statistical distribution of the model to that of the measurement data. The advanced statistical modelling methods described here are based on OSA's pioneering work [1-6].

In this paper we present two techniques for statistical modelling of active devices. The first, a two-stage, indirect method is based on multi-device parameter extraction followed by statistical postprocessing of the resulting sample of models. In the second, direct, technique the statistics of the model parameters are determined through direct matching of statistical distributions of the model and of the data. Matching of either cumulative probability distribution (CPD) functions or histograms can be employed [1,2]. The two techniques for statistical modelling have been implemented in OSA's HarPE [1], a powerful parameter extraction and device characterization CAD software system. An example of various graphical plots generated by HarPE is shown in figure 1, including DC I-V curves, the Smith charts and polar plots of S parameters, histograms and a Monte Carlo sweep.

Indirect statistical modelling

The indirect statistical modelling technique consists of two stages. In the first stage each of the devices represented by the multi-device measurement data is modeled individually in a deterministic fashion. Parameter extraction optimization is invoked for each of the outcomes to fit the simulated responses to the corresponding measurement data set. Such parameter extraction is carried out for all data sets in the multi-device measurements and results in as many device models as the number of data sets in the measurement data. They form a sample of device models, with different parameter values for different outcome models. Statistical postprocessing of those different parameter values leads to a single, consolidated model whose parameter values are described by the means, standard deviations and the correlation matrix. For non-Gaussian distributions, a discrete distribution function (DDF) approximation to the marginal distributions [6] is used to enhance model accuracy. Figure 2 illustrates the two stages of indirect statistical modelling.

Novel statistical modelling

In direct statistical modelling, the model parameter statistics are obtained directly instead of from postprocessing a set of individually extracted models. The whole multi-device measurement data is utilized simultaneously in a statistical fashion by generating the distribution of the measured device responses. The statistics of the model parameters are determined by fitting the distributions (CPDs or histograms) of the model responses to those of the measurement data. To obtain the distributions of the model responses we employ Monte Carlo simulation. Model parameters are randomly generated according to the parameter distribution and the resulting distribution of model responses is found. The fitting is carried out in a single optimization. Direct statistical modelling using CPD fitting is depicted in figure 3.

The optimization variables include the parameter statistics, for example, the mean values and standard deviations in the case of normal distributions, or the nominal values and tolerances in the case of uniform distributions (other types of distributions can also be applied). The initial parameter statistics and distributions need to be assumed at the starting point. At the solution we obtain the parameter statistics leading to the best match of the corresponding CPDs or histograms.

Each of the two approaches can be applied alternatively to carry out complete statistical modelling independently. However, each has its own advantages and disadvantages.

Indirect statistical modelling is straightforward and easy to use. No initial statistics need to be guessed at. However, it relies on the uniqueness of the parameter extraction process and, therefore, the resulting statistical models may not reflect the actual distribution of measurement data, even if the fit of the simulated responses to the corresponding measurements for individual device models is excellent. Direct statistical modelling, on the other hand, is based on a solid mathematical foundation and, therefore, should prove more reliable and robust than the indirect method. However, the initial parameter statistics need to be assigned and the parameter distribution types guessed. It may be justified to assume normal distributions in the case of physical or process parameters, but it may not be correct for equivalent circuit models. All of that may affect the solution. Also, a good starting point is important to reduce the computation time and assure successful optimization.

A practical approach is to combine the two methods. We use the indirect method first to obtain an initial statistical model and then apply the direct method to improve the model accuracy. For efficiency, the initial modelling may be carried out with a small number of devices.

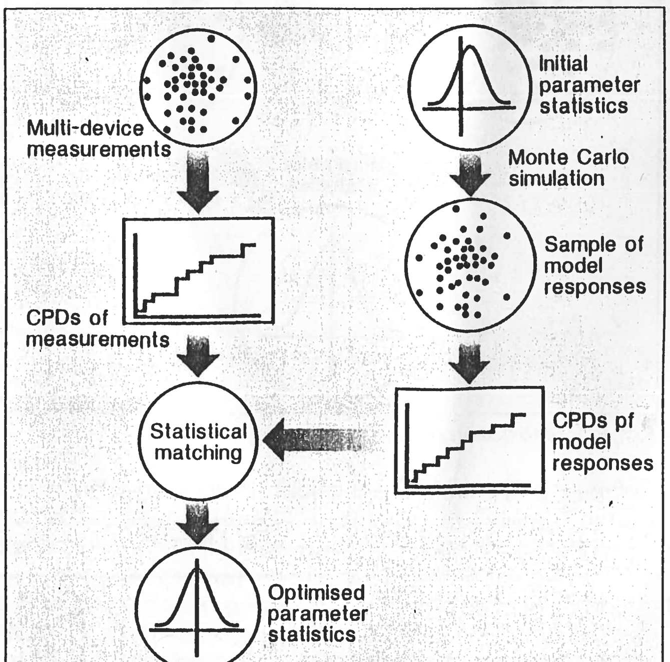


Figure 3: Direct statistical modelling using CPD matching.

Statistical model verification

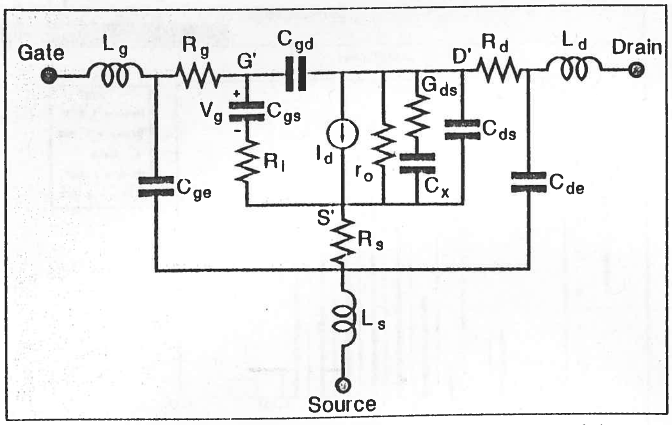
Two practical methods for model verification are statistical comparison [4,5] and yield verification [2]. The former compares the statistics of the model responses generated by Monte Carlo simulation with the statistics of the measurement data. The latter checks the consistency between yield predicted by statistical models and the yield estimated by the actual device data.

Visual comparisons of distributions may also be used for effective and quick model validation. HarPE provides useful graphical displays where the histograms or CPDs of the model responses and the corresponding data including the mean values and standard deviations can be shown in the same diagram.

Statistical modelling of GaAs MESFETs

Consider statistical modelling of a GaAs MESFET using a physics-oriented model which we call the KTL (Khatibzadeh-Trew-Ladbrooke) model. The KTL model combines the advantages of the Khatibzadeh and Trew model [7] and the small-signal Ladbrooke model [8] while overcoming their respective shortcomings. Its attractive statistical properties have already been presented in [2,3].

The KTL small-signal equivalent circuit follows the Ladbrooke model, shown in figure 4. The model includes the intrinsic FET parameters, L, Z, a, N_d, V_{b0}, v_{sat}, μ_0, ε, L_{G0}, a_0, r_{01}, r_{02}, r_{03}, and the linear extrinsic elements, L_g, R_g, L_d, R_d, L_s, R_s, G_{ds}, C_{ds}, C_{ge}, C_{de}, where L is the gate length, Z the gate width, a the channel thickness, N_d the doping density, V_{b0} the zero-bias barrier potential, v_{sat} the saturation value of electron drift velocity, μ_0 the low-field mobility of GaAs, ε the dielectric constant, L_{G0} the inductance from gate bond wires and pads, a_0 the proportionality coefficient, and r_{01}, r_{02} and



Parameter	Mean	σ(%)	Parameter	Mean	σ(%)
$L(\mu m)$	0.4685	3.57	$C_{ds}(pF)$	0.0547	1.58
$a(\mu m)$	0.1308	5.19	$C_{ge}(pF)$	0.0807	5.92
$N_d(m^{-3})$	2.3×10^{23}	3.25	$C_{de}(pF)$	0.0098	6.22
$V_{sat}(m/s)$	10.5×10^{4}	2.27	$C_x(pF)$	2.4231	4.03
$\mu_O(m^2/Vns)$	6.5×10^{10}	2.16	$Z(\mu m)$	300	*
$L_{GO}(nH)$	0.0396	10.9	ε	12.9	*
$R_d(\Omega)$	1.2867	4.32	$V_{bO}(V)$	0.6	*
$R_s(\Omega)$	3.9119	1.91	$r_{O1}(\Omega/V^2)$	0.35	*
$R_g(\Omega)$	8.1718	0.77	$r_{O2}(V)$	7.0	*
$L_d(nH)$	0.0659	5.74	$r_{O3}(\Omega)$	2003	*
$L_s(nH)$	0.0409	5.49	a_O	1.0	*
$G_{ds}(1/\Omega)$	3.9×10^{-3}	1.78			

σ denotes standard deviation

* Assumed fixed (non-statistical) parameters

Figure 4 (above): The small-signal equivalent circuit of the KTL model, where $I_d = g_m V_g e^{j\omega\tau}$.
Table 1: Optimized KTL model parameter statistics

r_{O3} the fitting coefficients [1,3,5].

The bias-dependent small-signal parameters, namely, g_m, C_{gs}, C_{gd}, R_i, L_g, r_o and τ, as shown in figure 4, are derived using the modified Ladbrooke formulae once the DC operating point is obtained using the Khatibzadeh and Trew model [3].

A sample of GaAs MESFET data which is obtained by aligning wafer measurements to a consistent bias condition [3] is used for statistical modelling. There are 35 data sets (devices) containing the small-signal S parameters measured at the frequencies from 1 to 21GHz with a 2GHz step under the bias condition of V_{gs} = -0.7V and V_{ds} = 5V.

We first use multi-device parameter extraction and statistical postprocessing based on 15 devices to obtain the initial parameter statistics including the mean values, standard deviations and the correlation matrix. Then, the initial model is optimized using the direct approach of CPD fitting. We consider 16 statistical parameters with normal distributions. This results in 32 optimization variables, namely all the means and standard deviations. The statistical KTL model parameter values after optimization are listed in table 1. The CPDs of the real part of S_{21} (RS21) at 11GHz from the data and from the statistical KTL model before and after optimization are shown in figure 5. We can see that after optimization the CPD matching between the data and the KTL model is significantly improved. The histograms of the imaginary part of S_{21} (IS21) at 11GHz from the data and from the statistical KTL model before and after optimization are shown in figure 6. We can see that after optimization the histogram matching between the data and the KTL model is also improved.

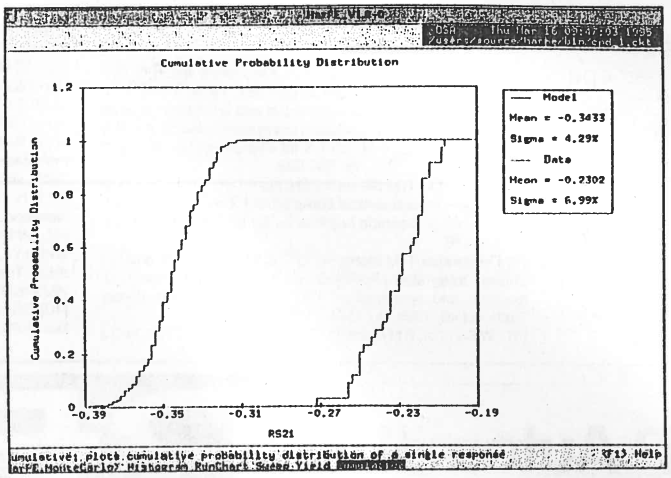
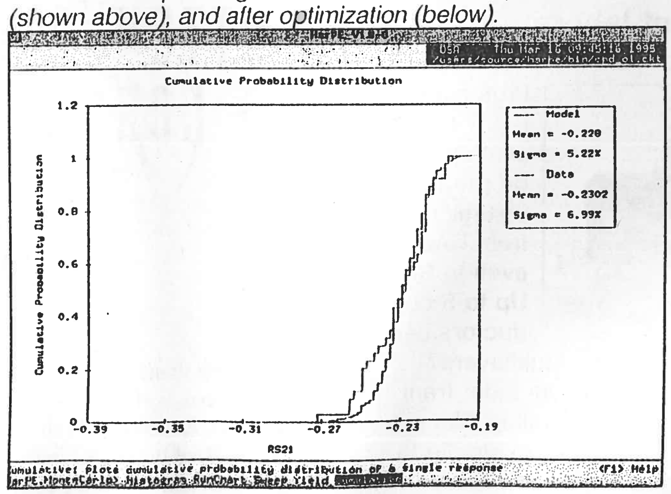


Figure 5: CPDs of the simulated RS21 (the real part of S_{21}) and the corresponding data at 11GHz, before optimization (shown above), and after optimization (below).



Conclusions

Statistical fluctuations in the manufacturing process cause variations in device parameter values, and consequently in device performance. The ultimate purpose of statistical modelling is to characterize devices for accurate yield and cost analysis and optimization.

We have presented two approaches to statistical modelling: indirect and direct. By combining the two methods we can obtain accurate statistical device models. This has been demonstrated on the example of statistical modelling of a GaAs MESFET using the KTL model: Model verification has been illustrated by comparing the distributions (CPDs and histograms) of the model responses and those of the data.

It should be pointed out that measurement errors may significantly affect accuracy of the resulting statistical model. If the measurement data contains some wild points (eg, due to faulty devices) they may severely degrade the resulting model and should be removed. A robust approach using the Huber function has been proposed to automatically handle such errors [9].

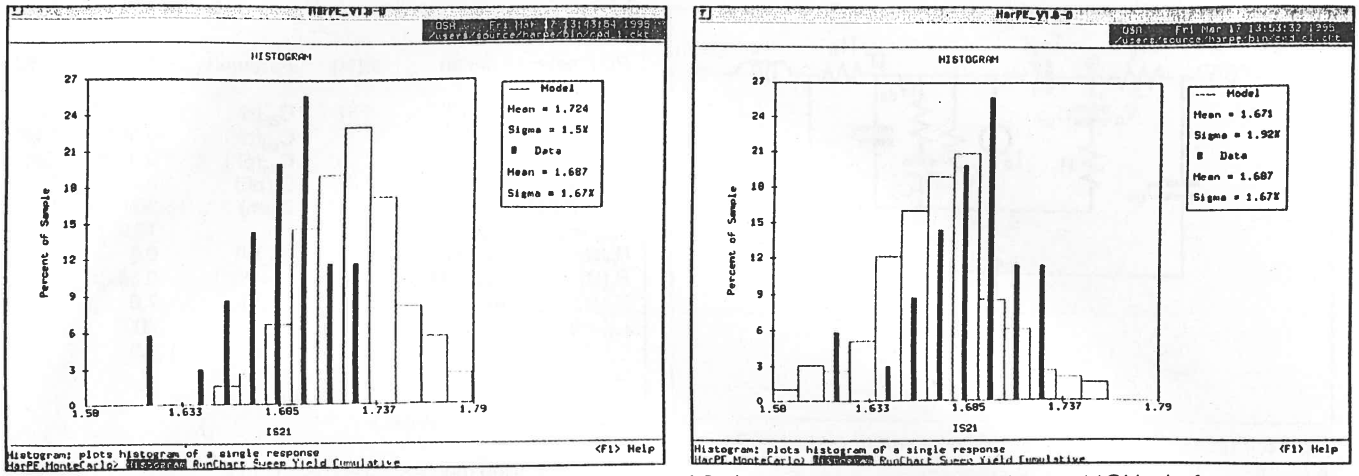


Figure 6: Histograms of the simulated IS21 (the imaginary part of S_{21}) and the corresponding data at 11GHz, before optimization (shown left), and after optimization (right).

Our advanced algorithms together with a number of state-of-the-art optimizers including l_1, l_2 (the least squares) and Huber, and aided by useful statistical displays implemented in HarPE, provide a complete environment for statistical modelling. The extracted statistical models can be used, for example, in OSA90/hope [10] for yield-driven and cost-driven circuit design.

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