# Modeling Of Microwave Circuits Exploiting Space Derivative Mapping

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Abstract—We present a novel approach to microwave circuit modeling, Space Derivative Mapping (SDM). SDM assumes the existence of an empirical model of the structure under consideration. It enables the construction of a space mapping-based locally valid model exploiting, for the first time, both the empirical simulations and the response sensitivity information. Parameter extraction uniqueness is no longer important. Statistical analysis of microwave circuits illustrates SDM.

#### I. INTRODUCTION

Full-wave simulations of microwave structures are CPU intensive. Developing fast and accurate models for simulating microwave circuits that can be utilized for design purposes over wide ranges of the parameter space is crucial. Space Mapping (SM) was introduced [1, 2] to address this problem.

In this paper we present a novel technique for microwave circuit modeling based on SM. SM assumes the existence of "coarse" and "fine" models for the circuit under consideration. The coarse model is fast but not necessarily very accurate (equivalent circuits, empirical formulas, etc.). The fine model is accurate but CPU intensive. Aggressive Space Mapping (ASM) [2] optimization, for example, iteratively establishes a mapping between the spaces of the parameters of the two models. After the optimization is completed a matrix **B** represents a valid mapping in the vicinity of the optimal design. On

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the other hand, the originally proposed SM approach [1] establishes a mapping between both spaces by individually extracting a set of coarse model points corresponding to a given set of fine model points as a prerequisite to optimization. The nonuniqueness of any of the extracted coarse model points is likely to result in an inaccurate mapping between the two spaces.

The SDM Modeling (SDMM) technique proposed here creates a locally valid mapping-based model in the vicinity of a point of interest in the fine model space. The technique is based on a novel lemma that estimates the mapping between the two spaces using only a single parameter extraction. The uniqueness of the extracted parameters is not a serious issue. Sensitivity information in both spaces is utilized in the construction of the mapping. The SDMM technique exploits the constructed mapping and coarse model simulations.

## II. BRIEF BACKGROUND

We refer to the vector of fine model parameters and the vector of coarse model parameters as  $x_{em}$  and  $x_{os}$ , respectively. The aim of SM optimization is to obtain the set of fine model parameters  $\overline{x}_{em}$  whose fine model response matches the optimal coarse model response evaluated at  $x_{os}^*$ , the optimal coarse model design. The ASM algorithm constructs the mapping iteratively, returning the final design  $\overline{x}_{em}$  and matrix  $\boldsymbol{B}$  representing the mapping at  $\overline{x}_{em}$ . A perturbation of  $\Delta x_{em}$  in the fine model space is mapped to a perturbation of  $\Delta x_{os}$  in the coarse model space by [2]

$$\Delta \mathbf{x}_{os} = \mathbf{B} \Delta \mathbf{x}_{em} \tag{1}$$

such that the fine model point  $\overline{x}_{em} + \Delta x_{em}$  and the coarse model point  $x_{os}^* + \Delta x_{os}$  have matched responses. The mapping (1) together with the coarse model essentially builds a fast and accurate model for the circuit under consideration in the vicinity of the final design  $\overline{x}_{em}$ . It can be used in additional analyses, e.g., statistical analysis.

#### III. THE NEW TECHNIQUE

The SDMM technique is based on the following novel lemma.

Lemma Assume that  $x_{os}$  corresponds to  $x_{em}$  through a parameter extraction process. Then the Jacobian  $J_{em}$  of the fine model responses at  $x_{em}$  and the Jacobian  $J_{os}$  of the coarse model responses at  $x_{os}$  are related by

$$\boldsymbol{J}_{em} = \boldsymbol{J}_{os} \boldsymbol{B} \tag{2}$$

where B is a valid mapping between the two spaces at  $x_{os}$  and  $x_{em}$  that satisfies (1). The proof of this novel lemma is omitted here for the sake of brevity.

It follows from (2) that

$$\boldsymbol{B} = \left(\boldsymbol{J}_{os}^T \boldsymbol{J}_{os}\right)^{-1} \boldsymbol{J}_{os}^T \boldsymbol{J}_{em} \tag{3}$$

Relation (3) assumes that  $J_{os}$  is a full rank matrix and  $m \ge n$ , where n is the number of parameters and m is the number of responses. It shows that B can be obtained by multiplying the Jacobian of the fine model responses with the pseudoinverse of  $J_{os}$ . A similar formula can be obtained using singular value decomposition [3] if  $J_{os}$  is not full rank

Suppose it is required to obtain a fast and accurate approximation to the fine model response in the vicinity of a particular point  $x_{em}^*$ . We denote by  $J_{em}^*$  the Jacobian of the fine model responses at  $x_{em}^*$ . The first step is to obtain the point  $\overline{x}_{os}$  corresponding to  $x_{em}^*$  through the parameter extraction problem

$$\overline{\mathbf{x}}_{os} = arg \left\{ \min_{\mathbf{x}_{os}} \left\| \mathbf{R}_{em}(\mathbf{x}_{em}^{*}) - \mathbf{R}_{os}(\mathbf{x}_{os}) \right\| \right\}$$
(4)

The Jacobian  $\overline{J}_{os}$  of the coarse model responses at  $\overline{x}_{os}$  may be estimated by perturbation. Both the parameter extraction step (4) and the evaluation of the Jacobian of the coarse model responses should add no significant overhead since the coarse model is assumed to be much faster than the fine model. The matrix B is then calculated by applying (3) as

$$\boldsymbol{B} = \left(\overline{\boldsymbol{J}}_{os}^{T} \overline{\boldsymbol{J}}_{os}\right)^{-1} \overline{\boldsymbol{J}}_{os}^{T} \boldsymbol{J}_{em}^{*} \tag{5}$$

Once B is available the SDM model is provided by the simple formula

$$R_{am}(x_{am}) \approx R_{os}(\overline{x}_{os} + B(x_{em} - x_{em}^*)) \tag{6}$$

This model is expected to enjoy a wide region of validity as the two models are assumed to share the same physical structure. The similarity in the nonlinear behavior of the two models makes this model superior to linear response approximation in the fine model space.

The uniqueness of the parameter extraction problem (4) should not affect the SDM model. If a different extracted point  $\bar{x}_{os}$  is obtained, a different mapping B given by (5) will be a valid mapping at the two points  $x_{em}^*$  and  $\bar{x}_{os}$ . The SDM model given by (6) is still an accurate model.

## IV. TWO-SECTION WAVEGUIDE TRANSFORMER

The SDMM technique is tested on the statistical analysis of a two-section waveguide transformer [4]. The coarse model is an "ideal" analytical model which neglects the junction discontinuity effects while the fine model is a more accurate "nonideal" analytical model which includes the junction discontinuity effects [4]. The design constraints for this problem are

$$vswr \le 1.04 \text{ for } 5.8 \text{ GHz} \le f \le 6.6 \text{ GHz}$$
 (7)

Optimizable parameters are the height and the length of each waveguide section.

The fine model is optimized using the minimax optimizer available in OSA90/hope [5]. An estimate for the Jacobian of the fine model responses is then obtained. Parameter extraction is applied to get  $\overline{x}_{os}$ . Fig. 1. shows the optimal fine model response and the coarse model response at  $\overline{x}_{os}$ . The Jacobian  $\overline{J}_{os}$  is estimated using perturbation. An estimate for  $\boldsymbol{B}$  is calculated using (5). The designable parameters are assumed to be uniformly distributed with equal relative tolerances. We apply SDMM to carry out space-mapped statistical analysis using (6) with 100 samples. Tolerances considered are 1% and 4%. Fine model responses were used to verify the accuracy of the SDMM. Corresponding responses are shown in Figs. 2 and 3.

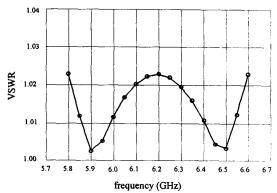


Fig. 1. The optimal fine model response (o) and the response (—) at the corresponding coarse model point for the twosection waveguide transformer.

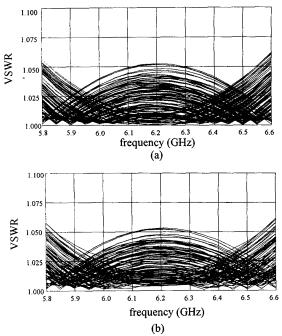
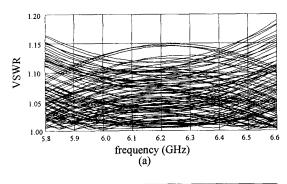


Fig. 2. Statistical analysis for the two-section waveguide transformer assuming uniform distribution with relative tolerances of 1.0%, (a) using the SDMM, and (b) using fine model simulations.



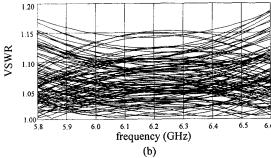


Fig. 3. Statistical analysis for the two-section waveguide transformer assuming uniform distribution with relative tolerances of 4.0%, (a) using the SDMM, and (b) using fine model simulations.

## V. SIX-SECTION H-PLANE WAVEGUIDE FILTER [6, 7]

A waveguide with a cross-section of 3.485 cm by 1.5 cm is used. As shown in Fig. 4, the six sections at separated by seven H-plane septa, which have a finith thickness of 0.508 mm

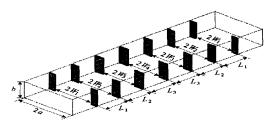


Fig. 4. The six-section H-plane waveguide filter [6, 7].

Optimizable parameters are the septa widths  $W_1$ ,  $W_3$  and  $W_4$  and the lengths  $L_1$ ,  $L_2$  and  $L_3$ . The coars model consists of lumped inductances and dispersive transmission line sections simulated by OSA90/hope [5] For the equivalent inductances of the H-plane septa we utilize formulas by Marcuvitz [8]. The fine model exploits  $HP\ HFSS$  [9] through  $HP\ Empipe3D$  [10].

The fine model is optimized using the minimax opt mizer available in OSA90/hope. Parameter extraction then applied to obtain  $\overline{x}_{os}$ . The corresponding response are shown in Fig. 5.

The linear interpolation formulas given in [11] a utilized to estimate  $J_{em}^*$  using the database generate during optimization.  $\overline{J}_{os}$  is estimated by perturbatio The mapping B is then calculated using (5).

Figs. 6 and 7 compare statistically generated responsible obtained using 100 random points, uniformly distribute with 1% and 4% relative tolerances using SDMM and fir model simulations.

### VI. CONCLUSIONS

We present a novel technique for the fast and accura modeling of microwave circuits. The technique exploits Space Derivative Mapping (SDM) approach in the co struction of a space-mapping based model. We introdu a novel lemma that enables the establishment of the mapping between the designable input parameters to a electromagnetic optimizer and the parameters of a corr sponding empirical model with no additional overhead electromagnetic simulations. SDM Modeling (SDM) alleviates the extraction uniqueness problem involved prior SM algorithms and the necessity of applying S optimization in the ASM algorithm. Statistical analysis microwave circuits exemplifies our technique.

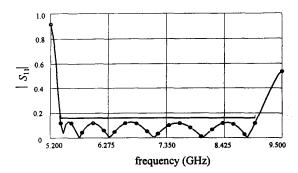


Fig. 5. The fine model response (o) and the response (—) at the extracted coarse model point for the six-section waveguide filter.

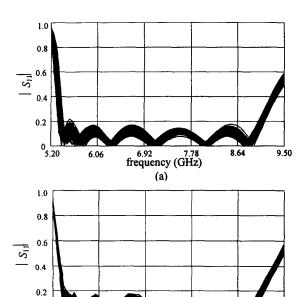


Fig. 6. Statistical analysis for the six-section waveguide filter assuming uniform distribution with relative tolerances of 1%, (a) using the SDMM, and (b) using fine model simulations.

6.92 7.78 frequency (GHz)

(b)

8.64

9.50

0 5.20

6.06

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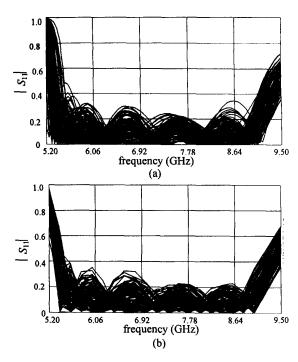


Fig. 7. Statistical analysis for the six-section waveguide filter assuming uniform distribution with relative tolerances of 4%, (a) using the SDMM, and (b) using fine model simulations.

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