(*c*) above holds to a good approximation for a practical transducer of this type. Thus, if one of the electrical ports is connected to a load and the other to an impedance equal to the load impedance, no reflections will occur from such an output transducer, and echoes will be eliminated.

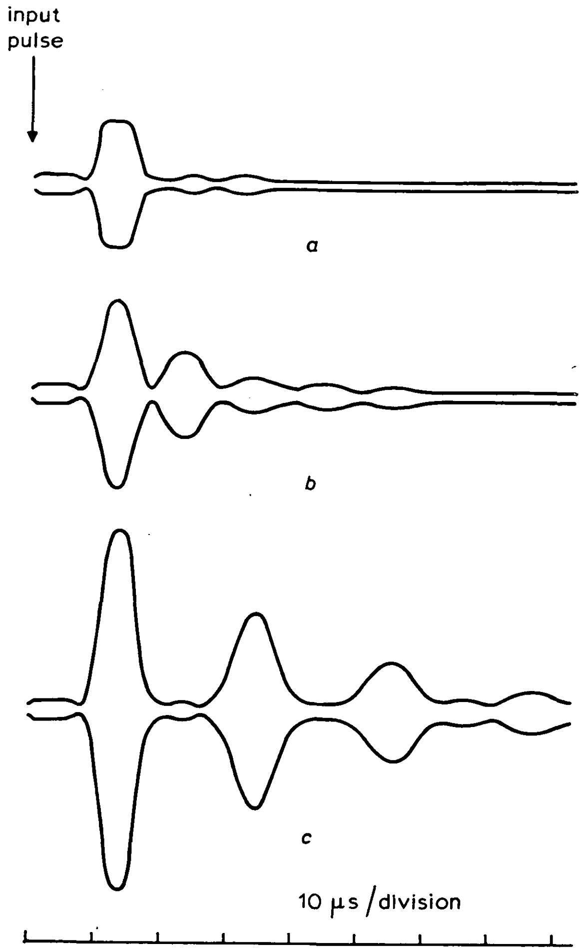

Fig. 3 *Signals received at output transducer*

A further experiment was performed to measure the directivity of the hybrid junction. With shunt coils for electrical matching, and with the two electrical ports driven 90° out of phase, the observed directivity exceeded 17 dB over a 16% bandwidth centred at 5 MHz.

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COMPUTATION OF EQUIVALENT WAVE SOURCE USING THE ADJOINT NETWORK

Indexing terms: Network analysis, Equivalent circuits

A method for computing the equivalent wave source at some port of an excited reciprocal or nonreciprocal network using one analysis of the appropriately terminated adjoint network is presented. The procedure should be useful in simplifying the scattering-matrix analysis of complicated networks.

A scattering-wave counterpart of Thévenin's and Norton's theorems has been formulated by Nemoto and Wait.[1] They show that, at a given port, an arbitrary network can be replaced by an equivalent wave source. They indicate how the equivalent wave and reflection coefficient might be computed, and suggest applications.

This letter will show how the equivalent wave source at some port of an excited reciprocal or nonreciprocal network (see Fig. 1A) may be computed using only one analysis of the appropriately terminated adjoint network[2–4] (see Fig. 1B).

We need to invoke a theorem for scattering variables presented and discussed by Bandler and Seviora.[4] For our present purposes, we may state that

$$\boldsymbol{b}_E{}^T \boldsymbol{\alpha}_E - \boldsymbol{a}_E{}^T \boldsymbol{\beta}_E = \begin{bmatrix} \boldsymbol{b}_E \\ \boldsymbol{b}_I \end{bmatrix}^T \begin{bmatrix} \boldsymbol{\alpha}_E \\ \boldsymbol{\alpha}_I \end{bmatrix} - \begin{bmatrix} \boldsymbol{a}_E \\ \boldsymbol{a}_I \end{bmatrix}^T \begin{bmatrix} \boldsymbol{\beta}_E \\ \boldsymbol{\beta}_I \end{bmatrix} \qquad (1)$$

where a and α denote normalised incident waves in the given and topologically similar adjoint networks, respectively, b and β denote the corresponding normalised reflected waves, the subscript E denotes vectors associated with external or excited ports and the subscript I denotes vectors associated with the remaining or internal ports. Eqn. 1 holds for arbitrary complex normalisation, but corresponding ports of the two networks are assumed to be similarly normalised. It is further assumed that there are no changes in normalisation across external junctions. Eqn. 1 arises by adding terms equal to its left-hand side (quantities subscripted by E) to both sides of a valid equation (involving quantities subscripted by I) derived as a special case of a more general equation.[4]

If every element in the given network is described by an equation of the form

$$\boldsymbol{b} = \boldsymbol{S}\boldsymbol{a} \qquad (2)$$

where $\boldsymbol{S}$ is a scattering matrix, $\boldsymbol{a}$ the vector of incident waves and $\boldsymbol{b}$ the vector of reflected waves, every element in the adjoint network is described by a corresponding equation of the form[4]

$$\boldsymbol{\beta} = \boldsymbol{S}^T \boldsymbol{\alpha} \qquad (3)$$

The terms on the right-hand side of eqn. 1 relating to any specific element, i.e.

$$\boldsymbol{b}^T \boldsymbol{\alpha} - \boldsymbol{a}^T \boldsymbol{\beta} \qquad (4)$$

reduce to

$$\boldsymbol{a}^T \boldsymbol{S}^T \boldsymbol{\alpha} - \boldsymbol{a}^T \boldsymbol{S}^T \boldsymbol{\alpha} = 0 \qquad (5)$$

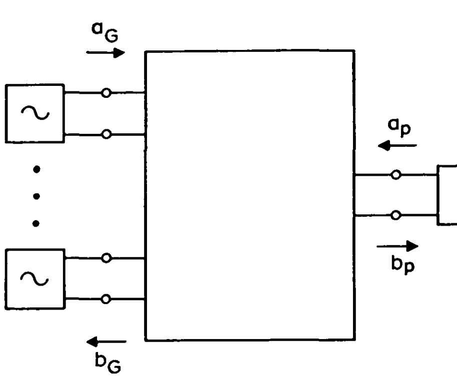

Fig. 1A *Given excited network for which an equivalent wave source is required looking to the left at the terminals of port p* so that the right-hand side of eqn. 1 is reduced to zero, giving

$$\boldsymbol{b}_E{}^T \boldsymbol{\alpha}_E - \boldsymbol{a}_E{}^T \boldsymbol{\beta}_E = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Let subscript G denote vectors associated with the excited ports of the given network and p the waves at the port of interest. Then eqn. 6 can be written

$$\boldsymbol{b}_G{}^T \boldsymbol{\alpha}_G - \boldsymbol{a}_G{}^T \boldsymbol{\beta}_G + b_p \alpha_p - a_p \beta_p = 0 \quad . \quad . \quad . \quad . \quad (7)$$

Suppose that the adjoint network is terminated as shown in Fig. 1B; i.e. the G ports are terminated in their normalisation numbers resulting in $\boldsymbol{\alpha}_G = \mathit{0}$ and a unit wave is applied at p. If port p in the given network is terminated in its normalisation number $a_p = 0$, and eqn. 7 is reduced to

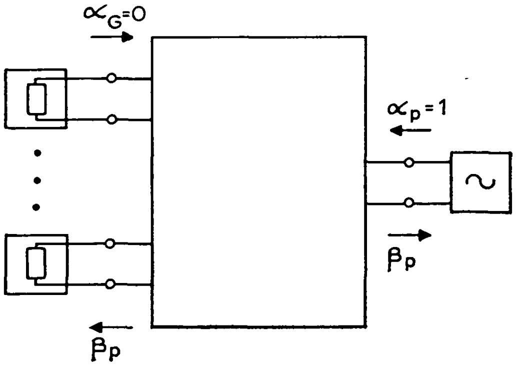

Fig. 1B *Appropriately terminated and excited adjoint netwok*

$$b_{eq} = b_p = \boldsymbol{a}_G{}^T \boldsymbol{\beta}_G \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where b_{eq} is the equivalent wave.

Now, suppose that the excitations in the given network are set to zero, i.e. $\boldsymbol{a}_G = \mathit{0}$, but port p is excited. Let the terminations and excitation of the adjoint network remain unchanged. In this case, eqn. 7 is reduced to

$$b_p \alpha_p - a_p \beta_p = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

so that

$$\rho_{eq} = \frac{b_p}{a_p} = \frac{\beta_p}{\alpha_p} = \beta_p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where ρ_{eq} is the equivalent reflection coefficient.

Observe that both b_{eq} and ρ_{eq} are evaluated from the results of just one analysis of the suitably terminated adjoint network. The given network does not have to be analysed. It might be remarked that, if the given network is reciprocal and hence identical to its adjoint, the same analysis can be used to yield the sensitivities of ρ_{eq} with respect to parameters in the given network, for example[4]

$$\frac{\partial \rho_{eq}}{\partial \phi_j} = \boldsymbol{\alpha}_i{}^T \frac{\partial \boldsymbol{S}_i}{\partial \phi_j} \boldsymbol{\alpha}_i$$

where ϕ_j is a jth parameter contained in an ith element of the scattering matrix $\boldsymbol{S}_i$ and $\boldsymbol{\alpha}_i$ represents the incident waves on the ports of the corresponding element in the adjoint. A second analysis of the given network would yield the sensitivities of b_{eq}.

It is expected that the procedure presented here can be used to advantage in simplifying the scattering-matrix analysis of complicated networks.

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ERRATUM

RASHIDI, K.: 'Pulse response of 0° Z cut ADP modulators', *Electron. Lett.*, 1971, **7**, pp. 114–115

The author wishes to make the following correction to his paper:

Lines 6 and 7 of the second column on p. 115 should read:

($16 \cdot 25 \times 10^{-11}$ m^2/N for ADP), and ρ is the density of the material ($1 \cdot 804 \times 10^3$ kg/m^3 for ADP).

Use of SI units in IEE papers

Authors of papers for the *Proceedings IEE*, of contributions to *Electronics Letters*, and of articles for other IEE publications will be required to use SI units (Système International d'Unités) in manuscripts submitted for the first time as from the 1st January 1972. On and after that date, any manuscripts using units other than SI will be returned to the authors for revision. In the meantime, authors of papers and articles submitted for consideration before 1st January 1972 are strongly requested to use SI units in their manuscripts.

Authors are particularly referred to the BSI publication PD5686: 'The use of SI units'. It should be noted that the use of the decibel and neper is permitted.