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### ABSTRACT

## Invited Paper

Computer-oriented circuit optimization techniques are reviewed. The preparation of the circuit design problem for computer optimization is discussed in some detail. Approximation methods, including least pth and minimax, are also discussed in detail. In particular, generalized least pth and minimax objectives are proposed as the most meaningful for circuit optimization. The adjoint network method of evaluating derivatives of circuit responses with respect to circuit parameters both in terms of voltages and currents and also in terms of wave variables is reviewed. A bibliography of relevant papers, including state-of-the-art reviews, is appended.

### INTRODUCTION

The prospect of fully automated optimal design of electrical circuits is an exciting one. Many research workers are currently developing techniques which should ultimately lead in this direction. Problem areas of relevance include development of efficient circuit analysis methods, development of efficient methods of generating circuit sensitivities and development of efficient methods of optimization.

The purpose of this paper is to review computer-oriented circuit optimization techniques. Good preparation of the circuit design problem as an optimization problem is essential if meaningful results are to be obtained. One section is devoted to this topic. Another section concerns least pth and minimax approximation. In particular, it is recommended that generalized objectives be used more often than is currently done. A section on the adjoint network method of evaluating the required derivatives for use in the optimization process is included. Finally, a bibliography of selected relevant papers is appended. Included are state-of-the-art reviews which the computeroriented circuit designer should find useful as an introduction to the subject.

# STEPS IN CIRCUIT DESIGN

We may outline ten fairly distinct steps involved in the computer-aided design of circuits using numerical optimization methods as follows. 1) Possible design goals must be considered in general terms. The designer should then clearly understand what his problem is and what the circuit to be designed is to achieve.

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3) An objective function for minimization has to be formulated. It must adequately reflect the design goal and should preferably not contain scale factors or constants which can not be easily fixed in advance. In circuit design the objective is usually to realize a least pth or minimax approximation.

4) Constraints should be formulated at this stage accounting, for example, for cost, size, stability, parasitic effects, parameter bounds, and so on. Too many designers appear to ignore this problem and then complain that optimization does not yield a realizable design.

5) A circuit analysis scheme for the chosen configuration has to be selected. Its choice will depend, for example, on whether the circuit is linear or nonlinear, lumped or distributed, and whether frequency- or time-domain responses are to be computed.

6) An important factor in choosing the analysis scheme is the availability or otherwise of partial derivatives of the response functions with respect to those parameters which are to be optimized. The use of the adjoint network method, for example, should be contemplated at this stage.

7) An optimization method has to be chosen. It may be a direct search method if derivatives are not readily available. If derivatives are readily available an efficient gradient method is to be preferred. The optimization method may itself involve iterative use of another optimization method, for example, linear programming.
8) The design problem should now be ready for

programming. Available computer programs should be used when possible. Programs should, if necessary, be adapted to the problem rather than the other way round.

9) An initial design, preferably feasible, has to be found to serve as a starting point in the optimization process. Generally speaking, the better this initial guess the faster the process will be.

10) If all the foregoing steps have been carefully implemented the optimal design should now be obtained by running the program. Unfortunately, however, snags may occur. False (as distinct from local) minima are often obtained due to poor scaling. Nonconvexity of the objective function may also result in the failure of the process to converge to the proper solution. As a rule of thumb, the program should be run starting from as many different feasible starting points as there are variable parameters, for a modest number of parameters. If at least a few terminating points are found to be the same it is reasonable to assume that a local minimum has been found. In the author's experience, the widely-held conviction that every point at which an optimization process terminates in a circuit design problem is a local minimum is usually unfounded. The corresponding design may be acceptable, but it need not even correspond to a local minimum.

# APPROXIMATION METHODS

The approximation of desired frequencyor time-domain responses is probably the most common type of problem a circuit designer has to solve. Both least pth (a generalization of least squares) and minimax (or Chebyshev) approximation techniques are employed. In this section the two types of approximation will be discussed.

### Least pth Approximation

the form

Minimization of an objective function of

$$U = \sum_{i \in I} |e_i(\psi)|^p \tag{1}$$

where  $e_i(\phi)$  is (in general) a complex nonlinear error function of the variable parameters  $\phi$ , I is a set of indices i and p is usually greater than or equal to 1, is called discrete least pth approximation. A sufficient number of sample points is chosen so that the discrete problem adequately represents the continuous problem.

Temes and Zail have presented a method of solving least pth approximation problems and applied it to the design of an active equalizer<sup>1,2</sup>. Assuming that partial derivatives of the objective function are available, any suitable efficient gradient method of minimization can be used to solve the problem<sup>3-6</sup>. A necessary condition for optimality is that the gradient vector of the objective function is zero. Sufficiency results if the function is also convex.

Bandler and Charalambous have recently presented generalized least pth objectives<sup>7</sup>. Design problems in which it is desired to meet or exceed certain upper and lower response specifications should be capable of solution using these objectives.

If the specification is violated it is recommended that

$$U = \sum_{i \in J_{n}} \left[ e_{ui}(\phi) \right]^{p} + \sum_{i \in J_{\rho}} \left[ -e_{\ell i}(\phi) \right]^{p}$$
(2)

where

$$J_{u} \triangleq \{i | e_{ui}(\phi) \ge 0, i \in I_{u}\}$$

$$J_{\ell} \triangleq \{i | -e_{\ell i}(\phi) \ge 0, i \in I_{\ell}\}$$
(3)

where subscript u denotes quantities related to an upper specification, i.e., one for which it is desired to force the circuit response below, and subscript  $\ell$  denotes quantities related to a lower specification, i.e., one for which it is desired to force the circuit response above. The error functions in this case are assumed to be real. Briefly, only those parts of the response which violate the specification are included in the objective function. For p>1 and under some mild restrictions the objective function and its first derivatives are continuous and the matrix of second partial derivatives is positive-semidefinite. Good results are, therefore, to be expected using gradient methods.

If the specification is satisfied it is recommended that

$$U = \sum_{i \in I_{u}} [-e_{ui}(\phi)]^{-p} + \sum_{i \in I_{\ell}} [e_{\ell i}(\phi)]^{-p}$$
(4)

where

$$-\mathbf{e}_{ui}(\phi) > 0 \qquad i \in \mathbf{I}_{u} \qquad (5)$$
$$\mathbf{e}_{li}(\phi) > 0 \qquad i \in \mathbf{I}_{l}$$

Thus, all parts of the response satisfy the specification. For  $p \ge 1$  and under some mild restrictions the objective function and its first derivatives are continuous and the matrix of second partial derivatives is positive-semi-definite.

#### Minimax Approximation

Minimization of an objective function of the form

$$U = \max |e_i(\phi)|$$
(6)  
i \varepsilon I

where the error functions may again be complex, is called discrete minimax approximation. It is easily shown that minimizing the objective function of (1) for a sufficiently large value of p results in a solution very close to the minimax solution. p=10 is often sufficiently large for practical purposes.

More general minimax objectives suitable for solving circuit design problems involving upper and lower specifications have been discussed<sup>8</sup>. Consider the objective function

$$\mathbf{U} = \max_{i} [\mathbf{e}_{ui}(\mathbf{x}), -\mathbf{e}_{li}(\mathbf{x})] \tag{7}$$

to be minimized, where, as previously, the error functions are assumed to be real. Letting  $U=\min U$ , if U>0 the maximum (weighted) amount by which the response fails to meet the specification has been minimized, and if U<0 then the minimum (weighted) amount by which the response exceeds the specification has been maximized.

Lasdon and Waren<sup>9</sup> have presented a method for minimax approximation and have applied it to the design of crystal filters and other problems<sup>9</sup>,10. Their approach requires the reformulation of the problem in terms of inequality constraints<sup>8</sup> and solution via the sequential unconstrained minimization technique<sup>11</sup>,12.

Ishizaki and Watanabe<sup>13</sup> have developed a method employing linear programming iteratively and have applied it to the design of attenuation and group delay equalizers. Their method at a particular stage involves linearization of the nonlinear error functions and the solution of an appropriate linear programming problem by the simplex method<sup>14</sup>. The Osborne and Watson method of minimax approximation appears to be basically the same<sup>15</sup>. A method based on it has been tested on transmission-line impedance matching circuits 16

Bandler and Macdonald developed the

razor search method<sup>17</sup> for direct minimax approximation for situations when partial derivative information is not readily available. Their method is based on pattern search  $^{18}$  and involves a few random moves to negotiate paths of discontinuous derivatives created by using an objective function of the form of (6). More recently, a gradient razor search method has been presented by Bandler and Lee-Chan<sup>19</sup> exploiting the adjoint network method of evaluating derivatives. Downhill paths of discontinuous derivatives are sought by solving an appropriate set of linear simultaneous equations. A generalization of the gradient razor search method is the method due to Bandler and Srinivasan<sup>16</sup>, involving the solution of linear programming problems at various stages during the optimization process. Transmission-line impedance matching circuits and filters have been optimized to test these algorithms<sup>16</sup>,17,19.

Conditions for optimal approximations in the minimax sense have been formulated<sup>20</sup>. Their use in tests for optimality of proposed circuit designs has been demonstrated.

## Discussion

An ideal response is usually impossible to achieve in practice. The most logical design specifications, in this author's opinion, are those which indicate acceptable deviations from the ideal. The optimization procedure would then be used to force the circuit response to meet or exceed these specifications. The most meaningful objective functions, in general, from the point of view of practical circuit design are, therefore, of the form of (2), (4) and (7). Unfortunately, however, least squares approximation of ideal responses by circuit responses are still widely employed. This practice is not usually consistent with step 3) of the previous section. Designs obtained by least squares approximation may sometimes be acceptable, but they may not be the best that the corresponding configurations can realize.

### THE ADJOINT NETWORK METHOD

The adjoint network method of evaluating partial derivatives of circuit responses with respect to circuit parameters has undoubtedly stimulated renewed interest in the possibility of fully automating circuit design as well as the application of efficient gradient methods of minimization.

Originally proposed by Director and Rohrer as a useful tool for computer-aided circuit design<sup>21</sup>,<sup>22</sup>, the method has since received considerable attention and further developments and applications have become widespread<sup>23-35</sup>.

The main result of using the adjoint network method is that all required first-order partial derivatives of the objective function can be obtained using at most the results of two complete analyses of the circuit regardless of the number of parameters involved and without actually perturbing them. At best, one can obtain the first partial derivatives using the results of only the analysis of the given circuit. This is true, for example, in the one-port design of reciprocal circuits using the reflection coefficient. Second-order derivatives may also be obtained efficiently using the adjoint network method.

Director and Rohrer presented their fundamental paper on the generalized adjoint network and network sensitivities<sup>21</sup>. They applied it to design in the frequency domain of lumped linear time-invariant circuits, employing a least squares type of objective function<sup>22</sup>. It has also been shown how least pth and minimax types of objective functions can be used<sup>23</sup>,<sup>24</sup>,<sup>25</sup>. An extension of the adjoint network method to secondorder sensitivities has been published<sup>26</sup>. Recent applications of the adjoint network concept include the simultaneous automated a.c. and d.c. design of linear integrated circuit amplifiers<sup>27</sup>. Of interest in that paper are the ideas on "growing" transistors.

Bandler and Seviora have extended the usefulness of the adjoint network method to gradient computations for circuits containing a variety of uniformly distributed elements<sup>24</sup>,<sup>25</sup>. Mokari-Bolhassan and Trick<sup>28</sup> have also considered distributed elements and have applied these ideas to the design of a lumped-distributed-active RC filter.

Seviora and Sablatash<sup>29</sup> have considered the adjoint network approach from the point of view of discrete networks, in particular, the adjoint digital filter.

Monaco and Tiberio have considered adjoint networks using wave variables<sup>30</sup>. Bandler and Seviora have also dealt with wave variables, showing how first- and second-order sensitivities can be efficiently computed for linear timeinvariant (lumped and distributed) circuits<sup>31,32</sup>. Tables of useful sensitivity formulas have also been presented.

Group delay computations using the concept of the adjoint network have been considered<sup>33,34</sup>.

#### CONCLUSIONS

Out of the profusion of literature that now exists on subjects relevant to computer-aided circuit design, which papers might a circuit designer interested in applying computer-oriented optimization techniques benefit from?

Introductions to optimization techniques have been published by Temes<sup>2</sup> and Bandler<sup>8</sup>. Another useful article, dealing specially with minimax approximation, has been published by Waren <u>et. al.<sup>10</sup></u>. More recently, Bandler and Seviora have reviewed the application of the adjoint network method for design in the frequency domain<sup>24</sup>.

Rather up-to-date survey papers on optimization methods have been published by Powell<sup>36</sup> and Fletcher<sup>37</sup>.

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