

Space Mapping Interpolating Surrogates for Highly Optimized EM-Based Design of Microwave Devices

John W. Bandler, *Fellow, IEEE*, Qingsha S. Cheng, Sameh A. Dakroury, Daniel M. Hailu, Kaj Madsen, Ahmed S. Mohamed and Frank Pedersen

McMaster University, Hamilton, Canada L8S 4K1, www.sos.mcmaster.ca

Abstract — We present a powerful optimization algorithm that incorporates Space Mapping (SM) and the new Output Space Mapping (OSM) to yield highly optimized results in a handful of fine model evaluations. Our new method employs an SM-based interpolating surrogate (SMIS) framework that aims at matching the surrogate with the fine model locally. Accuracy and convergence properties are demonstrated using a seven-section capacitively-loaded impedance transformer. A highly optimized six-section H-plane waveguide filter design emerges after only four HFSS EM simulations, excluding necessary Jacobian estimations, using the new algorithm with sparse frequency sweeps.

Index Terms — Space mapping, EM optimization, microwave filter design, CAD algorithms.

I. INTRODUCTION

Space Mapping (SM) exploits coarse models (usually computationally fast circuit-based models) to align with fine models (typically CPU intensive full-wave EM simulations) [1]-[4]. The novel Output Space Mapping (OSM) [5] can further correct residual misalignment close to the optimal fine model solution, where a classical Taylor model is more accurate than SM.

We present an SM-based interpolating surrogate framework (SMIS). Highly accurate SM interpolating surrogate models are built for use in gradient-based optimization. The new SMIS interpolating surrogate is

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada under Grants OGP0007239, STR 234854 and STPGP 269760, through the Micronet Network of Centres of Excellence and Bandler Corporation.

J.W. Bandler is with the Simulation Optimization Systems Research Laboratory, Department of Electrical and Computer Engineering, McMaster University, Hamilton, ON, Canada L8S 4K1 and also with Bandler Corporation, P.O. Box 8083, Dundas, Ontario, Canada L9H 5E7.

Q.S. Cheng, S.A. Dakroury, D. M. Hailu and A.S. Mohamed are with the Simulation Optimization Systems Research Laboratory, Department of Electrical and Computer Engineering, McMaster University, Hamilton, ON, Canada L8S 4K1.

K. Madsen and F. Pedersen are with the Technical University of Denmark, DK-2800, Lyngby, Denmark.

required to match both the responses and derivatives of the fine model within a local region of interest [6].

The new SMIS framework is formulated in Section II. An algorithm based on it is outlined in Section III. Convergence is compared with the minimax algorithm available in the Matlab Optimization Toolbox on a seven-section capacitively-loaded impedance transformer. Finally, the SMIS algorithm is implemented on a six-section H-plane waveguide filter [7] (see Section IV).

II. SPACE MAPPING-BASED INTERPOLATING SURROGATES (SMIS) FRAMEWORK

A. Design Problem

The original design problem is

$$\mathbf{x}_f^* = \arg \min_{\mathbf{x}_f} U(\mathbf{R}_f(\mathbf{x}_f)) \quad (1)$$

Here, $\mathbf{R}_f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is the fine model response vector, e.g., $|S_{11}|$ at selected frequency points $\omega_i, i=1, \dots, m$; m is the number of sample points; the fine model point is denoted $\mathbf{x}_f \in \mathbb{R}^n$, where n is the number of design parameters. $U : \mathbb{R}^m \rightarrow \mathbb{R}$ is a suitable objective function, and $\mathbf{x}_f^* \in \mathbb{R}^n$ is the optimal design.

B. The Surrogate

The SM-based interpolating surrogate $\mathbf{R}_s^{(j)} : \mathbb{R}^n \rightarrow \mathbb{R}^m$, used in the j th iteration, aims at satisfying the interpolation conditions

$$\mathbf{R}_s^{(j)}(\mathbf{x}_f^{(j)}) = \mathbf{R}_f(\mathbf{x}_f^{(j)}) \quad (2)$$

$$\mathbf{J}_s^{(j)}(\mathbf{x}_f^{(j)}) = \mathbf{J}_f(\mathbf{x}_f^{(j)}) \quad (3)$$

where $\mathbf{J}_s^{(j)}(\mathbf{x}_f^{(j)})$ and $\mathbf{J}_f(\mathbf{x}_f^{(j)})$ are the Jacobians of the surrogate and fine model at $\mathbf{x}_f^{(j)}$, respectively.

The conditions (2) and (3), and global match condition are satisfied by transforming a coarse model $\mathbf{R}_c : \mathbb{R}^n \rightarrow \mathbb{R}^m$, through different linear input and output mappings to all responses. Fig. 1 illustrates the SMIS framework. Here $\mathbf{P}_i : \mathbb{R}^n \rightarrow \mathbb{R}^n$ [1][2] is an input mapping applied to the i th response, and $\mathbf{R}_{c,i} \circ \mathbf{P}_i$ is the mapped i th response of the coarse model. The i th

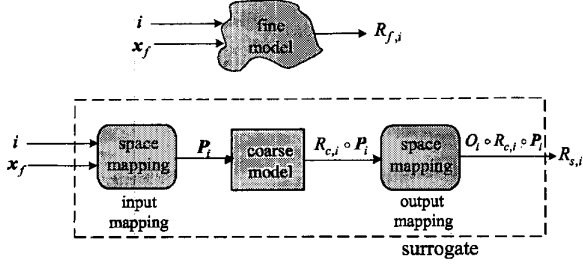


Fig. 1. Illustration of the SM-based interpolating surrogate (SMIS) concept. The aim is to calibrate the mapped coarse model (the surrogate) to match the fine model, using different input and output mappings for each sampled response.

response of the surrogate is a composed mapping $R_{s,i} = O_i \circ R_{c,i} \circ P_i$, where $O_i : \mathbb{R} \rightarrow \mathbb{R}$, and is defined as

$$R_{s,i}^{(j)}(\mathbf{x}_f) = \alpha_i^{(j)}(R_{c,i}(P_i^{(j)}(\mathbf{x}_f)) - R_{c,i}(P_i^{(j)}(\mathbf{x}_f^{(j)}))) + R_i^{(j)}, \quad i = 1, \dots, m, \text{ and } j = 0, 1, \dots \quad (4)$$

where $\alpha_i^{(j)} \in \mathbb{R}$, $R_i^{(j)} \in \mathbb{R}$, $i = 1, \dots, m$ are the output mapping parameters. The input mapping is defined as

$$P_i^{(j)}(\mathbf{x}_f) = \mathbf{B}_i^{(j)} \mathbf{x}_f + \mathbf{c}_i^{(j)} \quad (5)$$

where $\mathbf{B}_i^{(j)} \in \mathbb{R}^{n \times n}$, $\mathbf{c}_i^{(j)} \in \mathbb{R}^n$, $i = 1, \dots, m$ are the input mapping parameters.

The surrogate used in j th iteration is given by

$$\mathbf{R}_s^{(j)}(\mathbf{x}_f) = [R_{s,1}^{(j)}(\mathbf{x}_f) \dots R_{s,m}^{(j)}(\mathbf{x}_f)]^T \quad (6)$$

In each iteration, the surrogate is optimized to find the next iterate by solving

$$\mathbf{x}_f^{(j+1)} = \arg \min_{\mathbf{x}_f} U(\mathbf{R}_s^{(j)}(\mathbf{x}_f)) \quad (7)$$

In the first iteration, the mapping parameters $\mathbf{B}_i^{(0)} = \mathbf{I}$, $\mathbf{c}_i^{(0)} = \mathbf{0}$, $\alpha_i^{(0)} = 1$ and $R_i^{(0)} = R_{c,i}(\mathbf{x}_f^{(0)})$ are used, which ensure that $\mathbf{R}_s^{(0)}(\mathbf{x}_f) = \mathbf{R}_c(\mathbf{x}_f)$. For $j > 0$, the parameter $\mathbf{R}^{(j)} = \mathbf{R}_f(\mathbf{x}_f^{(j)})$ is utilized.

C. The Surface Fitting Approach for Parameter Extraction

Parameter extraction (PE) is a crucial step in any SM algorithm. In this paper, we employ a surface fitting approach for PE, which involves the minimization of residuals between the surrogate and fine model, and extracting the parameters $\mathbf{B}_i^{(j)}$, $\mathbf{c}_i^{(j)}$ and $\alpha_i^{(j)}$, $i = 1, \dots, m$.

The aim is to ensure matching responses (2) and derivatives (3), and a global match between the surrogate and the fine model, by satisfying the conditions $\mathbf{R}_s^{(j)}(\mathbf{x}_f^{(k)}) = \mathbf{R}_f(\mathbf{x}_f^{(k)})$, $k = 1, \dots, j+1$. Updating the surrogate from iteration j to $j+1$ involves the residual

$$\mathbf{r}_i^{(j+1)}(\alpha, \mathbf{B}, \mathbf{c}) \triangleq \begin{bmatrix} R_{s,i}^{(j+1)}(\mathbf{x}_f^{(1)}, \alpha, \mathbf{B}, \mathbf{c}) - R_{f,i}(\mathbf{x}_f^{(1)}) \\ \vdots \\ R_{s,i}^{(j+1)}(\mathbf{x}_f^{(j)}, \alpha, \mathbf{B}, \mathbf{c}) - R_{f,i}(\mathbf{x}_f^{(j)}) \\ J_{s,i}^{(j+1)}(\mathbf{x}_f^{(j+1)}, \alpha, \mathbf{B}, \mathbf{c}) - J_{f,i}(\mathbf{x}_f^{(j+1)}) \end{bmatrix} \quad (8)$$

where $J_{f,i}$, $J_{s,i}$ are the i th columns of \mathbf{J}_f^T and \mathbf{J}_s^T , respectively. The residual (8) is used during the PE optimization process

$$\{\alpha_i^{(j+1)}, \mathbf{B}_i^{(j+1)}, \mathbf{c}_i^{(j+1)}\} = \arg \min_{\alpha, \mathbf{B}, \mathbf{c}} \|\mathbf{r}_i^{(j+1)}(\alpha, \mathbf{B}, \mathbf{c})\| \quad (9)$$

which extracts the mapping parameters for the i th response, and for iteration $j+1$. Hence, we have the complete set of mapping parameters after m PE optimizations.

III. THE PROPOSED SMIS ALGORITHM

Our proposed algorithm starts with the coarse model as the initial surrogate. The algorithm incorporates explicit SM [1] and OSM [5] to speed up the convergence to the optimal solution.

- Step 1 Select a coarse model and a fine model.
- Step 2 Set $j = 0$, and initialize $\mathbf{x}_f^{(0)}$.
- Step 3 Optimize the surrogate (6) to find the next iterate $\mathbf{x}_f^{(j+1)}$ by solving (7).
- Step 4 Evaluate $\mathbf{R}_f(\mathbf{x}_f^{(j+1)})$, $\mathbf{J}_f(\mathbf{x}_f^{(j+1)})$.
- Step 5 Terminate if the stopping criteria are satisfied.
- Step 6 Update the input and output mapping parameters $\alpha_i^{(j+1)}$, $\mathbf{B}_i^{(j+1)}$, $\mathbf{c}_i^{(j+1)}$, $i = 1, \dots, m$ through PE by solving (9).
- Step 7 Set $j = j + 1$, and go to Step 3.

As stopping criteria for the algorithm in Step 5, the relative change in the solution vector, or the relative change in the objective function could be used.

IV. EXAMPLES

A. Seven-Section Capacitively-Loaded Impedance Transformer

We apply the proposed SMIS algorithm to the seven-section capacitively-loaded impedance transformer [8]. Here $U_f = \max_{1 \leq i \leq m} |S_{11,i}|$. We consider a ‘‘coarse’’ model as an ideal seven-section TL, where the ‘‘fine’’ model is a capacitively-loaded TL with capacitors $C_{1..8} = 0.025$ pF. Design parameters are normalized lengths $\mathbf{x}_f = [L_1 L_2 L_3 L_4 L_5 L_6 L_7]^T$, with respect to the quarter-wave length L_q at the center frequency 4.35 GHz. Design specifications are

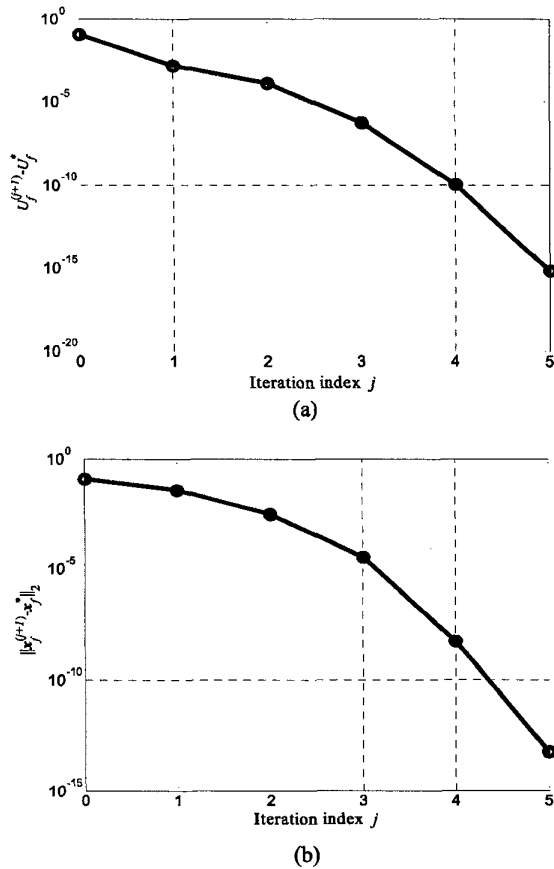


Fig. 2. (a) The difference between the fine model objective function U_f obtained using the SMIS algorithm and the fine model objective function at the fine model minimax solution U_f^* obtained by direct optimization. (b) The corresponding difference between the designs.

$$|S_{11}| \leq 0.07, \text{ for } 1 \text{ GHz} \leq \omega \leq 7.7 \text{ GHz}$$

with 68 points per frequency sweep.

The characteristic impedances for the seven-section capacitively-loaded impedance transformer are fixed as in [8]. The Jacobians of both the coarse and fine model were obtained analytically using the Adjoint Network Method [9]. We solve the PE problem using the nonlinear least squares Levenberg-Marquardt algorithm available in the Matlab Optimization Toolbox [10].

Direct optimization of the fine model starting from an arbitrary point was unsuccessful. Starting from the coarse minimax optimum (the first step in the SM process), it takes 14 iterations (153 fine model evaluations) to reach the fine model direct minimax optimization solution [8] [10]. Our SMIS algorithm took 5 fine model evaluations or 4 iterations to reach the same accurate solution. Both approaches employ exact gradients. The difference

between the minimax objective function at the optimal minimax fine model response and the response obtained using the SMIS algorithm is shown in Fig. 2.

B. Six-Section H-plane Waveguide Filter

The six-section H-plane waveguide filter is shown in Fig. 3 [7]. The design parameters are the lengths and widths $x_f = [L_1 \ L_2 \ L_3 \ W_1 \ W_2 \ W_3 \ W_4]^T$. Design specifications are

$$|S_{11}| \leq 0.16, \text{ for } 5.4 \text{ GHz} \leq \omega \leq 9.0 \text{ GHz}$$

$$|S_{11}| \geq 0.85, \text{ for } \omega \leq 5.2 \text{ GHz}$$

$$|S_{11}| \geq 0.5, \text{ for } \omega \geq 9.5 \text{ GHz}$$

with 23 points per frequency sweep.

A waveguide with a cross-section of 1.372 inches by 0.622 inches (3.485 cm by 1.58 cm) is used. The six

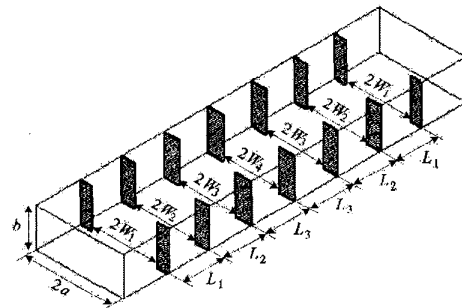


Fig. 3. The six-section H-plane waveguide filter [7] example.

sections are separated by seven H-plane septa, which have a finite thickness of 0.02 inches (0.508 mm). The coarse model consists of lumped inductances and dispersive

TABLE I
OPTIMIZABLE PARAMETER VALUES OF THE SIX-SECTION H-PLANE WAVEGUIDE FILTER

Parameter	Initial solution	Solution by SMIS algorithm
W_1	0.48583	0.51397
W_2	0.43494	0.47244
W_3	0.40433	0.44501
W_4	0.39796	0.44627
L_1	0.65585	0.63142
L_2	0.65923	0.63922
L_3	0.67666	0.65705
all values are in inches		

V. CONCLUSIONS

We present a powerful algorithm based on our novel SMIS framework that delivers the accuracy expected from direct optimization using sequential linear programming, yet converges in a handful of iterations. It aims at matching a surrogate (mapped coarse model) with the fine model within a local region of interest by introducing more degrees of freedom into the space mapping. Convergence is demonstrated through a seven-section capacitively-loaded impedance transformer. A highly optimized, H-plane filter design emerges after only four EM simulations (three iterations), excluding necessary Jacobian estimations, using the new algorithm with sparse frequency sweeps.

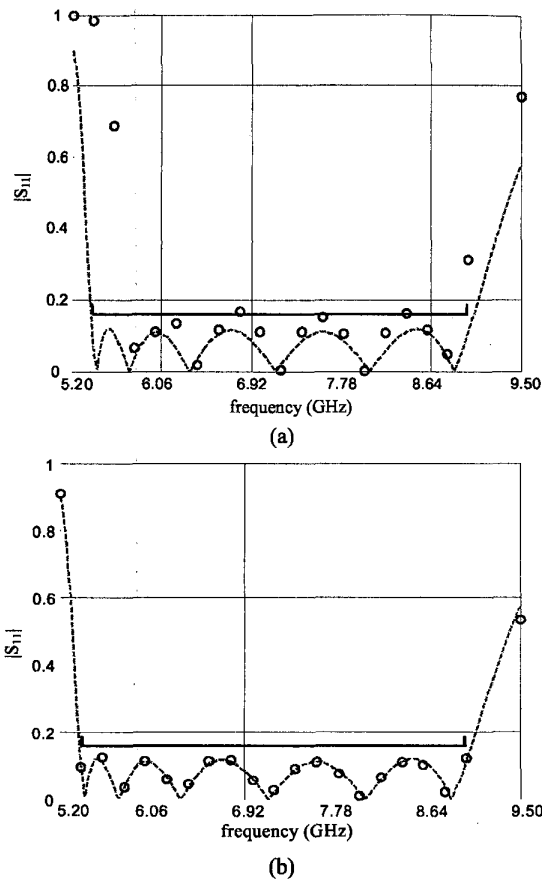


Fig. 4. H-plane filter optimal coarse model response (—) and the HFSS (fine model) response (a) at the initial solution (○), and (b) at the SMIS algorithm solution reached after 3 iterations (○).

transmission line sections. There are various approaches to calculate the equivalent inductive susceptance of an H-plane septum. We use a simplified version of a formula due to Marcuvitz [11]. The coarse model is simulated using Matlab [10]. The fine model exploits the Agilent HFSS [12] simulator. One frequency sweep takes 2.5 minutes on an Intel Pentium 4 (3 GHz) machine with 1 GB RAM and running in Windows XP Pro. Seven fine model simulations, due to the seven 0.01 inch perturbations, are required to find the fine model Jacobian off-line using the finite difference method. Thus, the time taken for fine model and Jacobian calculation is 21 minutes per iteration on an Intel P4 machine. Fig. 4(a) shows the fine model response at the initial solution. Fig. 4(b) shows the fine model response after running our SMIS algorithm using HFSS. The total time taken was 126 minutes on an Intel P4 3 GHz machine. Table I shows the initial and optimal design parameter values of the six-section H-plane waveguide filter.

REFERENCES

- [1] J.W. Bandler, R.M. Biernacki, S.H. Chen, P.A. Grobelny and R.H. Hemmers, "Space mapping technique for electromagnetic optimization," *IEEE Trans. Microwave Theory Tech.*, vol. 42, 1994, pp. 2536-2544.
- [2] J.W. Bandler, R.M. Biernacki, S.H. Chen, R.H. Hemmers and K. Madsen, "Electromagnetic optimization exploiting aggressive space mapping," *IEEE Trans. Microwave Theory Tech.*, vol. 43, 1995, pp. 2874-2882.
- [3] A.M. Pavio, "The electromagnetic optimization of microwave circuits using companion models," Workshop on Novel Meth. for Device Modeling and Circuit CAD, *IEEE MTT-S IMS*, Anaheim, CA, 1999.
- [4] J. Snel, "Space mapping models for RF components," Workshop on Statistical Design and Modeling Tech. For Microwave CAD, *IEEE MTT-S IMS*, 2001.
- [5] J.W. Bandler, Q.S. Cheng, D.H. Gebre-Mariam, K. Madsen, F. Pedersen and J. Søndergaard, "EM-based surrogate modeling and design exploiting implicit, frequency and output space mappings," *IEEE MTT-S IMS Digest*, Philadelphia, PA, 2003, pp. 1003-1006.
- [6] F. Pedersen, "Space mapping optimization using interpolating surrogates," ITC, McMaster University, Hamilton, ON, Canada, May 15, 2003.
- [7] M.H. Bakr, "Advances in space mapping optimization of microwave circuits," Ph.D. Thesis, Department of Electrical and Computer Engineering, McMaster University, Hamilton, ON, Canada, 2000.
- [8] M.H. Bakr, J.W. Bandler, K. Madsen and J. Søndergaard, "An introduction to the space mapping technique," *Optimization and Engineering*, vol. 2, 2001, pp. 369-384.
- [9] J.W. Bandler and R. E. Seviara, "Computation of sensitivities for noncommensurate networks," *IEEE Trans. Circuit Theory*, vol. CT-18, 1971, pp. 174-178.
- [10] Matlab™, Version 6.5, The MathWorks, Inc., 3 Apple Hill Drive, Natick MA 01760-2098, 2002.
- [11] N. Marcuvitz, *Waveguide Handbook*. New York: McGraw-Hill, First Edition, 1951, p. 221.
- [12] Agilent HFSS™ Version 5.6, HP EESof, Agilent Technologies, 1400 Fountaingrove Parkway, Santa Rosa, CA 95403-1799, 2000.