

# Recent Trends in Space Mapping Technology

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**Abstract**—We review recent trends in the art of Space Mapping (SM) technology for modeling and design of engineering devices and systems. The SM approach aims at achieving a satisfactory solution with a handful of computationally expensive so-called “fine” model evaluations. SM procedures iteratively update and optimize surrogates based on fast physically-based “coarse” models. Parameter extraction is an essential SM subproblem. It is used to align the surrogate (enhanced coarse model) with the fine model. Recent developments including TLM-based modeling and design using SM and the SM-based interpolating surrogates framework are discussed. Some practical applications are reviewed.

**Index Terms**—CAD, design automation, EM simulation, optimization, parameter extraction, space mapping.

## I. INTRODUCTION

TRADITIONAL optimization techniques for engineering design [1,2,3] exploit simulated responses and possible derivatives w.r.t. design parameters. Schemes combining the speed and maturity of circuit simulators with the accuracy of EM solvers are possible. Through a Space Mapping (SM), a suitable surrogate can be obtained: faster than an EM-based “fine” model and at least as accurate as an empirical “coarse” model on which it is based.

We review the state of the art of SM, conceived by Bandler in 1993. Bandler *et al.* [4,5] demonstrated how SM intelligently links companion “coarse” (simplified, fast or low-fidelity) and “fine” (accurate, practical or high-fidelity) models of different complexities. For example, an EM simulator could serve as a fine model. A low fidelity EM simulator or an empirical circuit model could be a coarse model (see Fig. 1).

## II. HISTORY OF SPACE MAPPING

The first algorithm was introduced in 1994 [4]. A linear mapping between the coarse and fine parameter

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spaces is evaluated by a least squares solution of the equations which result from associating points (data) in the two spaces. The corresponding surrogate is a piecewise linearly mapped coarse model.

The Aggressive Space Mapping (ASM) approach [5] exploits each fine model iterate immediately. This iterate, determined by a quasi-Newton step, in effect optimizes the corresponding surrogate model.

Parameter Extraction (PE) is key to establishing mappings and updating surrogates. PE attempts to locally align a surrogate with a given fine model, but nonuniqueness may cause breakdown of the algorithm [6]. Multi-point PE [6,7], a statistical PE [7], a penalty PE [8], aggressive PE [9] and a gradient PE approach [10] attempt to improve uniqueness.

The trust region aggressive SM algorithm [11] exploits trust region (TR) strategies [12] to stabilize optimization iterations. The hybrid aggressive SM algorithm [13] alternates between (re)optimization of a surrogate and direct response matching. The surrogate model based SM [14] algorithm combines a mapped coarse model with a linearized fine model and defaults to direct optimization of the fine model.

Neural space mapping approaches [15,16,17] utilize Artificial Neural Networks (ANN) in EM-based modeling and design of microwave devices. A full review of ANN applications in microwave circuit design including the SM technology is found in [18].

Several SM-based model enhancement approaches have been proposed: the SM tableau approach [19], space derivative mapping [20], and SM-based neuromodeling [15].

A comprehensive review of SM is presented in [21].

In Implicit SM (ISM) [22] an auxiliary set of preassigned parameters, e.g., dielectric constants or substrate heights, is extracted to match the surrogate with the fine model. The resulting calibrated coarse

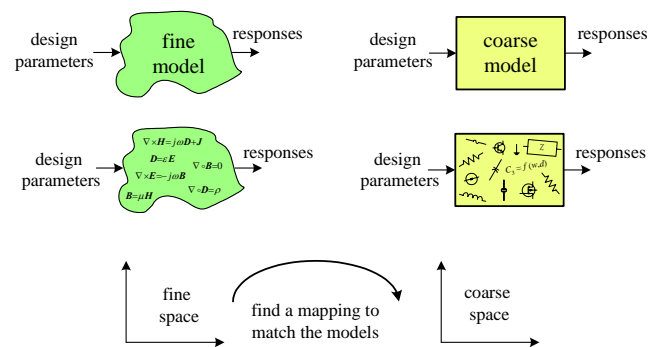


Fig. 1. Linking companion coarse (empirical) and fine (EM) models through a mapping.

model is then reoptimized to predict the next fine model. ISM is effective for microwave circuit modeling and design using EM simulators and is more easily implemented than [23].

The coarse model deviates from the fine model. We proposed Output SM (OSM) [24] to tune the residual response misalignment between the fine model and its surrogate.

We currently utilize ISM and OSM within the TLM simulation environment. We design a CPU intensive fine-grid TLM structure by utilizing a coarse-grid TLM surrogate model. The dielectric constant is employed to compensate for the coarseness of this surrogate.

We propose an SM-based interpolating surrogate (SMIS). Highly accurate SMIS models are built for use in gradient-based optimization. The SMIS surrogate is forced to match both the responses and derivatives of the fine model within a local region of interest [25].

### III. TLM-BASED DESIGN AND MODELING USING SM

We construct a surrogate of the fine model iteratively

$$\mathbf{R}_s^{(j)}(\mathbf{x}_f, \mathbf{x}^{(j)}, \boldsymbol{\alpha}^{(j)}, \boldsymbol{\beta}^{(j)}) \triangleq \boldsymbol{\alpha}^{(j)} \mathbf{R}_c^{(j)}(\mathbf{x}_f, \mathbf{x}^{(j)}) + \boldsymbol{\beta}^{(j)} \quad (1)$$

$$\mathbf{R}_f(\mathbf{x}_f) \approx \mathbf{R}_s^{(j)}(\mathbf{x}_f, \mathbf{x}^{(j)}, \boldsymbol{\alpha}^{(j)}, \boldsymbol{\beta}^{(j)})$$

where, at the  $j$ th iteration, the preassigned parameter vector is  $\mathbf{x}^{(j)} \in \mathbb{R}^p$ . The diagonal matrix  $\boldsymbol{\alpha}^{(j)}$  and the shifting vector  $\boldsymbol{\beta}^{(j)}$  are output mapping parameters. The surrogate and the coarse model responses are denoted  $\mathbf{R}_s^{(j)}$  and  $\mathbf{R}_c^{(j)} \in \mathbb{R}^m$ .

#### A. Parameter Extraction (Surrogate Calibration)

We extract the parameters of the surrogate (1) to match the fine model by varying the preassigned dielectric constant  $\epsilon_r$  (i.e.,  $\mathbf{x} = \epsilon_r$ ). We also tune  $\boldsymbol{\alpha}$  and  $\boldsymbol{\beta}$  to improve the surrogate. The PE step is given by

$$\left[ \mathbf{x}^{(j)}, \boldsymbol{\alpha}^{(j)}, \boldsymbol{\beta}^{(j)} \right] \triangleq \arg \min_{\mathbf{x}, \boldsymbol{\alpha}, \boldsymbol{\beta}} \left\| \mathbf{r}^{(j)}(\mathbf{x}_f^{(j)}, \mathbf{x}, \boldsymbol{\alpha}, \boldsymbol{\beta}) \right\|, \quad (2)$$

$$\mathbf{r}^{(j)}(\mathbf{x}_f^{(j)}, \mathbf{x}, \boldsymbol{\alpha}, \boldsymbol{\beta}) = \mathbf{R}_s^{(j)}(\mathbf{x}_f^{(j)}, \mathbf{x}, \boldsymbol{\alpha}, \boldsymbol{\beta}) - \mathbf{R}_f^{(j)}(\mathbf{x}_f^{(j)})$$

#### B. Surrogate Optimization (Prediction)

We optimize the objective function of the surrogate (1) in an effort to find the optimal fine model design. We utilize TR strategies to find the step  $\mathbf{h}^{(j)}$  [11,23,26]

$$\mathbf{h}^{(j)} \triangleq \arg \min_{\mathbf{h}} U(\mathbf{R}_s^{(j)}(\mathbf{x}_f^{(j)} + \mathbf{h}, \mathbf{x}^{(j)}, \boldsymbol{\alpha}^{(j)}, \boldsymbol{\beta}^{(j)})), \quad (3)$$

$$\|\mathbf{h}\| \leq \delta^{(j)}$$

where  $\delta^{(j)}$  is the trust region size at the  $j$ th iteration. The tentative step is accepted as a successful step in the fine space if there is a reduction of the objective function of the fine model otherwise it is rejected. The TR radius is updated according to [26].

We consider a single-resonator filter (Fig. 2). Design

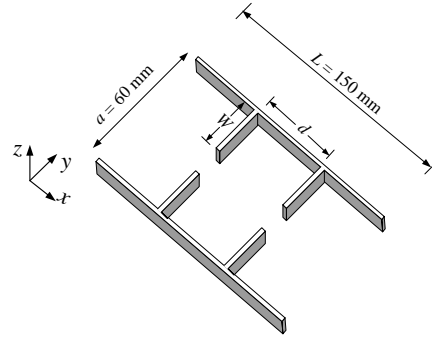


Fig. 2. Topology of the single-resonator filter.

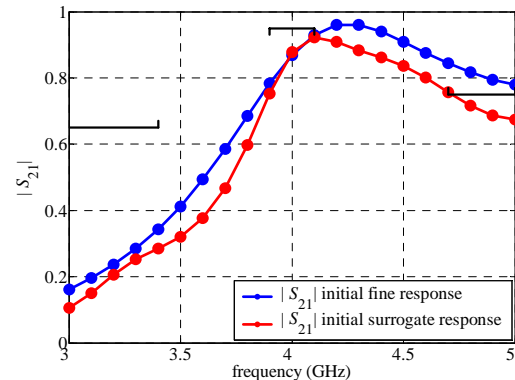


Fig. 3. The fine model and (—●—) the surrogate (---●---) responses at the initial design using linear interpolation.

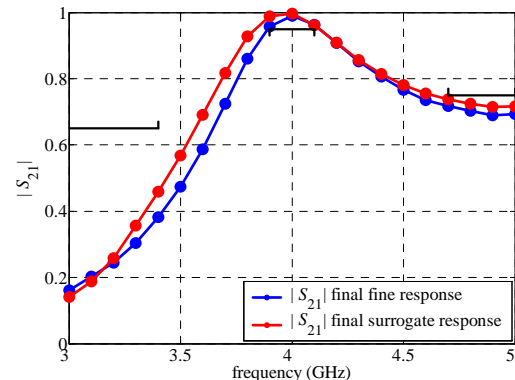


Fig. 4. The fine model and (—●—) the surrogate (---●---) responses at the final design using linear interpolation.

parameters are  $W$  and  $d$ . The fine model has a square cell  $\Delta x = \Delta y = 1.0$  mm, while the coarse model utilizes  $\Delta x = \Delta y = 5.0$  mm. The frequency range is  $3.0\text{GHz} \leq \omega \leq 5.0\text{GHz}$ . The coarse model simulates 23 times faster than the fine model. We utilize a minimax objective function with upper and lower design specifications.

The Matlab [27] least-squares Levenberg-Marquardt algorithm solves the PE problem. The TR subproblem (3) is solved by the minimax routine described in [28]. A linear interpolation scheme is used [29].

The algorithm converges in 5 iterations to an optimal fine model response although the coarse model exhibits a poor response at the initial design (see Fig. 3). Fig. 4 depicts the fine-grid TLM responses at the final design using linear interpolation. The optimal design is given by  $W = 14.56$  mm and  $d = 32.97$  mm.

#### IV. SPACE MAPPING INTERPOLATED SURROGATES (SMIS) FRAMEWORK

##### A. The Surrogate

The SM-based interpolating surrogate [25]  $\mathbf{R}_s^{(j)}: \mathbb{R}^n \rightarrow \mathbb{R}^m$ , used in the  $j$ th iteration, aims at satisfying the interpolation conditions

$$\mathbf{R}_s^{(j)}(\mathbf{x}_f^{(j)}) = \mathbf{R}_f(\mathbf{x}_f^{(j)}) \quad (4)$$

$$\mathbf{J}_s^{(j)}(\mathbf{x}_f^{(j)}) = \mathbf{J}_f(\mathbf{x}_f^{(j)}) \quad (5)$$

where  $\mathbf{J}_s^{(j)}(\mathbf{x}_f^{(j)})$  and  $\mathbf{J}_f(\mathbf{x}_f^{(j)})$  are the Jacobians of the surrogate and fine model at  $\mathbf{x}_f^{(j)}$ , respectively.

The conditions (4) and (5), and global match condition are satisfied by transforming a coarse model  $\mathbf{R}_c: \mathbb{R}^n \rightarrow \mathbb{R}^m$ , through various linear input and output mappings.  $\mathbf{P}_i: \mathbb{R}^n \rightarrow \mathbb{R}^n$  [5] is an input mapping applied to the  $i$ th response, and  $\mathbf{R}_{c,i} \circ \mathbf{P}_i$  is the mapped  $i$ th response of the coarse model. The corresponding response of the surrogate is a composed mapping  $\mathbf{R}_{s,i} = \mathbf{O}_i \circ \mathbf{R}_{c,i} \circ \mathbf{P}_i$ , where  $\mathbf{O}_i: \mathbb{R} \rightarrow \mathbb{R}$

$$\begin{aligned} \mathbf{R}_{s,i}^{(j)}(\mathbf{x}_f) &= \alpha_i^{(j)}(\mathbf{R}_{c,i}(\mathbf{P}_i^{(j)}(\mathbf{x}_f)) - \\ &\quad \mathbf{R}_{c,i}(\mathbf{P}_i^{(j)}(\mathbf{x}_f^{(j)}))) + \mathbf{R}_i^{(j)}, \quad (6) \\ &i = 1, \dots, m, \text{ and } j = 0, 1, \dots \end{aligned}$$

where  $\alpha_i^{(j)} \in \mathbb{R}$ ,  $\mathbf{R}_i^{(j)} \in \mathbb{R}$ ,  $i = 1, \dots, m$  are the output mapping parameters. The input mapping is defined as

$$\mathbf{P}_i^{(j)}(\mathbf{x}_f) = \mathbf{B}_i^{(j)}\mathbf{x}_f + \mathbf{c}_i^{(j)} \quad (7)$$

where  $\mathbf{B}_i^{(j)} \in \mathbb{R}^{n \times n}$ ,  $\mathbf{c}_i^{(j)} \in \mathbb{R}^n$ ,  $i = 1, \dots, m$  are the input mapping parameters.

The surrogate used in the  $j$ th iteration is given by

$$\mathbf{R}_s^{(j)}(\mathbf{x}_f) = [\mathbf{R}_{s,1}^{(j)}(\mathbf{x}_f) \dots \mathbf{R}_{s,m}^{(j)}(\mathbf{x}_f)]^T \quad (8)$$

The surrogate is optimized to find the next iterate by solving

$$\mathbf{x}_f^{(j+1)} = \arg \min_{\mathbf{x}_f} U(\mathbf{R}_s^{(j)}(\mathbf{x}_f)) \quad (9)$$

In the first iteration, the mapping parameter values  $\mathbf{B}_i^{(0)} = \mathbf{I}$ ,  $\mathbf{c}_i^{(0)} = \mathbf{0}$ ,  $\alpha_i^{(0)} = 1$  and  $\mathbf{R}_i^{(0)} = \mathbf{R}_{c,i}(\mathbf{x}_f^{(0)})$  ensure that  $\mathbf{R}_s^{(0)}(\mathbf{x}_f) = \mathbf{R}_c(\mathbf{x}_f)$ . For  $j > 0$ , the parameter  $\mathbf{R}^{(j)} = \mathbf{R}_f(\mathbf{x}_f^{(j)})$  is utilized.

##### B. The Surface Fitting Approach for PE

We employ a surface fitting approach for PE, which involves the minimization of residuals between the surrogate and fine model, and extracting the parameters  $\mathbf{B}_i^{(j)}$ ,  $\mathbf{c}_i^{(j)}$  and  $\alpha_i^{(j)}$ ,  $i = 1, \dots, m$ .

We require a strict match of responses (4) and derivatives (5), and aim at a global match between the surrogate and the fine model by satisfying

$\mathbf{R}_s^{(j)}(\mathbf{x}_f^{(k)}) = \mathbf{R}_f(\mathbf{x}_f^{(k)})$ ,  $k = 1, \dots, j+1$ . Updating the surrogate from iteration  $j$  to  $j+1$  involves a residual vector. A residual defined in [25] is used during the PE optimization process

$$\{\alpha_i^{(j+1)}, \mathbf{B}_i^{(j+1)}, \mathbf{c}_i^{(j+1)}\} = \arg \min_{\alpha, \mathbf{B}, \mathbf{c}} \left\| \mathbf{r}_i^{(j+1)}(\alpha, \mathbf{B}, \mathbf{c}) \right\| \quad (10)$$

to obtain the mapping parameters for the  $i$ th response, and for iteration  $j+1$ . Hence, we have the complete set of mapping parameters after  $m$  PE optimizations.

##### C. The SMIS Algorithm

The SMIS algorithm is developed in [25]. Accuracy and convergence are demonstrated through a seven-section capacitively-loaded impedance transformer. Direct optimization of the fine model starting from an arbitrary point is unsuccessful [25]. Starting from the coarse minimax optimum (the first step in the SM process), it took 14 iterations (153 fine model evaluations) to reach the fine model direct minimax optimization solution using Matlab's 'fminimax' routine [27]. Our SMIS algorithm took 5 fine model evaluations or 4 iterations to reach the same accurate solution. Both approaches employed exact gradients.

#### V. RECENT CONTRIBUTIONS TO SPACE MAPPING

Ismail *et al.* [30] exploit SM-optimization in the design of dielectric-resonator filters and multiplexers. Jansson *et al.* [31] apply the SM technique and surrogate models together with response surfaces in structural optimization and vehicle crashworthiness problems. Devabhaktuni *et al.* [32] propose a technique for generating microwave neural models of high accuracy using less accurate data. Wu *et al.* [33,34] apply the aggressive SM approach to LTCC RF passive circuit design. Feng *et al.* [35] employ the ASM technique for the design of antireflection coatings for photonic devices, such as the semiconductor optical amplifiers. Feng *et al.* also [36] utilize a generalized SM for modeling of photonic devices such as an optical waveguide facet. A full review of practical applications of SM in the literature is found in [21].

#### VI. CONCLUSIONS

The SM approach and the SM-based modeling concepts for engineering design and modeling are reviewed. This CAD methodology embodies the traditional experience and intuition of engineers. Different algorithms of SM including various approaches for the PE subproblem are stated.

OSM is currently utilized in TLM-based design exploiting SM and SM-based interpolating surrogates. OSM is utilized to achieve an optimal design in spite of a poor initial surrogate response.

The SMIS approach reaches a highly accurate solution in a handful of iterations.

Recent SM applications are reviewed. They indicate

that the SM technology is applicable not only in the RF and microwave engineering arena, which it was originally applied to, but also in other fields.

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