Optimization of Spiral Inductor on Silicon using Space Mapping

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***Abstract*—We present an efficient method for the optimal design of spiral inductors used in RF circuits. The optimization process exploits the EM simulator Sonnet *em* and space mapping (SM) technology. A straightforward geometric programming formulation of the spiral inductor optimization is implemented in the surrogate model optimization. An EM-validated optimal spiral inductor design emerges in ten minutes.**

***Index Terms*—CAD, geometric programming, inductors, integrated circuit design, optimization methods, space mapping.**

I. Introduction

As an important component in radio-frequency integrated circuits (RF-ICs), such as Low Noise Amplifiers (LNA) and Voltage Controlled Oscillators (VCO), the spiral inductor is critical to the performance of RF and analog systems.

Previous optimization methods for spiral inductors include exhaustive enumeration, geometric programming (GP) [1]–[2], sequential quadratic programming (SQP) [3] and Mesh-Adaptive Direct Search (MADS) [4]. These methods are usually based on circuit models. Although efficient, the results depend on the quality of the circuit model they use. It is likely that the design does not meet the specification or is unsatisfactory when validated by electromagnetic (EM) solvers. On the other hand, EM solvers, such as Sonnet *em*, are accurate at the expense of time. Direct optimization based on EM solvers is desirable but expensive.

Space mapping (SM) technology [5]-[6] incorporates the computational efficiency of (cheap) circuit models and the accuracy of (expensive) EM simulations. It performs optimization on a cheap model (coarse model) and calibrates it using EM simulator (fine model). A satisfactory design can usually be obtained in a few EM simulations.

We apply implicit space mapping (ISM) [6] to spiral inductor optimization. Our strategy is based on the geometric programming formulation of spiral inductor optimization proposed in [1] and [2]. By regarding several coefficients in the circuit model as ISM parameters (preassigned parameters), we (re)calibrate the circuit model with EM simulations during the optimization process. Using this method, a satisfactory EM-validated spiral inductor design emerges in ten minutes.

We also propose a simplified geometric programming formulation based on [1] and [2]. Because space mapping is used to calibrate the circuit model, this simplification does not affect our result, but makes the problem easier to solve.

II. Implicit Space Mapping Technology

Space mapping technology assumes the availability of two physically-based models: a coarse model (computationally fast circuit-based model or low-fidelity EM simulation) and a fine model (typically a cpu-intensive full-wave EM simulation). As in [6], we define the fine model response at a point $\boldsymbol{x}_f$ in the design space by $\boldsymbol{R}_f(\boldsymbol{x}_f)$. The design problem is to obtain

$$\boldsymbol{x}_f^* \triangleq \arg\min_{\boldsymbol{x}_f \in X_f} U(\boldsymbol{R}_f(\boldsymbol{x}_f)) \tag{1}$$

where U is the objective function and X_f is the design variable domain. We assume that it is expensive to solve (1) by direct optimization if full-wave EM simulation is used.

We define the coarse-model based surrogate response at a point $\boldsymbol{x}_c$ by $\boldsymbol{R}_c(\boldsymbol{x}_c, \boldsymbol{x}_p)$, where $\boldsymbol{x}_p$ is a set of preassigned parameters, for example, empirical model coefficients or the dielectric constant of a substrate. ISM optimization involves two principal iteration steps: ISM modeling through parameter extraction and ISM prediction through surrogate optimization.

The aim of ISM modeling is to match the surrogate to the fine model by adjusting selected preassigned parameters $\boldsymbol{x}_p$. The data used in this step comes from the fine model response obtained in previous iterations. As in [6], we denote $\boldsymbol{x}_c^{*(j)}$ as the surrogate optimal point at the jth iteration and $\boldsymbol{x}_c^{*(0)}$ as the initial point (coarse model optimum). ISM modeling at the jth iteration is to find

$$\boldsymbol{x}_p^{(j)} \triangleq \arg\min_{\boldsymbol{x}_p} \left\| \begin{bmatrix} \boldsymbol{e}_0^T & \boldsymbol{e}_1^T & \cdots & \boldsymbol{e}_{j-1}^T \end{bmatrix}^T \right\| \tag{2}$$

where

$$\boldsymbol{e}_i^T = \boldsymbol{R}_f(\boldsymbol{x}_c^{*(i)}) - \boldsymbol{R}_c(\boldsymbol{x}_c^{*(i)}, \boldsymbol{x}_p) \ (i = 0, \cdots, j-1) \,. \tag{3}$$

After ISM modeling, we optimize the (re)calibrated coarse model (surrogate model) in ISM prediction, i.e., we find

$$\boldsymbol{x}_c^{*(j)} \triangleq \arg\min_{\boldsymbol{x}_c \in X_c} U(\boldsymbol{R}_c(\boldsymbol{x}_c, \boldsymbol{x}_p^{(j)})) \tag{4}$$

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where X_c is the design variable domain of the surrogate model.

By continuing ISM modeling and ISM prediction, we hope to find a good fine model solution.

III. Spiral Inductor Optimization Using Geometric Programming

As pointed out in Section II, ISM prediction involves the optimization of the circuit model. To do this, we propose a simplified geometric programming formulation of the problem based on [1] and [2].

Fig. 1 shows the layout of a square spiral inductor. The design parameters are number of turns n, the width of metal trace w, the turn spacing s, the outer diameter d_{out} and the average diameter $d_{avg} = 0.5(d_{out} + d_{in})$. Only four of these five design parameters are independent, but in the GP formulation of spiral inductor optimization, all five parameters are used. We intend to achieve the highest quality factor Q and a certain inductance at the target frequency.

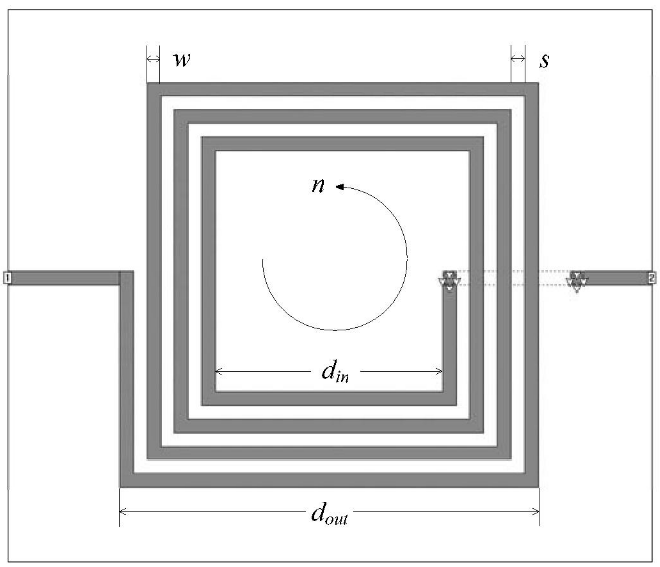


Fig. 1. Square spiral inductor layout and geometry.

Fig. 2 shows models of the spiral inductor. Following [1], all circuit elements could be written as posynomials (sums of monomials) of design variables and factors k_i dependent on technology and frequency. In particular, the expression for inductance is the monomial function [1]

$$L_s = \beta d_{out}^{\alpha_{Ls1}} w^{\alpha_{Ls2}} d_{avg}^{\alpha_{Ls3}} n^{\alpha_{Ls4}} s^{\alpha_{Ls5}} \tag{5}$$

with inductance in nH and dimensions in μm. The coefficients $\beta = 1.66 \cdot 10^{-3}$, $\alpha_{Ls1} = -1.33$, $\alpha_{Ls2} = -0.125$, $\alpha_{Ls3} = 2.50$, $\alpha_{Ls4} = 1.83$ and $\alpha_{Ls5} = -0.022$ are extracted from a large family of inductors.

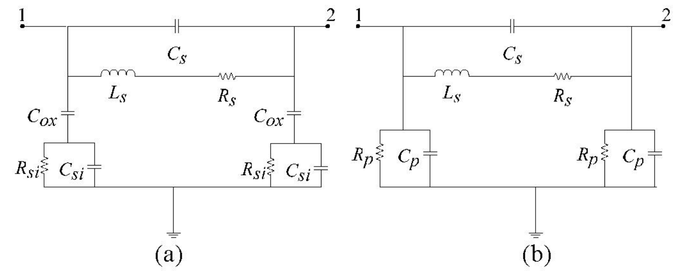


Fig. 2. Circuit models of the spiral inductor [1]: (a) π model, (b) simplified model.

The quality factor Q can be written as [1]

$$Q = \frac{\omega L_s}{R_s} \cdot \frac{R_p \left(1 - \frac{R_s^2 C_{tot}}{L_s} - \omega^2 L_s C_{tot}\right)}{R_p + \left[\left(\frac{\omega L_s}{R_s}\right)^2 + 1 \right] R_s} \tag{6}$$

where

$$C_{tot} = C_p + C_s . \tag{7}$$

Unfortunately, the expression for Q shown in (6) is not GP compatible. In [1], this problem is solved by introducing a new variable and turning (6) into a posynomial inequality constraint. In [2], a different approach is used. By noticing that $[(\omega L_s / R_s)^2 + 1] R_s$ is much smaller than R_p, the quality factor is approximated by

$$Q = \frac{\omega L_s}{R_s} - \omega R_s C_{tot} - \frac{\omega^3 L_s^2 C_{tot}}{R_s} . \tag{8}$$

We notice that (8) is still not compatible with standard GP, because the objective function is not a posynomial function. Although it can be solved using the algorithm mentioned in [2], it cannot be solved by commercial optimization software such as MOSEK [8].

We further develop the approach in [2]. We notice that maximizing Q is equivalent to minimizing $1/Q$ and the second and the third term in (8) is much smaller than the first term. Thus $1/Q$ can be approximated as a posynomial function of the design parameters

$$\begin{aligned} \frac{1}{Q} &\approx \frac{R_s}{\omega \cdot L_s}\left(1 + \omega \cdot R_s \cdot C_{tot} \cdot \frac{R_s}{\omega \cdot L_s} + \frac{\omega^3 \cdot L_s^2 \cdot C_{tot}}{R_s} \cdot \frac{R_s}{\omega \cdot L_s}\right) \\ &= \frac{k_1}{\omega\beta} d_{out}^{-\alpha_{Q1}} w^{-\alpha_{Q2}-1} d_{avg}^{-\alpha_{Q3}+1} n^{-\alpha_{Q4}+1} s^{-\alpha_{Q5}} \\ &+ \frac{k_1^3 k_7}{\omega\beta^2} d_{out}^{-2\alpha_{Q1}} w^{-2\alpha_{Q2}-2} d_{avg}^{-2\alpha_{Q3}+4} n^{-2\alpha_{Q4}+4} s^{-2\alpha_{Q5}} \\ &+ \frac{k_1^3 k_3}{\omega\beta^2} d_{out}^{-2\alpha_{Q1}} w^{-2\alpha_{Q2}-1} d_{avg}^{-2\alpha_{Q3}+3} n^{-2\alpha_{Q4}+4} s^{-2\alpha_{Q5}} \\ &+ \omega k_1 k_7 n^2 d_{avg}^2 + \omega k_1 k_3 n^2 d_{avg} w \end{aligned} \tag{9}$$

In (9), k_1 , k_3 and k_7 are technology-dependent coefficients [1]. A new set of coefficients, α_{Qi} (i=1, 2, $\cdots$, 5), is used. In the coarse model, they are the same as α_{Lsi} ($i=1, 2, \cdots, 5$) . But in the surrogate model, they will be treated as different preassigned parameters and extracted separately to calibrate Q and L_s respectively, as discussed in the next section.

The final GP formulation is shown in (10). The second constraint is the relaxation of the equality constraint on d_{avg} [1]. The third constraint ensures that the inductor layout physically exists.

$$
\begin{aligned}
&\min\ 1/Q \\
&s.t.\ L_{s\min} \le \beta d_{out}^{\alpha_{Ls1}} w^{\alpha_{Ls2}} d_{avg}^{\alpha_{Ls3}} n^{\alpha_{Ls4}} s^{\alpha_{Ls5}} \le L_{s\max} \\
&\quad d_{avg} + ns + nw \le d_{out} \\
&\quad (2n+1)(s+w) \le d_{out} \\
&\quad d_{out\min} \le d_{out} \le d_{out\max} \\
&\quad w_{\min} \le w \le w_{\max} \\
&\quad s_{\min} \le s \le s_{\max} \\
&\quad n_{\min} \le n \le n_{\max}
\end{aligned}
\quad . \qquad (10)
$$

IV. Spiral Inductor Optimization By Space Mapping

We use the circuit model discussed in Section III as the coarse model and Sonnet ***em*** as the fine model. We define $\boldsymbol{R}_c = [1/Q_c\ L_{sc}]^T$ as the response of the coarse model, where $1/Q_c$ and L_{sc} are given in (9) and (5). We define $\boldsymbol{R}_f = [1/Q_f\ L_{sf}]^T$ as the response of the fine model, where [7]

$$Q_f = -\frac{\mathrm{Im}(Y_{11})}{\mathrm{Re}(Y_{11})} \qquad (11)$$

$$L_{sf} = -\frac{1}{2\pi f}\mathrm{Im}(\frac{1}{Y_{12}}) . \qquad (12)$$

Y_{11} and Y_{12} are Y parameters of the spiral inductor obtained from EM simulation and f is frequency.

We define β , α_{Lsi} ($i=1, 2, \cdots, 5$) , k_1 , k_3 , k_7 , and α_{Qi} ($i=1, 2, \cdots, 5$) as preassigned parameters

$$\boldsymbol{x}_p = [\beta\ \alpha_{L_s1} \cdots \alpha_{L_s5}\ k_1\ k_3\ k_7\ \alpha_{Q1} \cdots \alpha_{Q5}]^T . \qquad (13)$$

The ISM algorithm can be summarized as follows.

Step 1 Set j=0 and pick an initial design parameter $\boldsymbol{x}_c^{*(0)}$.
Step 2 Simulate the fine model at $\boldsymbol{x}_c^{*(j)}$ and increment j.
Step 3 Extract the preassigned parameters $\boldsymbol{x}_p^{*(j)}$ by solving (2) (ISM modeling).
Step 4 Optimize the (re)calibrated coarse model (surrogate model) to obtain $\boldsymbol{x}_c^{*(j)}$ by solving (10).
Step 5 Terminate if a stopping criterion (e.g., convergence) is satisfied.
Step 6 Go to Step 2.

We solve (10) with the MOSEK optimization toolbox [8]. However, in a practical design the number of turns n should be discrete. We address this problem by first considering n as continuous and solving (10) to get the optimal n^*. Then we round n^* to the two nearest discrete values n_1^* and n_2^*. Fixing n to n_1^* and n_2^*, we perform another two optimizations. Finally, we choose the better result of these two optimizations as the result of step 4.

V. Example

We apply ISM to optimizing the spiral inductor in the following sample CMOS process shown in Fig. 3. The conductivity of the Si substrate is 5 S/m. Two metal layers of 1 μm thickness, M1 and M2, are used for the spiral inductor and the underpass. The conductivity of the metal layers are 3×10^7 S/m. We intend to achieve the highest Q and 4 nH inductance at 3 GHz. The tolerance for the inductance is 5%, which means that the L_s should range from 3.8 nH to 4.2 nH. The constraints on the design parameters are listed in Table I. The number of turns n is restricted to discrete values as $k+0.5$, where k is a positive integer.

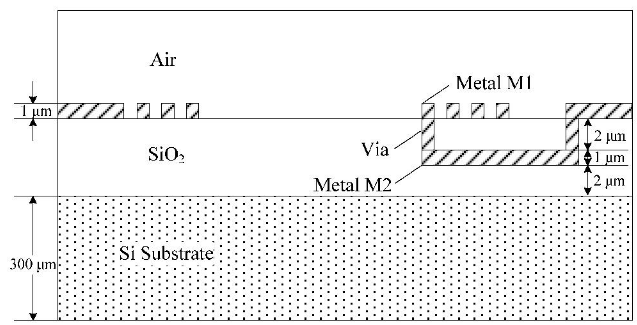
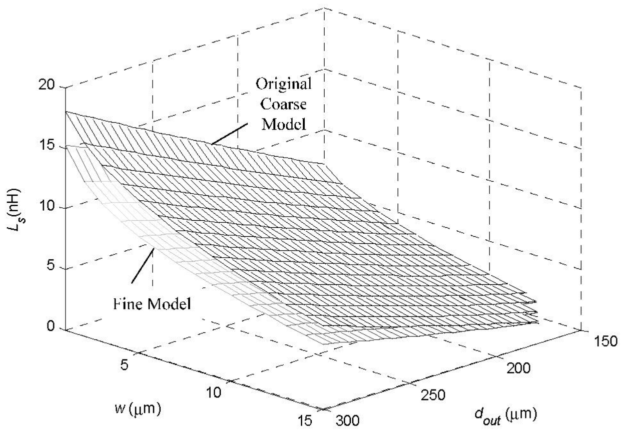


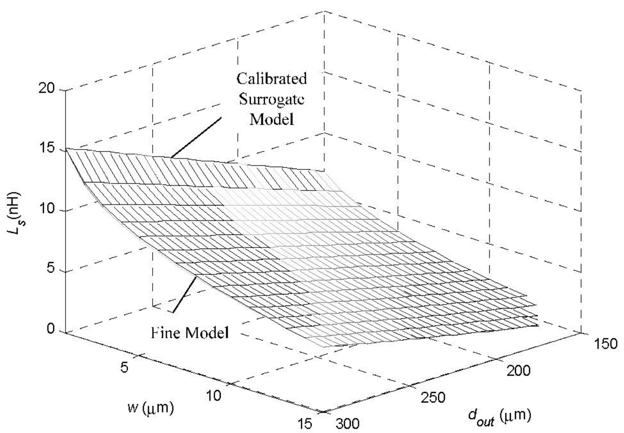
Fig. 3. Sectional view of the spiral inductor.

In Table II we compare the results obtained by our ISM algorithm, circuit-model-based geometric programming [1] and enumeration of the fine model. In enumeration, the sampling steps in the design region are 5 μm for d_{out}, 1 μm for w, one turn for n and 2 μm for s. The Q and L_s shown in the table are all obtained from EM simulations. With the ISM algorithm, a satisfactory design emerges in ten EM simulations. In comparison, the result given by the circuit-model based GP [1] does not meet the specification when validated by the EM simulator. Enumeration of the fine model gives a result very close to that of the ISM algorithm, but takes much longer time (several days).

In Fig. 4 we compare the inductance L_s of the coarse model and the surrogate model in the last iteration with the fine model over the design region (n is fixed to 4.5 and s is fixed to 2 μm). It can be seen that the surrogate model is successfully calibrated. A similar result is obtained for the quality factor Q.



(a)



(b)

Fig. 4. L_s over the design region (n = 4.5, s = 2 μm): (a) the original coarse and fine models, (b) the calibrated surrogate model in the last iteration and the fine model.

TABLE I
CONSTRAINTS ON DESIGN PARAMETERS

Parameter	Minimum Value	Maximum Value
d_{out}	150 μm	300 μm
w	1 μm	15 μm
n	2.5	7.5
s	2 μm	10 μm

TABLE II
COMPARISON OF DIFFERENT OPTIMIZATION METHODS

Method	Optimal Design ($[d_{out}\ w\ n\ s]^T$ in μm)	Q	L_s (nH)	EM Simulations
ISM	$[203\ 10\ 4.5\ 2]^T$	4.9	3.8	10
Circuit Model GP	$[252\ 15\ 3.5\ 2]^T$	5.2	3.1	0*
Enumeration	$[205\ 10\ 4.5\ 2]^T$	4.9	3.9	13950

* One EM simulation is taken to validate the design. It shows that the specification is not met.

VI. CONCLUSIONS

We present a new spiral inductor optimization method based on space mapping technology. We show that the new method can provide an EM-validated optimal design in very few full-wave EM simulations.

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