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PRACTICAL LEAST pTH APPROXIMATION WITH EXTREMELY LARGE VALUES OF p

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Abstract

Some computational experience relevant to computer-aided design using the ideas of least pth approximation with extremely large values of p is reported. Values of p up to 1,000,000,000,000 have been successfully employed in conjunction with efficient gradient minimization algorithms such as the Fletcher-Powell method and the recent method due to Fletcher.

1. INTRODUCTION

This paper describes some computational experience of the computer-aided design of actual circuits and systems using the ideas of least pth approximation with extremely large values of p. The obvious reason why large values of p are desirable is that the corresponding optimal approximations tend to become minimax (or Chebyshev) approximations. Thus, least pth approximation using the Fletcher-Powell method[1] and a new method due to Fletcher[2] and the values of p up to 1,000,000,000,000 have been tried on a number of test examples.

2. THE METHOD

We restrict ourselves here to a discussion of rather conventional discrete least pth approximation problems. Consider the minimization of

$$U(\phi) = \Big(\sum_{i\in I} |e_i(\phi)|^p\Big)^{\frac{1}{p}} > 0 \quad \text{for } 1<p<\infty \tag{1}$$

where $e_i(\phi)$, in general, represents a weighted error or deviation between a complex specified function (desired response) and a complex approximating function (actual response) at some sample point i of a finite set I, and ϕ represents the k variable parameters, i.e.,

$$\phi \triangleq [\phi_1 \quad \phi_2 \quad \cdots \quad \phi_k]^T \tag{2}$$

Assuming that the $e_i(\phi)$ for $i\in I$ are continuous with continuous derivatives,

$$\nabla U(\phi) = \Big(\sum_{i\in I} |e_i(\phi)|^p\Big)^{\frac{1-p}{p}} \sum_{i\in I} |e_i(\phi)|^{p-2} \mathrm{Re}\{e_i^*(\phi)\nabla e_i(\phi)\} \quad \ldots \tag{3}$$

where

$$\nabla \triangleq \Big[\frac{\partial}{\partial\phi_1} \quad \frac{\partial}{\partial\phi_2} \quad \cdots \quad \frac{\partial}{\partial\phi_k}\Big]^T \tag{4}$$

and * denotes the complex conjugate.

In an effort to alleviate the ill-conditioning resulting from the evaluation of $|e_i(\phi)|^p$ for very large values of p, we let

$$M(\phi) \triangleq \max_{i\in I} |e_i(\phi)| \tag{5}$$

and rewrite (1) and (3), respectively, in the form

$$U(\phi) = M(\phi)\Big(\sum_{i\in I} \Big|\frac{e_i(\phi)}{M(\phi)}\Big|^p\Big)^{\frac{1}{p}} \tag{6}$$

and

$$\nabla U(\phi) = \Big(\sum_{i\in I} \Big|\frac{e_i(\phi)}{M(\phi)}\Big|^p\Big)^{\frac{1-p}{p}} \sum_{i\in I} \Big|\frac{e_i(\phi)}{M(\phi)}\Big|^{p-2} \mathrm{Re}\Big\{\frac{e_i^*(\phi)}{M(\phi)} \nabla e_i(\phi)\Big\} \quad \ldots \tag{7}$$

noting that $M(\phi)>0$ and, again, that $U(\phi)>0$ and $1<p<\infty$.

3. EXAMPLES

Space does not permit a full discussion of all the examples attempted, so we will briefly consider test problems for which the minimax solutions are known [3-6].

Consider the design of 10Ω to 1Ω transmission-line transformers for a relative bandwidth of 100%. Let e_i in (6) be ρ_i, the reflection coefficient, sampled uniformly at 11 points on the relative frequency interval [0.5, 1.5] GHz for 2-section designs and at {0.5, 0.6, 0.7, 0.77, 0.9, 1.0, 1.1, 1.23, 1.3, 1.4, 1.5} GHz for 3-section designs.

Appropriate gradient vectors with respect to length and characteristic impedance of transmission lines are calculated by the adjoint network method[7].

The progress of the two algorithms used on a CDC 6400 computer from indicated starting points is summarized in Tables 1 and 2 and Figures 1 to 5. For convenience we show results for different values of n where $p=10^n$. The figures are plots of M of (5) against N, the number of function evaluations at the beginning of an iteration. One function evaluation includes the evaluation of appropriate gradients.

4. CONCLUSIONS

Our experience to-date indicates that, by and large, the smaller the value of p, the faster will $M(\phi)$ be minimized, always assuming that the value of p is sufficiently large so that a specified M can be attained. No attempts at modifying the minimization methods to improve convergence for extremely large values of p nor a detailed study of other possible effects of numerical ill-conditioning have so far been carried out. But, if the success we have had using our present approach together with efficient gradient methods are widely repeatable, then far-reaching consequences are foreseen not only in nonlinear approximation, but in the closely related field of nonlinear programming[8]. Applications in the important area of filter design which can be readily carried out using least pth objectives[9], are also envisaged.

5. ACKNOWLEDGEMENTS

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TABLE 1

OPTIMIZATION OF A 2-SECTION 10:1 QUARTER-WAVE TRANSFORMER OVER 100 PERCENT BANDWIDTH WITH VARIABLE CHARACTERISTIC IMPEDANCES Z_1 AND Z_2

Fig.	Starting Point Z_1	Starting Point Z_2	n where $p=10^n$	Number of Function Evaluations N* Fletcher [2]	Number of Function Evaluations N* Fletcher-Powell [1]
			2	22	31
			3	28	49
1	1.0	3.0	6	33	56
			9	33	56
			12	33	56
			2	30	26
			3	58	50
2	1.0	6.0	6	†	133
			9	†	172
			12	†	198
			2	15	23
			3	†	41
3	3.5	6.0	6	44	101
			9	102	118
			12	102	118
			2	14	16
			3	19	56
4	3.5	3.0	6	21	75
			9	21	85
			12	21	310

*The number N listed are those required to bring M within 0.01 percent of the known optimum value, namely, 0.42857.

†Missing entries are due to parameters becoming negative - constraints were not imposed during optimization.

TABLE 2

OPTIMIZATION OF A 3-SECTION 10:1 TRANSFORMER OVER 100 PERCENT BANDWIDTH WITH VARIABLE LENGTHS AND CHARACTERISTIC IMPEDANCES

Starting Point: $Z_1=1.5$, $Z_2=3.0$, $Z_3=6.0$, $\ell_1/\ell_q=0.8$ $\ell_2/\ell_q=1.2$, $\ell_3/\ell_q=0.8$, where ℓ_q is the quarter-wavelength at center frequency.

n where $p=10^n$	Number of Function Evaluations N to reach the value of M shown in brackets; the optimum value of M is 0.19729 — Fletcher [2]		Fletcher-Powell [1]	
3	57 ~~88~~	(0.19734)	115 ~~143~~	(0.19734)
4	86 ~~104~~	(0.19730)	378	(0.19729)
6	418	(0.19729)	702	(0.19740)
9	634 ~~637~~	(0.19730)	661	(0.19740)
12	668 ~~684~~	(0.19736)	645	(0.19851)

6. REFERENCES

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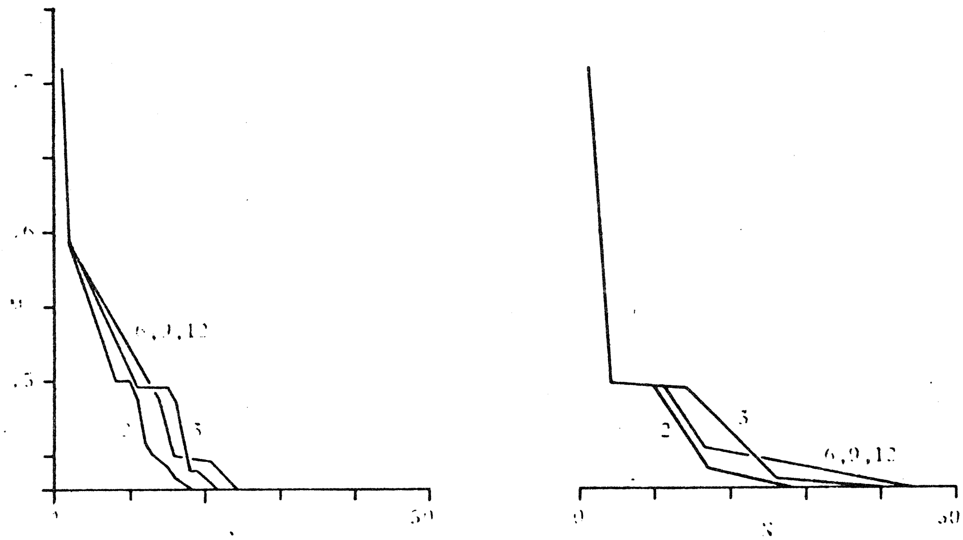
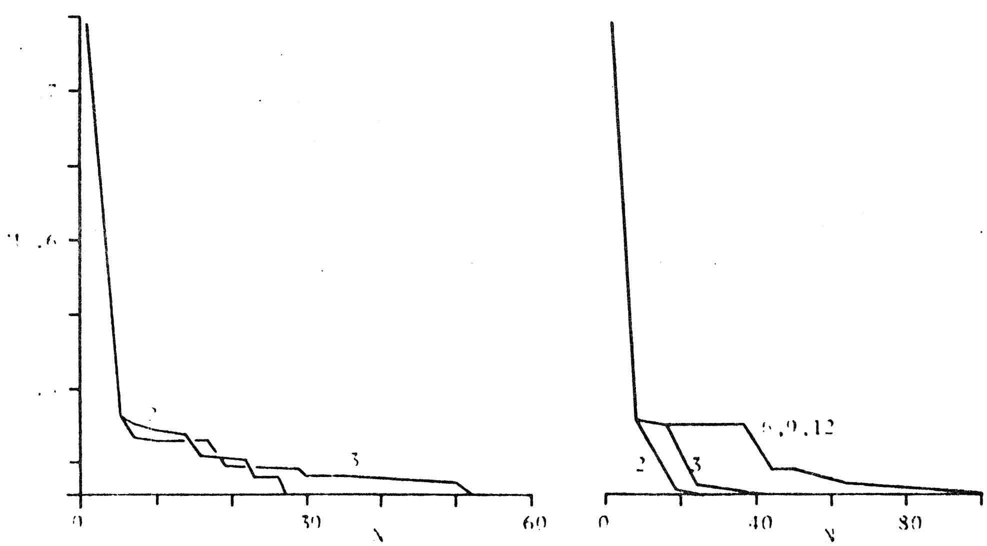


(a) Fletcher (b) Fletcher-Powell

Fig. 1



(a) Fletcher (b) Fletcher-Powell

Fig. 2

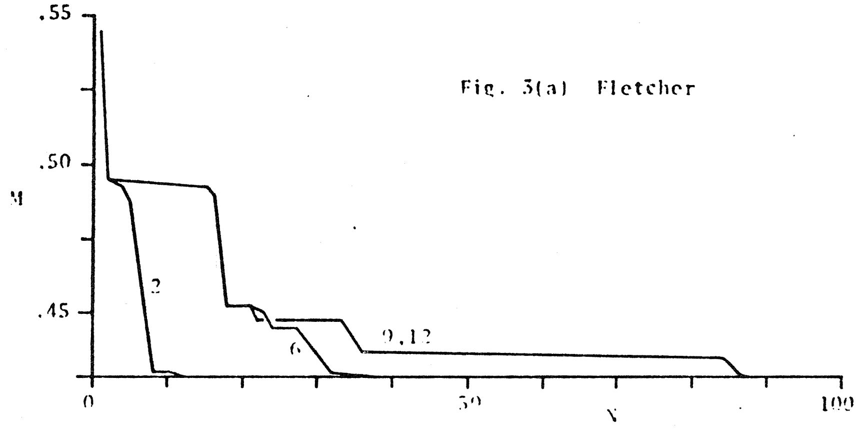


Fig. 3(a) Fletcher
.55
.50
.45
M
2
6
9,12
0
50
100
N

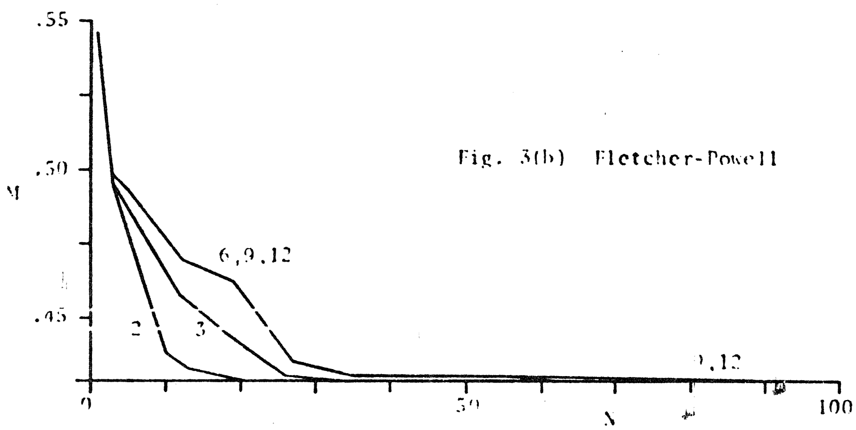


Fig. 3(b) Fletcher-Powell
.55
.50
.45
M
6,9,12
2
3
9,12
0
50
100
N

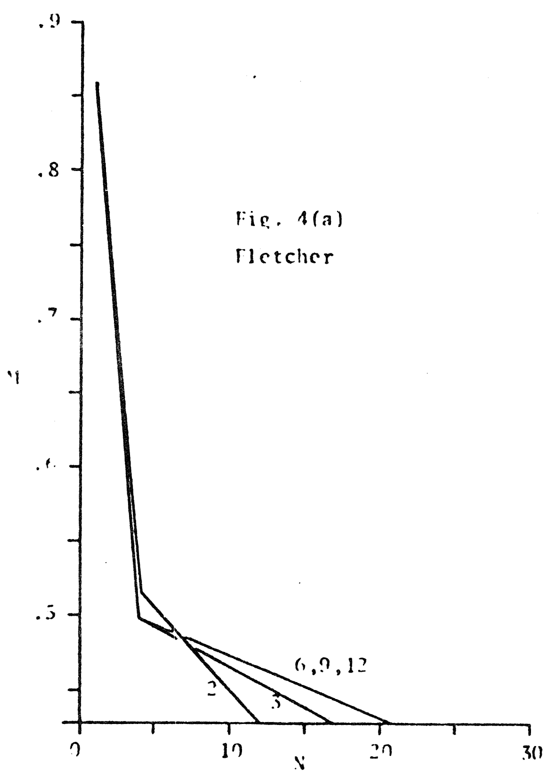


Fig. 4(a)
Fletcher
.9
.8
.7
.6
.5
M
6,9,12
2
3
0
10
20
30
N

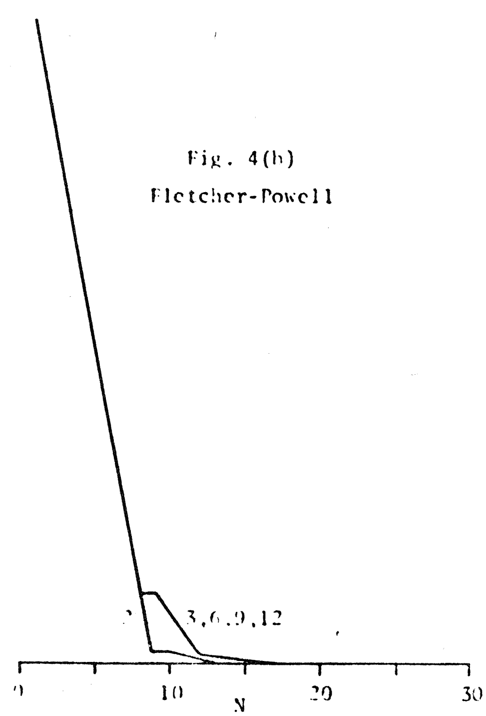
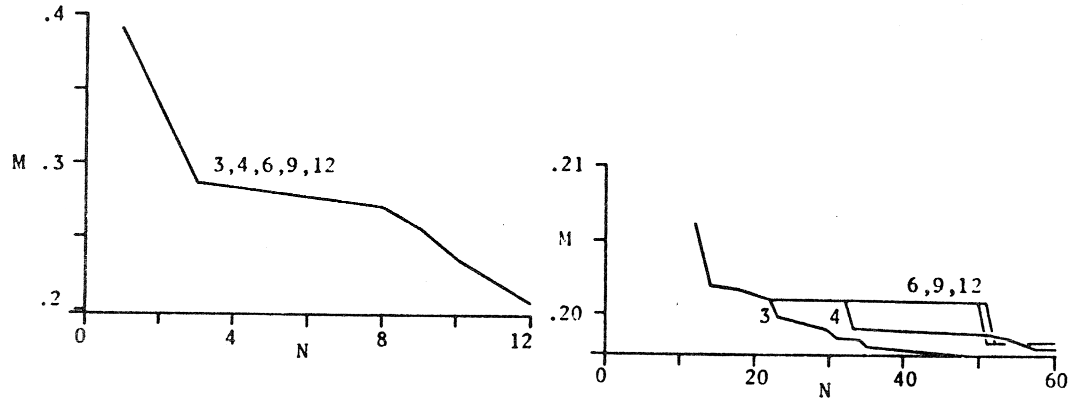
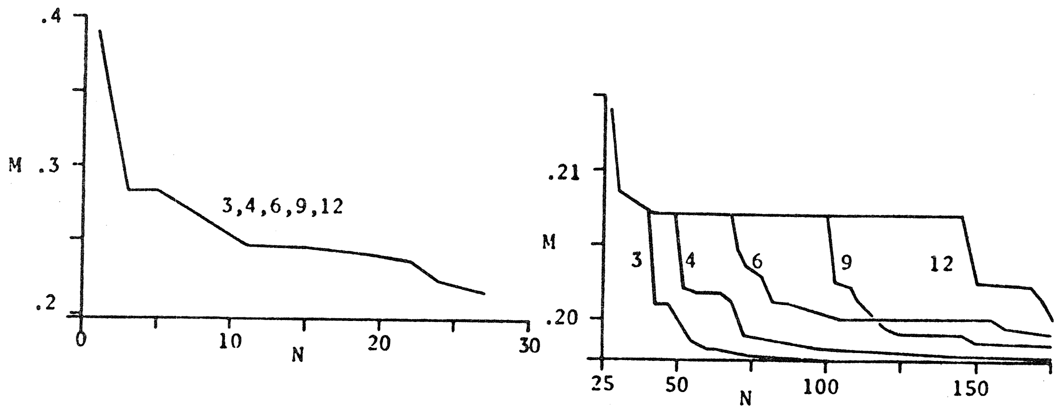


Fig. 4(b)
Fletcher-Powell
2
3,6,9,12
0
10
20
30
N



(a) Fletcher



(b) Fletcher-Powell

Fig. 5