Microwave Device Modeling Using Space-Mapping and Radial Basis Functions

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***Abstract*—A novel surrogate modeling methodology is presented that utilizes space mapping combined with radial basis function interpolation. The method has advantages both over the standard space mapping modeling methodology and the recently published space mapping modeling with variable weight coefficients. In particular, it provides accuracy comparable or better than the latter method and computational efficiency as good as the standard space mapping modeling procedure. Examples illustrating the performance of the new modeling method as well as a comparison with previously published approaches are given.**

***Index Terms*—Computer-aided design (CAD), EM modeling, space mapping, surrogate modeling, radial basis functions.**

I. Introduction

Full-wave EM simulations of microwave structures are CPU intensive. Statistical analysis and yield optimization, crucial for manufacturability-driven designs in a time-to-market development environment, demand accurate and fast models. The space mapping (SM) concept [1] addresses this issue.

SM assumes "fine" and "coarse" models. The "fine" model may be a high fidelity CPU-intensive EM simulator, undesirable for direct statistical analysis and design. The "coarse" model can be a simplified representation such as an equivalent circuit with empirical formulas. SM modeling [2]-[6] exploits the speed of the coarse model and the accuracy of the fine model to develop fast, accurate enhanced models (surrogates) valid over a wide range of parameter values.

The standard SM modeling approach is based on setting up the surrogate model using a small amount of fine-model data (usually a so-called star distribution: $2n+1$ points, where n is the number of design variables) and performing extraction of model parameters over the whole set of this data [4], [5]. This simple methodology gives reasonable accuracy especially for low-dimensional problems. In order to further improve the modeling performance one needs to involve more fine model information. Unfortunately, SM is not suitable for handling a large amount of fine model data by itself, i.e., increasing the number of base points does not help if the number of degrees of freedom of the model remains unchanged.

A recently published space mapping modeling technique with variable weight coefficients [6] aimed at overcoming these limitations. It provides better accuracy than the standard method, however, at the expense of significant increase of the evaluation time, which is due to a separate parameter extraction required for each evaluation of the surrogate model. This limits potential applications of the method.

In this paper, we present a new approach that combines the standard space mapping modeling methodology with radial basis function interpolation. This combination gives modeling accuracy comparable to or better than the method [6] without compromising computational cost.

II. Surrogate Modeling with Space Mapping and Radial Basis Function Interpolation

Let $\boldsymbol{R}_f: X_f \to R^m$ and $\boldsymbol{R}_c: X_c \to R^m$ denote the fine and coarse model response vectors, where $X_f \subseteq R^n$ and $X_c \subseteq R^n$ are design variable domains of the fine and coarse models, respectively. For example, $\boldsymbol{R}_f(\boldsymbol{x})$ and $\boldsymbol{R}_c(\boldsymbol{x})$ may represent the magnitude of a transfer function at m chosen frequencies.

We denote by $X_R \subseteq X_f$ the region of interest in which we want enhanced matching between the surrogate and the fine model. We assume that X_R is an n-dimensional interval in R^n with center at reference point $\boldsymbol{x}^0 = [x_{0.1} \ldots x_{0.n}]^T \in R^n$:

$$X_R = [\boldsymbol{x}^0 - \boldsymbol{\delta}, \boldsymbol{x}^0 + \boldsymbol{\delta}] = [x_{0.1} - \delta_1, x_{0.1} + \delta_1] \times \ldots \times [x_{0.n} - \delta_n, x_{0.n} + \delta_n] \quad (1)$$

where $\boldsymbol{\delta} = [\delta_1 \ldots \delta_n]^T$ determines the size of X_R. We use $X_R(\boldsymbol{x}^0, \boldsymbol{\delta})$ to denote the region of interest defined by $\boldsymbol{x}^0$ and $\boldsymbol{\delta}$. Let $X_B = \{\boldsymbol{x}^1, \boldsymbol{x}^2, \ldots, \boldsymbol{x}^N\} \subset X_R(\boldsymbol{x}^0, \boldsymbol{\delta})$ be the base set, where N is the number of base points, such that the fine model response is known at all points $\boldsymbol{x}^j$, $j = 1, 2, \ldots, N$. We do not assume any particular location of the base points.

We define a generic surrogate model $\bar{\boldsymbol{R}}_s : X_R \times M_{m\times m} \times M_{n\times n} \times M_{n\times 1} \to R^m$ as

$$\bar{\boldsymbol{R}}_s(\boldsymbol{x}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{c}) = \boldsymbol{A} \cdot \boldsymbol{R}_c(\boldsymbol{B} \cdot \boldsymbol{x} + \boldsymbol{c}) \quad (2)$$

with matrices $\boldsymbol{A} = diag\{a_1, \ldots, a_m\}$, $\boldsymbol{B} \in M_{n\times n}$, and $\boldsymbol{c} \in M_{n\times 1}$ ($M_{k\times l}$ denotes the set of $k\times l$ real matrices) found using the parameter extraction process

$$(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{c}) = \arg \min_{(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma})} \sum_{k=1}^{N} \| \boldsymbol{R}_f(\boldsymbol{x}^k) - \bar{\boldsymbol{R}}_s(\boldsymbol{x}^k, \boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}) \| \quad (3)$$

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Apart from model (2), (3), an optional frequency scaling can be implemented that works in such a way that the coarse model is evaluated at a different frequency than the fine model using the transformation: $\omega \rightarrow f_0 + f_1\omega$, where $\boldsymbol{F} = [f_0\ f_1] \in R^2$ is obtained together with matrices $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{c}$ using a parameter extraction process similar to (3). More general space mapping surrogate models can be found in [4]-[7].

On the top of the standard SM surrogate, we use radial basis function (RBF) interpolation [8], [9] of the difference between the fine model $\boldsymbol{R}_f$ and standard surrogate $\bar{\boldsymbol{R}}_s$, denoted as $\tilde{\boldsymbol{R}}_s : X_R \rightarrow R^m$. Let $\boldsymbol{R}_f(\boldsymbol{x}) = [R_{f.1}(\boldsymbol{x})\ \ldots\ R_{f.m}(\boldsymbol{x})]^T$ and $\bar{\boldsymbol{R}}_s(\boldsymbol{x}) = [\bar{R}_{s.1}(\boldsymbol{x})\ \ldots\ \bar{R}_{s.m}(\boldsymbol{x})]^T$. $\tilde{\boldsymbol{R}}_s$ is defined as

$$\tilde{\boldsymbol{R}}_s(\boldsymbol{x}) = \begin{bmatrix} \sum_{j=1}^{N} \lambda_{1.j}\phi(\|\boldsymbol{x}-\boldsymbol{x}^j\|/\gamma) \\ \ldots \\ \sum_{j=1}^{N} \lambda_{m.j}\phi(\|\boldsymbol{x}-\boldsymbol{x}^j\|/\gamma) \end{bmatrix} \quad (4)$$

where $\|\ .\ \|$ denotes the Euclidean norm. The parameters $\lambda_{k.j}$ are calculated so that they satisfy

$$\Phi\boldsymbol{\lambda}_k = \boldsymbol{F}_k \qquad k = 1, 2, \ldots, m \quad (5)$$

where $\boldsymbol{\lambda}_k = [\lambda_{k.1}\ \ \lambda_{k.2}\ \ldots\ \lambda_{k.N}]^T$,

$$\boldsymbol{F}_k = \begin{bmatrix} R_{f.k}(\boldsymbol{x}^1) - \bar{R}_{s.k}(\boldsymbol{x}^1) \\ \vdots \\ R_{f.k}(\boldsymbol{x}^N) - \bar{R}_{s.k}(\boldsymbol{x}^N) \end{bmatrix} \quad (6)$$

and Φ is $N \times N$ matrix with elements

$$\Phi_{ij} = \phi(\|\boldsymbol{x}^i - \boldsymbol{x}^j\|/\gamma) \quad (7)$$

$\gamma = \gamma(\boldsymbol{\delta}, N)$ is a characteristic distance of the base set defined as

$$\gamma(\boldsymbol{\delta}, N) = \frac{2}{nN^{1/n}} \sum_{i=1}^{n} \delta_i \quad (8)$$

Parameter γ is, in fact, an average distance between base points and it is used in (4) as a normalization factor.

In this paper we use a Gaussian basis function defined as

$$\phi(r) = e^{-cr^2} \qquad r \geq 0 \qquad c > 0 \quad (9)$$

Other choices of basis functions can be found in the literature [9].

The combined surrogate model $\boldsymbol{R}_s : X_R \rightarrow R^m$ is defined as

$$\boldsymbol{R}_s(\boldsymbol{x}) = \bar{\boldsymbol{R}}_s(\boldsymbol{x}) + \tilde{\boldsymbol{R}}_s(\boldsymbol{x}) \quad (10)$$

Once coefficients λ are found, evaluation of (4) is very fast, which means that evaluation cost of model (10) is not significantly larger than evaluation cost of the standard space mapping surrogate model (2). This is in contrast with the modeling technique [6] requiring a separate parameter extraction for each evaluation of the surrogate model, which involves a number of coarse model evaluations (typically hundreds or even thousands).

III. Examples

In this section, we compare the modeling accuracy for the standard SM modeling methodology [4], SM modeling with variable weight coefficients [6], and the new modeling approach (2)-(10) described in Section II.

A. Test Problem Description

Problem 1: Two-section capacitively-loaded impedance transformer [10]. The coarse model and the fine model are, respectively, an ideal two-section transmission line (TL) and a capacitively-loaded TL as shown in Fig. 1. Both models are implemented in Matlab. The design parameters are $\boldsymbol{x} = [L_1\ L_2]^T$. The frequency range is 0.5GHz $\leq \omega \leq$ 1.5GHz. The reference point is $\boldsymbol{x}^0 = [74.25^\circ\ 79.24^\circ]^T$. The region of interest is defined by a 10% deviation from $\boldsymbol{x}^0$. We consider $|S_{11}|$ as the model response.

Problem 2: Seven-section capacitively-loaded impedance transformer [10]. The coarse and fine models are shown in Fig. 2 (both implemented in Matlab). The frequency range is 1.0GHz $\leq \omega \leq$ 7.7GHz. The design parameters are $\boldsymbol{x} = [L_1\ L_2\ L_3\ L_4\ L_5\ L_6\ L_7]^T$. The reference point is $\boldsymbol{x}^0 = [1\ 1\ 1\ 1\ 1\ 1\ 1]^T$. We consider the region of interest defined by a 5% deviation from $\boldsymbol{x}^0$.

Problem 3: Microstrip right-angle bend [3]. The fine model, Fig. 3(a), is analyzed by Sonnet's ***em***™. The coarse model, Fig. 3(b), is an equivalent circuit with parameters calculated from Kirschning *et al.* [12]. The design parameters are $\boldsymbol{x} = [W\ H\ \varepsilon_r]^T$. The frequency range is 1 to 31 GHz. The reference point is $\boldsymbol{x}^0$ = [25 12 9]^T, and the region size is $\boldsymbol{\delta}$ = [6 4 1]^T.

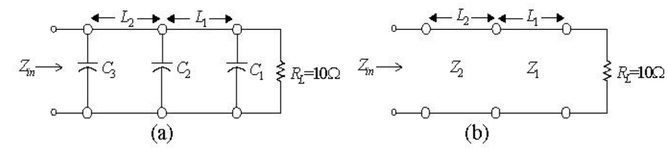

Fig.1. Two-section capacitively-loaded impedance transformer [10]: "fine" model (a) and "coarse" model (b).

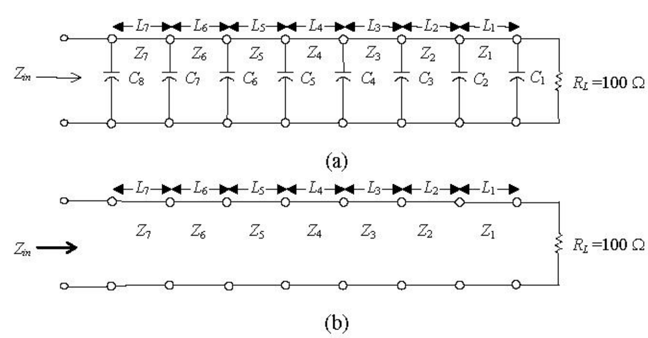

Fig. 2. Seven-section capacitively-loaded impedance transformer [10]: "fine" model (a) and "coarse" model (b).

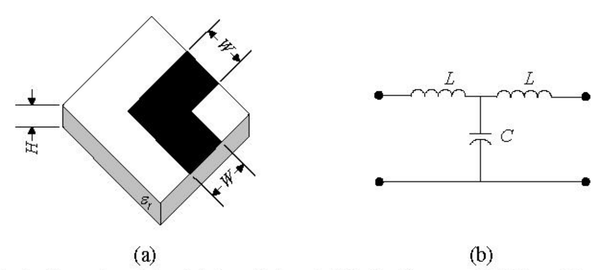

Fig.3. The microstrip right-angle bend [3]: the fine model (a) and the coarse model (b).

B. Experimental Setup

For each of the test problems, we performed a number of experiments using the standard SM surrogate model, SM model [6], pure RBF interpolation (i.e., RBF interpolating directly to fine model data) as well as the new SM model (2)-(10). Table I shows details of the base sets used in our experiments. The base sets have growing numbers of points (and decreasing characteristic distances γ) in order to examine the dependence of the modeling error on the amount of fine model data used to create the model. The standard SM model uses the star-distribution base set [4] and surrogate model (2) enhanced by frequency space mapping.

Accuracy was tested using 30 test points randomly distributed in the region of interest. The error measure used was the l_2 norm of the difference between the fine model response and the corresponding surrogate model response.

TABLE I
BASE SET DATA FOR TEST PROBLEMS 1-3

Test Problem	Base Set	Base Set Description	Number of Base Points	γ
1	X_{B1}	Uniform mesh of density 3	9	5.12
	X_{B2}	Uniform mesh of density 5	25	3.07
	X_{B3}	Uniform mesh of density 7	49	2.19
	X_{B4}	Uniform mesh of density 10	100	1.53
2	X_{B1}	Star distribution	15	0.136
	X_{B2}	Uniform mesh of density 2 + $\boldsymbol{x}^0$	129	0.100
	X_{B3}	Uniform mesh of density 3	2187	0.067
3	X_{B1}	Uniform mesh of density 2	8	3.66
	X_{B2}	Uniform mesh of density 3	27	2.44
	X_{B3}	Uniform mesh of density 4	64	1.83
	X_{B4}	Uniform mesh of density 5	125	1.47

C. Experimental Results and Discussion

Tables II, III, and IV show numerical results (error statistics) for the standard SM model, SM model [6], pure radial basis function interpolation, and the new model (2)-(10) with all the base sets considered. Figs. 4-6 show dependence of average modeling error on the characteristic distance γ. Table V presents the qualitative comparison of the surrogate model evaluation cost for all the considered methods.

The results show that the new model—the combination of space mapping and RBF interpolation—outperforms both the standard SM model and the SM model with variable weight coefficients [6], as well as the pure RBF interpolation. Moreover, the computational cost of the new method is significantly lower than the cost of the SM model [6] and almost the same as for the standard SM model. The pure RBF interpolation is the fastest method because it only requires evaluation of formula (4), however, it cannot compete with the new model (2)-(10) in terms of modeling accuracy.

TABLE II
MODELING RESULTS FOR TEST PROBLEM 1
VERIFICATION FOR 30 RANDOM TEST POINTS

Model Description	Base Set	Average Error	Maximum Error	Standard Deviation
Standard SM Model	star dist.	0.0404	0.0703	0.0130
SM Model [6]	X_{B1}	0.0218	0.0446	0.0080
SM Model [6]	X_{B2}	0.0133	0.0330	0.0064
SM Model [6]	X_{B3}	0.0087	0.0244	0.0048
SM Model [6]	X_{B4}	0.0058	0.0220	0.0043
Pure RBF Interpolation	X_{B1}	0.0310	0.0642	0.0165
Pure RBF Interpolation	X_{B2}	0.0073	0.0181	0.0045
Pure RBF Interpolation	X_{B3}	0.0042	0.0189	0.0044
Pure RBF Interpolation	X_{B4}	0.0033	0.0124	0.0028
New SM Model (2)-(10)	X_{B1}	0.0134	0.0259	0.0059
New SM Model (2)-(10)	X_{B2}	0.0057	0.0154	0.0036
New SM Model (2)-(10)	X_{B3}	0.0031	0.0067	0.0016
New SM Model (2)-(10)	X_{B4}	0.0016	0.0041	0.0011

TABLE III
MODELING RESULTS FOR TEST PROBLEM 2
VERIFICATION FOR 30 RANDOM TEST POINTS

Model Description	Base Set	Average Error	Maximum Error	Standard Deviation
Standard SM Model	star dist.	0.0136	0.0232	0.0039
SM Model [6]	X_{B1}	0.0151	0.0299	0.0061
SM Model [6]	X_{B2}	0.0105	0.0150	0.0020
SM Model [6]	X_{B3}	0.0053	0.0082	0.0013
Pure RBF Interpolation	X_{B1}	0.0821	0.2057	0.0509
Pure RBF Interpolation	X_{B2}	0.0679	0.1441	0.0244
Pure RBF Interpolation	X_{B3}	0.0365	0.0788	0.0144
New SM Model (2)-(10)	X_{B1}	0.0098	0.0195	0.0039
New SM Model (2)-(10)	X_{B2}	0.0088	0.0129	0.0019
New SM Model (2)-(10)	X_{B3}	0.0036	0.0057	0.0009

TABLE IV
MODELING RESULTS FOR TEST PROBLEM 3
VERIFICATION FOR 30 RANDOM TEST POINTS

Model Description	Base Set	Average Error	Maximum Error	Standard Deviation
Standard SM Model	star dist.	0.0116	0.0275	0.0062
SM Model [6]	X_{B1}	0.0492	0.1728	0.0389
SM Model [6]	X_{B2}	0.0089	0.0541	0.0096
SM Model [6]	X_{B3}	0.0036	0.0093	0.0020
SM Model [6]	X_{B4}	0.0023	0.0066	0.0014
Pure RBF Interpolation	X_{B1}	0.0824	0.1050	0.0198
Pure RBF Interpolation	X_{B2}	0.0306	0.0726	0.0158
Pure RBF Interpolation	X_{B3}	0.0316	0.0891	0.0219
Pure RBF Interpolation	X_{B4}	0.0273	0.0337	0.0136
New SM Model (2)-(10)	X_{B1}	0.0062	0.0115	0.0028
New SM Model (2)-(10)	X_{B2}	0.0013	0.0036	0.0008
New SM Model (2)-(10)	X_{B3}	0.0011	0.0027	0.0009
New SM Model (2)-(10)	X_{B4}	0.0009	0.0030	0.0006

TABLE V

QUALITATIVE COMPARISON OF COMPUTATIONAL COST OF EVALUATING SURROGATE MODEL

Modeling Method	Main Sources of Computational Cost	Relative Evaluation Cost
Standard SM Model	Coarse model evaluation	Similar to coarse model
SM Model [6]	Parameter extraction	Much higher than coarse model
RBF Interpolation	Evaluation of formula (4)	Lower than coarse model
SM Model (2)-(10)	Coarse model evaluation	Similar to coarse model

IV. CONCLUSION

A new SM-based modeling methodology is presented. It combines space mapping with radial basis function interpolation. This combination allows us to reduce the modeling error to a level not attainable by any other space mapping technique without compromising computational cost.

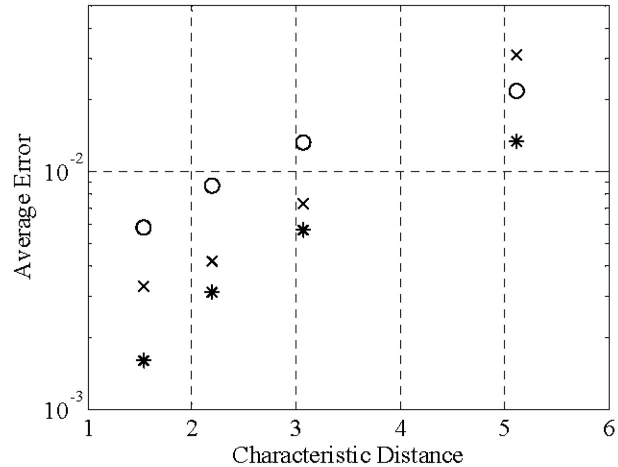

Fig.4. Average modeling error versus characteristic distance γ of the base set for test problem 1 (two-section impedance transformer). Data for SM model [6] (o), pure RBF interpolation (×), and new SM model (2)-(10) (*).

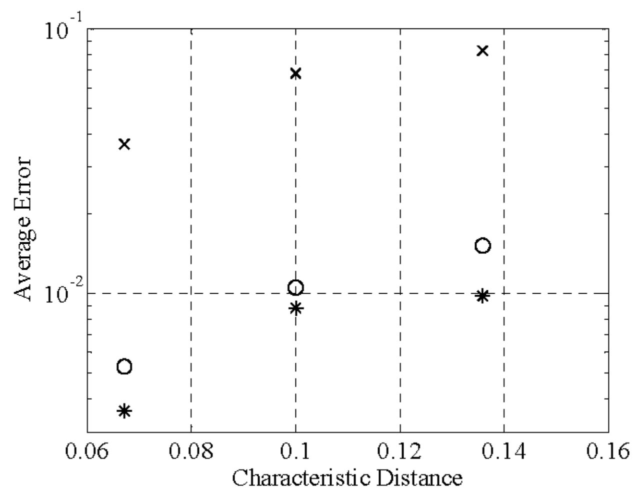

Fig.5. Average modeling error versus characteristic distance γ of the base set for test problem 2 (seven-section impedance transformer). Data for SM model [6] (o), pure RBF interpolation (×), and new SM model (2)-(10) (*).

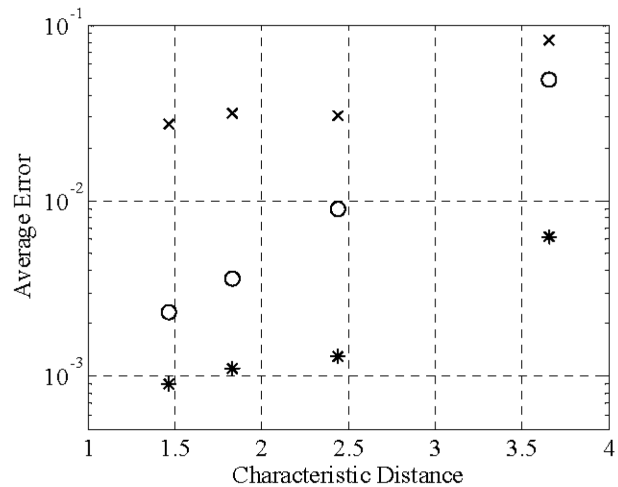

Fig.6. Average modeling error versus characteristic distance γ of the base set for test problem 1 (microstrip right-angle bend). Data for SM model [6] (o), pure RBF interpolation (×), and new SM model (2)-(10) (*).

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