Comparison of ρ Plane with H-Plane Negative Resistance Stability Criteria Using the Smith Chart

Abstract—**The Nyquist-type negative-resistance stability criterion is formulated directly in terms of a transformed reflection coefficient. It is compared with the immittance plane criterion, particular attention being paid to the "arbitrary closing loops." It is concluded that the immittance plane criterion is the least ambiguous and therefore the easiest to use.**

The purpose of this correspondence is to formulate the Nyquist-type criterion for the stability of negative-resistance circuits directly in terms of a transformed reflection coefficient, and to contrast the requirements with those for an immittance criterion. The use of a Smith chart with redefined scales to accommodate negative resistances has been reported [1]–[4]. This will provide a medium for the comparison. Particular attention will be paid to the so-called "arbitrary closing loops" of the Nyquist plots at high frequency, because, in the author's experience, they are often misinterpreted. The tunnel diode is a convenient example of negative resistance; lumped components are assumed for simplicity. It will be seen that an immittance (H) plane criterion (introduced in a previous paper by the author [1]) is the least ambiguous in terms of the Nyquist criterion and is, therefore, the easiest to implement.

The transformation describing the redefined Smith chart is

$$\rho'(j\omega) = \frac{1}{\rho^*(j\omega)}, \tag{1}$$

where ρ is the voltage reflection coefficient, and where $*$ signifies the complex conjugate. The reason for introducing this chart is that a single curve provides information simultaneously on gain and stability. Stability requires the right-half (RH) p-plane poles be absent from $\rho(p)$, where

$$\rho(p) = \frac{Z_L(p) - Z_C(-p)}{Z_L(p) + Z_C(p)} \tag{2}$$

and where Z_L and Z_C are defined in Fig. 1. If

$$H^*(j\omega) \equiv H(-j\omega), \tag{3}$$

then

$$\rho^*(j\omega) \equiv \rho(-j\omega). \tag{4}$$

Substituting p for $j\omega$ in (1),

$$\rho'(p) = \frac{1}{\rho(-p)} = \frac{Z_L(-p) + Z_C(-p)}{Z_L(-p) - Z_C(p)}. \tag{5}$$

For stability, therefore, $\rho'(p)$ must have no zeros in the left-half (LH) p plane.

To comply with the theorem on page 149 of Bode [5], the p-plane contour required for the present analysis must enclose the entire LH p plane (Fig. 2). The motion is positive up the

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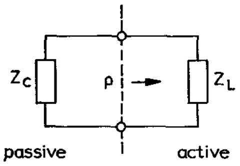

Fig. 1. The circuit under consideration.

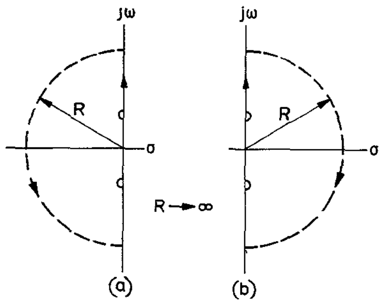

Fig. 2. The p plane, (a) a CCW encirclement in the LH, (b) a CW encirclement in the RH.

TABLE I
LIMITING CASES IN THE TRANSFORMATION FROM p TO H PLANE

Function	Limiting cases as $p \to \infty$		
$Z_T(p)$	$\frac{1}{pC}$	R	pL
$Y_T(p)$	$\frac{1}{pL}$	G	pC
Closing loop in RH H plane	Small CCW	Single point	Large CW

TABLE II
LIMITING CASES IN THE TRANSFORMATION FROM p TO ρ AND ρ' PLANES AS $p \to \infty$

Function	C	R	L
$\rho(p)$		$\frac{R-R'}{R+R'}$	
	-1		$+1$
$\rho'(p)$		$\frac{R+R'}{R-R'}$	

$j\omega$ axis, avoiding singularities on the axis by small semicircular indentations into the LH plane, but counterclockwise (CCW) around an infinite semicircle standing on the $j\omega$ axis, centered at the origin and lying in the LH plane. Relating the criterion for $\rho'(j\omega)$, which can be plotted on the redefined Smith chart, to the more familiar function ρ, we may state: *The plot of $\rho'(j\omega)$ must for stability encircle the ρ'-plane origin as many times in a clockwise (CW) manner as $\rho(p)$ has zeros in the RH p plane.* These zeros, not all necessarily "active," are given by zeros of the numerator of $\rho(p)$, since RH p-plane poles of $Z_L(p)+Z_C(p)$, contributed by Z_L, are common to the numerator and therefore cancel out.

The determination of stability by Nyquist plots can involve an arbitrary closing loop, e.g., for $\omega > \omega_R$ (the resistive cutoff frequency), in a tunnel-diode circuit [1], [6]. This loop presents some conceptual difficulties, but may be necessary to avoid the infinite semicircle in the p-plane (Fig. 2).

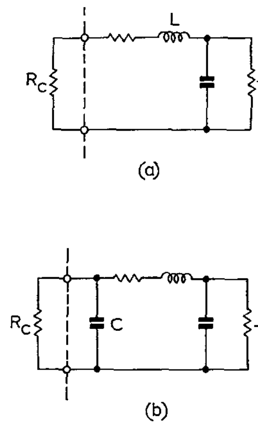

Fig. 3 Tunnel diode (a) without (b) with its package capacitance, facing a constant resistance or matched line.

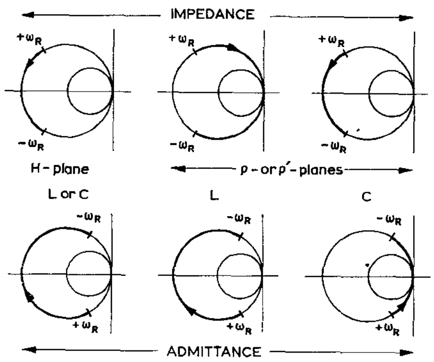

Fig. 4. Some typical arbitrary closing loops on the Smith chart applicable to Fig. 3. The loops are drawn as arcs of the unit circle for convenience. The L and C stand for the limiting cases relevant to Fig. 3 as $p \to \infty$. As indicated, the upper three charts are for impedance coordinates, and the lower three charts for admittance coordinates. The second and third column of charts show closing loops for a ρ- or ρ'-plane stability criterion, the second corresponding to Fig 3(a), the third to Fig 3(b).

Suppose

$$Z_T(p)|_{p\to\infty} \to \frac{1}{pC}, \tag{6}$$

where

$$Z_T = Z_L + Z_C; \tag{7}$$

then an infinite semicircular CW motion in the RH p plane corresponds to an infinitesimal semicircular CCW motion in the RH Z plane. Suppose

$$Z_T(p)|_{p\to\infty} \to R, \tag{8}$$

then the whole of the infinite semicircular arc in the RH p plane coalesces into a single point in the RH Z plane. Suppose

$$Z_T(p)|_{p\to\infty} \to pL, \tag{9}$$

then an infinite semicircular CW motion in the RH p plane corresponds to a similar infinite motion in the RH Z plane. Corresponding statements can be made about $Y_T(p)|_{p\to\infty}$, where $Y_T = 1/Z_L + 1/Z_C$ and not $1/Z_T$. These results are summarized in Table I.

Above ω_R no external negative resistance is exhibited by a tunnel diode by definition—in this case all $j\omega$ plots terminate somewhere in the RH H plane. It is therefore justified to draw an arbitrary loop *from* just above ω_R *to* just below $-\omega_R$ in the RH H plane (along a constant resistance or conductance line just to the right of the imaginary-axis, say) regardless of which limiting case of Table I applies.

Analogous limiting cases will now be found for the ρ' plane, for $R' = Z_C(p)|_{p\to\infty}$.

If $Z_L(p)|_{p\to\infty} \to \dfrac{1}{pC}$,

$$\rho'(p)|_{p\to\infty} \to -1. \tag{10}$$

If $Z_L(p)|_{p\to\infty} \to R$,

$$\rho'(p)|_{p\to\infty} \to \frac{R+R'}{R-R'}. \tag{11}$$

If $Z_L(p)|_{p\to\infty} \to pL$,

$$\rho'(p)|_{p\to\infty} \to +1. \tag{12}$$

The infinite CCW semicircle in the LH p plane of Fig. 2(a) will, therefore, coalesce into a single point on the real ρ or ρ' axis, according to Table II.

The question of where an arbitrary loop can be drawn without ambiguity presents some difficulty. It is important to distinguish between whether the Smith chart is being used with the stability criterion in terms of immittance or in terms of voltage reflection coefficient. Figure 3 illustrates the difficulty with the aid of Tables I and II, with Z_L representing the tunnel-diode equivalent circuit either in the form of Fig. 3(a) or (b), and with $Z_C(p)$ equal to a constant. Note that the H-plane criterion is associated with a closing loop through the region of the origin. The closing loops for the ρ'-plane criterion, however, pass through $\rho = \pm 1$ according to whether L or C, respectively, predominates as $p \to \infty$.

To summarize:

1) The numerator of $\rho(p)$ may have zeros in the RH p plane other than "active" ones which would have to be determined.
2) The arbitrary closing loop in the ρ' plane cannot be so easily dismissed (see Fig. 4) as in the H-plane.
3) No further information on *stability* is forthcoming by considering ρ' rather than H.

It is advisable, therefore, that when using the Smith chart for predicting simultaneously the stability and gain of reflection amplifiers, for example, the stability criterion itself should be formulated in terms of H to avoid ambiguity. The Smith chart plot is visualised as a distorted H-plane plot for this purpose.

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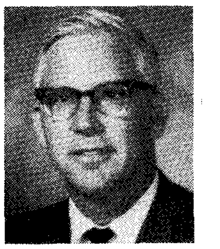

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