Accelerating Space Mapping Optimization with Adjoint Sensitivities

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***Abstract*—We propose a procedure for accelerating the space mapping optimization process. Exploiting both fine- and surrogate-model sensitivity information, a good mapping between the two model spaces is efficiently obtained. This results in a significant speed-up over direct gradient-based optimization of the original fine model and enhanced performance compared with other space mapping approaches. Our approach utilizes commercially available software with adjoint sensitivity analysis capabilities. It is illustrated through an example.**

***Index Terms*—Adjoint sensitivity analysis, aggressive space mapping (ASM), engineering optimization, implicit space mapping.**

I. Introduction

SPACE mapping (SM) optimization aims at shifting the optimization burden from an expensive "fine" (or high-fidelity) model to a cheap "coarse" (or low-fidelity) model by iteratively optimizing and updating a surrogate. Utilizing a mapping between the parameter spaces of the coarse and fine models, the optimization iterates are guided in the fine-model space by very few fine-model simulations. In the field of microwave circuit design, SM usually exploits full-wave electromagnetic (EM) solvers as fine models while circuit-based CAD tools are utilized as coarse models [1].

Several SM algorithms have been proposed since its inception in 1994 [1]. Aggressive SM (ASM) [2] exploits a quasi-Newton iteration with the classical Broyden formula [3] to estimate the mapping. Implicit space mapping (ISM) [2] exploits preassigned parameters of the fine model. In ISM, an auxiliary set of parameters (e.g., dielectric constant of a substrate) is used to match the coarse model to the fine model. The coarse model is then calibrated by these parameters and re-optimized to predict a better fine-model design. Output SM (OSM) [2] introduces a transformation of the coarse-model response. OSM is usually used in the final stage when combined with other SM methods.

The theory of adjoint sensitivity analysis has been extended to electromagnetic solvers (e.g., see [4]). Using at most one extra EM simulation, we obtain the gradient of the fine-model response with respect to all design parameters regardless of their number. Adjoint sensitivity analysis can be implemented with various full-wave solvers. The self-adjoint sensitivity analysis (SASA) has the advantage of eliminating the extra adjoint EM simulation, resulting in negligible overhead compared to the full-wave simulation itself. It is applicable to network responses such as S-parameters [4].

In this letter, we propose an algorithm for enhancing SM optimization. Using the self-adjoint sensitivities supplied by the EM solver and the cheap response sensitivities of the surrogate model, a mapping between the fine and coarse parameter spaces is accurately estimated. Results show that using available sensitivities dramatically decreases the number of iterations and, hence, the number of fine model evaluations.

II. Background

The SM algorithm [2] aims at establishing a mapping between the design parameters of the coarse and fine models

$$\boldsymbol{x}_c = \boldsymbol{B}\boldsymbol{x}_f + \boldsymbol{c} \tag{1}$$

such that

$$\|\boldsymbol{R}_c(\boldsymbol{x}_c) - \boldsymbol{R}_f(\boldsymbol{x}_f)\| \leq \varepsilon. \tag{2}$$

Here $\boldsymbol{x}_c$ and $\boldsymbol{x}_f$ are the coarse- and fine-model design (or input) parameters, respectively, and $\boldsymbol{R}_c$ and $\boldsymbol{R}_f$ are their corresponding responses. ε is a sufficiently small number. Once the linear input mapping parameters ($\boldsymbol{B}$) are known, an approximation of the fine-model optimal design can be obtained from

$$\boldsymbol{x}_c^* = \boldsymbol{B}^{(i)}\overline{\boldsymbol{x}}_f + \boldsymbol{c}^{(i)} \tag{3}$$

where $\boldsymbol{x}_c^*$ is the optimal coarse-model solution, $\boldsymbol{B}^{(i)}$ is the ith iteration linear mapping matrix, $\overline{\boldsymbol{x}}_f$ is an estimate of the optimal fine model, and $\boldsymbol{c}^{(i)}$ is the corresponding shift vector.

The original SM differs in the way $\boldsymbol{B}$ is estimated. In [1], two sets of points in the fine- and coarse-model parameter space are used to estimate the mapping parameters. This approach requires an overhead of expensive fine-model simulations. In the ASM algorithm, $\boldsymbol{B}$ is updated using the classical Broyden formula. An initial guess is usually the identity matrix. If the two models are significantly misaligned, the algorithm may fail to give a good result. A trust region methodology integrated with ASM limits the step taken in every iteration to guarantee the convergence of the algorithm [2]. In ISM the mapping is established inside the surrogate model.

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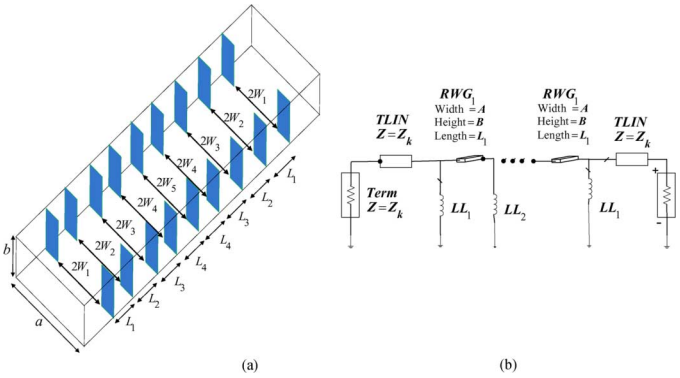

Fig. 1. Eight-section H-plane filter: (a) the fine model in HFSS with $a = 1.372$ inch and $b = 0.622$ inch; (b) the coarse model in Agilent ADS.

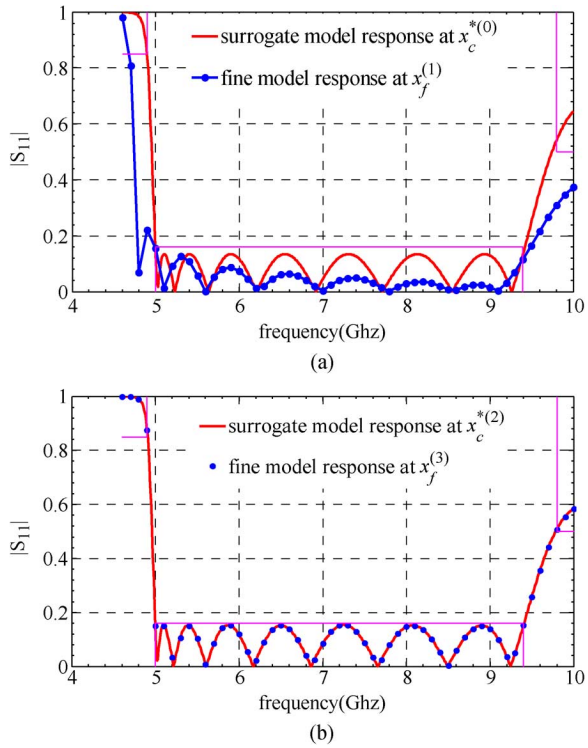

Fig. 2. Responses obtained during the optimization processes. The horizontal purple lines show the specifications.

III. SM Exploiting Jacobian Information and Preassigned Parameters

Based on the concept of ISM [2], at iteration $i = 1, 2, \ldots$, the surrogate model response can be defined as

$$\boldsymbol{R}_s^{(i)}(\boldsymbol{x}_c) = \boldsymbol{R}_c\left(\boldsymbol{x}_c, \boldsymbol{p}^{(i)}\right) \tag{4}$$

where

$$\boldsymbol{p}^{(i)} = \arg\min_{\boldsymbol{p}} \left\| \boldsymbol{R}_f\left(\boldsymbol{x}_c^{*(i)}\right) - \boldsymbol{R}_c\left(\boldsymbol{x}_c^{*(i)}, \boldsymbol{p}\right)\right\| \tag{5}$$

$\boldsymbol{p}$ is a vector of preassigned parameters which are different from the optimizable parameters $\boldsymbol{x}_c$. These parameters are optimized to ensure a better match between the fine and surrogate model responses. $\boldsymbol{x}_c^{*(i)}$ is the optimum surrogate response using $\boldsymbol{p}^{(i-1)}$. The main drawback of this approach is the requirement of enough preassigned parameters to match the fine and surrogate model responses at each iteration which are not always available. Here we have a different approach both in using preassigned parameters and exploiting surrogate sensitivities. In this approach, by using Jacobian information, we establish a mapping between the input of the surrogate and fine models and we use preassigned parameters to match the responses of the surrogate and fine models.

TABLE I
Design Parameter Values During the Optimization Processes, All Values Are in Inch

	$\boldsymbol{x}_c^{(0)}$	$\boldsymbol{x}_c^{*(0)}(\boldsymbol{x}_f^{(1)})$	$\boldsymbol{x}_c^{*(1)}$	$\boldsymbol{x}_f^{(2)}$	$\boldsymbol{x}_c^{*(2)}$	$\boldsymbol{x}_f^{(3)}$
W_1	0.5	0.57433	0.56723	0.53318	0.56278	0.53139
W_2	0.5	0.55880	0.56199	0.52337	0.56216	0.52243
W_3	0.5	0.54539	0.54984	0.50497	0.55107	0.50487
W_4	0.5	0.53965	0.54463	0.49759	0.54612	0.49720
W_5	0.5	0.53804	0.54317	0.49663	0.54505	0.49562
L_1	0.7	0.59479	0.69760	0.69200	0.75096	0.69930
L_2	0.7	0.63638	0.63286	0.63552	0.63490	0.63059
L_3	0.7	0.65139	0.65159	0.65101	0.65491	0.65051
L_4	0.7	0.65729	0 65833	0.65992	0.66130	0.65749

Contrary to (5) our parameter extraction (PE) process is performed via

$$\left(\boldsymbol{x}_c^{(i)}, \boldsymbol{p}^{(i)}\right) = \arg\min_{(\boldsymbol{x}_c, \boldsymbol{p})} \left\| \boldsymbol{R}_f\left(\boldsymbol{x}_f^{(i)}\right) - \boldsymbol{R}_c(\boldsymbol{x}_c, \boldsymbol{p})\right\| \tag{6}$$

where $\boldsymbol{x}_f^{(i)}$ and $\boldsymbol{R}_f(\boldsymbol{x}_f^{(i)})$ are the ith fine-model solution and fine model response, respectively. The initial point for performing (6) is $(\boldsymbol{x}_c^{*(i-1)}, \boldsymbol{p}^{(i-1)})$. Using (6) allows more degrees of freedom for matching the responses in comparison with ISM. Based on [2], the Jacobians of the surrogate and fine model responses at two corresponding points in the ith iteration are related by

$$\boldsymbol{J}_s\left(\boldsymbol{x}_c^{(i)}\right) \boldsymbol{B}^{(i)} \cong \boldsymbol{J}_f\left(\boldsymbol{x}_f^{(i)}\right). \tag{7}$$

The Jacobian of the fine-model response is estimated using SASA. If not available, the Jacobian of the surrogate-model response is obtained through finite differences (FD). The latter computation is fast because the coarse model is much faster than the fine model. The linear input mapping matrix is obtained by solving (7) as

$$\boldsymbol{B}^{(i)} \cong \left(\boldsymbol{J}_s\left(\boldsymbol{x}_c^{(i)}\right)^T \boldsymbol{J}_s\left(\boldsymbol{x}_c^{(i)}\right)\right)^{-1} \boldsymbol{J}_s\left(\boldsymbol{x}_c^{(i)}\right)^T \boldsymbol{J}_f\left(\boldsymbol{x}_f^{(i)}\right) \tag{8}$$

where $\boldsymbol{J}_s(\boldsymbol{x}_c^{(i)})$ is the Jacobian of the surrogate model. Then we re-optimize the new surrogate model to find

$$\boldsymbol{x}_c^{*(i)} = \arg\min_{\boldsymbol{x}} U\left(\boldsymbol{R}_s^{(i)}(\boldsymbol{x})\right) \tag{9}$$

where U is the given objective function. Finally, the fine-model solution is updated as

$$\boldsymbol{x}_f^{(i+1)} = \boldsymbol{B}^{(i)-1}\left(\boldsymbol{x}_c^{*(i)} - \boldsymbol{x}_c^{(i)}\right) + \boldsymbol{x}_f^{(i)}. \tag{10}$$

If we denote the simulation response accuracy by δ_f, the termination criterion for optimization is determined as

$$\left\| \boldsymbol{R}_f\left(\boldsymbol{x}_f^{(i+1)}\right) - \boldsymbol{R}_s^{(i)}\left(\boldsymbol{x}_c^{*(i)}\right)\right\|_\infty \leq \delta_f \tag{11}$$

TABLE II
PREASSIGNED PARAMETER VALUES, ALL VALUES ARE IN INCH

	a	b
$\boldsymbol{p}^{(0)}$	1.3720	0.6220
$\boldsymbol{p}^{(1)}$	1.3526	0.6778
$\boldsymbol{p}^{(2)}$	1.3333	0.7003

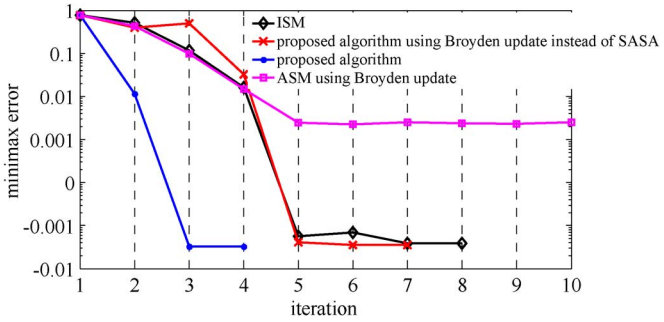

Fig. 3. Comparison of proposed algorithm with other SM algorithms.

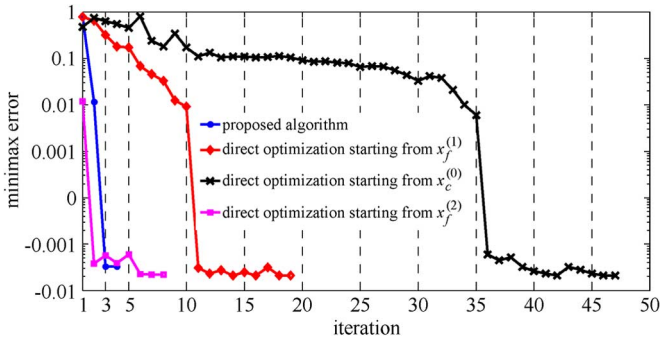

Fig. 4. Comparison of direct gradient optimization starting from different points with the proposed space mapping algorithm. One iteration corresponds to one fine model evaluation.

where $\|\cdot\|_\infty$ is the infinite norm operator. One can terminate the program if $\boldsymbol{R}_f(\boldsymbol{x}_f^{(i+1)})$ satisfies the specifications but this may prevent the optimizer from reaching the best possible solution. The proposed algorithm can be summarized as follows:

Step 0: Obtain $\boldsymbol{x}_c^{*(0)}$ through (9) starting from $\boldsymbol{x}_c^{(0)}, \boldsymbol{p}^{(0)}$.
Step 1: Set $i = 1$, set $\boldsymbol{x}_f^{(i)} = \boldsymbol{x}_c^{*(0)}$, obtain $\boldsymbol{R}_f(\boldsymbol{x}_f^{(i)})$, and obtain $\boldsymbol{J}_f(\boldsymbol{x}_f^{(i)})$ simultaneously using SASA.
Step 2: Find $\boldsymbol{x}_c^{(i)}$ and $\boldsymbol{p}^{(i)}$ using (6).
Step 3: If not available, obtain $\boldsymbol{J}_s(\boldsymbol{x}_c^{(i)})$ through FD.
Step 4: Calculate $\boldsymbol{B}^{(i)}$ using (8).
Step 5: Obtain $\boldsymbol{x}_c^{*(i)}$ through (9).
Step 6: Find $\boldsymbol{x}_f^{(i+1)}$ using (10), set $i = i + 1$.
Step 7: Obtain $\boldsymbol{R}_f(\boldsymbol{x}_f^{(i)})$, and obtain $\boldsymbol{J}_f(\boldsymbol{x}_f^{(i)})$ using SASA.
Step 8: If (11) is true, go to *Step* 9; else go to *Step* 2.
Step 9: Terminate.

IV. EXAMPLE

Here, we consider an eight-section H-plane filter, an extended version of a design presented in [5]. The design specifications are

$$|S_{11}| \le 0.16 \qquad 5.0\ \text{GHz} \le \omega \le 9.4\ \text{GHz}$$
$$|S_{11}| \ge 0.85 \qquad \omega \le 4.9\ \text{GHz}$$
$$|S_{11}| \ge 0.5 \qquad \omega \ge 9.8\ \text{GHz}.$$

The fine model is an EM model simulated using Ansoft HFSS ver. 13 on an X5670 3 GHz workstation with 48 GB of RAM. The simulation accuracy (Maximum Delta S) is set to 0.002 at 5.0 GHz with 50% mesh refinement and a magnetic wall is used (because of symmetry) to reduce the simulation time. The average simulation time is 240 min. As shown in Fig. 1, the coarse model is constructed by waveguide sections and inductances to model the septa. The a and b in Fig. 1 are the width and the height of the waveguide which are used as preassigned parameters. The values of the inductances are found from a simplified formula [6]. This coarse model is simulated using the model solver Agilent ADS. We used the MATLAB optimization toolbox for parameter extraction (using fminsearch), direct optimization and re-optimizing the surrogate model (using fminimax) (see Fig. 2).

The initial design parameters $\boldsymbol{x}_c^{(0)}$ as well as the fine- and surrogate-model parameters obtained during the optimization processes and preassigned parameters are shown, respectively, in Tables I and II. Fig. 3 shows that our enhanced SM algorithm converges in fewer fine-model simulations than required by the other SM algorithms [2]. Direct optimization needs 19 fine-model evaluations to converge even though it uses SASA and starts from the optimum coarse model which is the best possible starting point. Fig. 4 shows a comparison of the new algorithm with direct optimization using SASA starting from different starting points.

V. CONCLUSION

We propose an algorithm for the integration of sensitivity analysis with SM optimization. It is shown through an example that the number of fine-model evaluations is reduced compared to direct optimization, even when using Jacobian information in the direct optimization process and starting from the optimum coarse model solution. The proposed algorithm also converges in fewer fine model evaluations than other space mapping methodologies. The availability of parallel processing make the combination of space mapping and direct optimization more interesting as direct optimization can be performed by starting from different points obtained from space mapping optimization independently.

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