

# Fast Space Mapping Modeling with Adjoint Sensitivity

Qingsha S. Cheng<sup>†</sup>, *Senior Member, IEEE*, John W. Bandler<sup>†</sup>, *Life Fellow, IEEE*,  
Natalia K. Nikolova<sup>†</sup>, *Fellow, IEEE*, and Slawomir Koziel<sup>‡</sup>, *Senior Member, IEEE*

<sup>†</sup> Department of Electrical and Computer Eng., McMaster University, Hamilton, ON, Canada L8S 4K1

<sup>‡</sup> School of Science and Engineering, Reykjavik University, 101 Reykjavik, Iceland

**Abstract**—A space mapping technique is presented that utilizes recent commercially available adjoint sensitivity analysis procedures for modeling microwave and RF components. It is shown that using only one full-wave electromagnetic simulation along with an adjoint sensitivity evaluation one can build a good surrogate model. The proposed technique calibrates a surrogate by matching responses and corresponding adjoint sensitivities. This surrogate is accurate over an extended region of the parameter space. Using a three-section transformer and an H-plane filter, the technique is demonstrated to be straightforward to implement within available simulation software.

**Index Terms**—CAD, adjoint sensitivity, electromagnetic modeling, microstrip modeling, microwave filters, space mapping, surrogate modeling.

## I. INTRODUCTION

Microwave component modeling is essential in replacing time-consuming electromagnetic (EM) simulations for fast design optimization and verification purposes. To reduce the training data involved in the modeling process, knowledge is preferably embedded [1]. For example, modeling based on space mapping (SM) [2] addresses the rapid generation of an accurate surrogate model while using a minimal amount of training data.

The main idea of space mapping is to take advantage of available so-called fine and coarse models. A fine model is usually evaluated using an accurate but CPU-intensive EM simulator, such as that provided by Ansoft HFSS [3] or CST Microwave Studio [4]. The corresponding coarse model might be a simplified physics-based model, such as an equivalent circuit with attendant empirical formulas. SM establishes a relationship between these fine and coarse models through a traditional parameter extraction process. A corresponding mapped coarse model thus becomes a surrogate with respect to the expensive fine model.

Adjoint sensitivity formulations [5] are an efficient vehicle for evaluating design parameter sensitivities for full-wave EM simulations of microwave structures. Such evaluations are obtained with some overhead during the EM simulation. This facility recently found its way into popular commercial software packages such as Ansoft HFSS and CST Studio. Meanwhile, an adjoint neural network approach to sensitivity

modeling [6] has already found its way into neural-based microwave modeling and design techniques.

Adjoint sensitivities have been indicated for space-mapping-based optimization [7]. Implicit space mapping [8] was offered for SM-based modeling in connection with (or even within) available commercial software. The present paper proposes a simple algorithm that combines adjoint sensitivities with implicit space mapping to create an accurate space-mapping-based surrogate through available software packages. The method is implemented within Agilent ADS [9]. The EM model response and its sensitivities are obtained through Ansoft HFSS.

A single center point is considered in the design parameter region of interest. At this point, a mapping is calibrated that employs preassigned parameters by matching the responses and corresponding derivatives of the fine model to those of a suitable surrogate. It is shown that empirical models of a three-section transformer and an H-plane filter can be enhanced to almost the accuracy offered by surrogates created using multipoint star-distributions (center point plus positively and negatively perturbed points on each axis [8]).

## II. ADJOINT SENSITIVITY-ASSISTED SPACE MAPPING MODELING

The surrogate approach described in [8] is used: input SM [2] ( $\mathbf{B}$  and  $\mathbf{c}$ ), implicit SM [2] ( $\mathbf{B}_p$  and  $\mathbf{c}_p$ ) and output SM [2] ( $\mathbf{A}$  and  $\mathbf{d}$ ). The surrogate  $\mathbf{R}_s$  is formulated as

$$\mathbf{R}_s(\mathbf{x}_f, \mathbf{A}, \mathbf{B}, \mathbf{c}, \mathbf{B}_p, \mathbf{c}_p, \mathbf{d}) = \mathbf{A} \cdot \mathbf{R}_c(\mathbf{B} \cdot \mathbf{x}_f + \mathbf{c}, \mathbf{B}_p \cdot \mathbf{x}_f + \mathbf{c}_p) + \mathbf{d} \quad (1)$$

with matrices  $\mathbf{A} = \text{diag}\{a_1, \dots, a_m\}$ ,  $\mathbf{B} \in M_{n \times n}$ ,  $\mathbf{c} \in M_{n \times 1}$ ,  $\mathbf{B}_p \in M_{n \times p}$ ,  $\mathbf{c}_p \in M_{p \times 1}$  and  $\mathbf{d} \in M_{m \times 1}$  ( $M_{k \times l}$  denotes the set of  $k \times l$  real matrices).  $\mathbf{R}_c$  represents the coarse model response vector.  $\mathbf{x}_f$  denotes the design parameters.

To take advantage of adjoint sensitivity information in the fine model, the following parameter extraction (PE) procedure for  $q$  training points is proposed.

$$\begin{aligned} & (\bar{\mathbf{A}}, \bar{\mathbf{B}}, \bar{\mathbf{c}}, \bar{\mathbf{B}}_p, \bar{\mathbf{c}}_p, \bar{\mathbf{d}}) = \\ & \arg \min_{(\mathbf{A}, \mathbf{B}, \mathbf{c}, \mathbf{B}_p, \mathbf{c}_p, \mathbf{d})} (w_1 \sum_{k=1}^q \|\mathbf{R}_f(\mathbf{x}_f^{(k)}) - \mathbf{R}_s(\mathbf{x}_f^{(k)}, \mathbf{A}, \mathbf{B}, \mathbf{c}, \mathbf{B}_p, \mathbf{c}_p, \mathbf{d})\| \\ & + w_2 \sum_{k=1}^q \|\mathbf{J}_{R_f}(\mathbf{x}_f^{(k)}) - \mathbf{J}_{R_s}(\mathbf{x}_f^{(k)})\|) \end{aligned} \quad (2)$$

where  $\mathbf{J}_{R_f}$  and  $\mathbf{J}_{R_s}$  are the Jacobian matrices of the fine and surrogate models, respectively;  $w_1$  and  $w_2$  are weighting factors.

This work was supported in part by the Natural Sciences and Engineering Research Council of Canada under Grants RGPIN7239-06, STPGP381153-09, by Bandler Corporation, by the Icelandic Centre for Research (RANNIS) Grant 110034021, and by the Reykjavik University Development Fund under Grant T09009.

In ADS, the matching of the derivatives for the  $k$ th design is implemented as

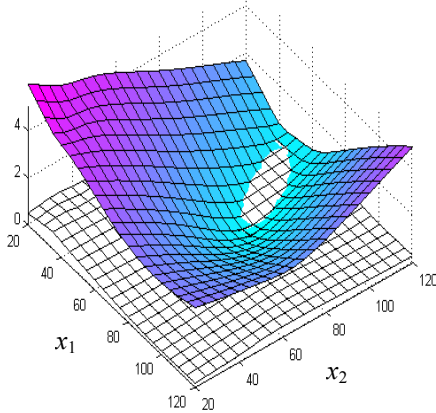
$$\| \mathbf{J}_{R_f}(\mathbf{x}_f^{(k)}) - \mathbf{J}_{R_s}(\mathbf{x}_f^{(k)}) \| \triangleq \sum_i w_i \left\| \frac{\partial R_f^{(k)}}{\partial x_{fi}} - \frac{\partial R_s^{(k)}}{\partial x_{fi}} \right\| \quad (3)$$

where  $w_i$  (e.g., 0.01) is a weighting factor for matching the partial derivative for the  $i$ th element  $x_{fi}$  of the design parameter vector  $\mathbf{x}_f$ .

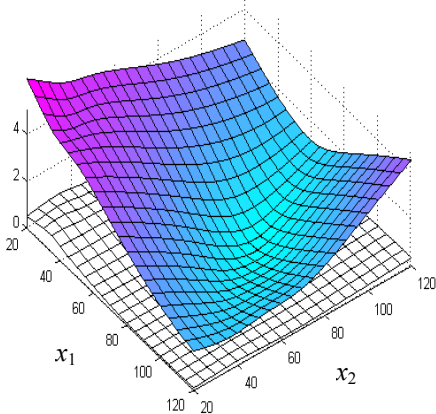
In Fig. 1, using a synthetic two-section transformer example [2], it is shown that for single-point parameter extraction, a surrogate calibration that includes sensitivity information performs better than one using only responses.

The surrogate is obtained as

$$\mathbf{R}_s(\mathbf{x}_f) = \mathbf{R}_s(\mathbf{x}_f, \bar{\mathbf{A}}, \bar{\mathbf{B}}, \bar{\mathbf{c}}, \bar{\mathbf{B}}_p, \bar{\mathbf{c}}_p, \bar{\mathbf{d}}). \quad (4)$$



(a)



(b)

Fig. 1. Error plots for a two-section capacitively loaded impedance transformer [2] which exploits a “physically-based” surrogate model (light grid) versus a classical Taylor approximation (dark grid). Shown is: (a) the surrogate calibrated with a single-point response only parameter extraction procedure; (b) the surrogate calibrated using additional derivative information.

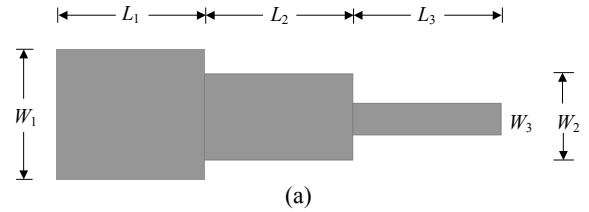
In a single-point parameter extraction, the  $\bar{\mathbf{d}}$  parameters are not used, since they will immediately match the surrogate responses to fine model without consideration of model physicality. The modeling algorithm is implemented in Agilent ADS. Since adjoint sensitivities are not available here, finite differences estimate the surrogate derivatives  $\mathbf{J}_{R_s}$ .

### III. EXAMPLES

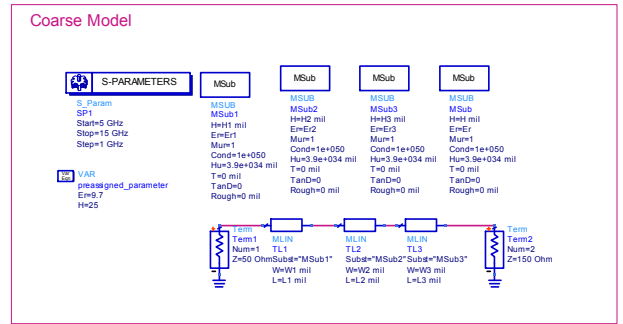
#### A. Three-Section Microstrip Transformer

An ADS implementation is demonstrated for a three-section microstrip impedance transformer [Fig. 3(a)] [8].

As the candidate fine model, both Sonnet *em* [10] and Ansoft HFSS are considered. The coarse model, Fig. 2(b), comprises cascaded ADS microstrip empirical models. Three MSub components (MSub1, MSub2 and MSub3) introduce the preassigned parameters  $\{\varepsilon_1, h_1, \varepsilon_2, h_2, \varepsilon_3, h_3\}$ , the dielectric constant and the substrate height for each microstrip line.



(a)



(b)

Fig. 2. The three-section 3:1 microstrip impedance transformer [8]: (a) structure and dimensions; (b) ADS coarse model.

The design parameters are  $\mathbf{x}_f = [W_1 L_1 W_2 L_2 W_3 L_3]^T$ . The region of interest is  $12.7 \text{ mil} \leq W_1 \leq 15.5 \text{ mil}$ ,  $105 \text{ mil} \leq L_1 \leq 129 \text{ mil}$ ,  $5 \text{ mil} \leq W_2 \leq 6.2 \text{ mil}$ ,  $108 \text{ mil} \leq L_2 \leq 132 \text{ mil}$ ,  $1.5 \text{ mil} \leq W_3 \leq 1.9 \text{ mil}$ , and  $109 \text{ mil} \leq L_3 \leq 133 \text{ mil}$ . The frequency range used is 5 GHz to 15 GHz with a step of 1 GHz (11 points). The reflection coefficient  $S_{11}$  is used to match the responses of the implicit space mapping-based surrogate and the fine model. The reference point is  $\mathbf{x}^0 = [14.1 \ 117 \ 5.6 \ 120 \ 1.7 \ 121]^T \text{ mil}$ . The thickness of the dielectric substrate is 0.635 mm (25 mil) and its relative permittivity is 9.7. The approach of [8] is modified to implement the new mapping by adding new goals for derivative matching, simple derivative calculation formulas and perturbed simulations. Sonnet *em* sensitivities are estimated for the reference point using a finite difference method. Ansoft HFSS sensitivity is obtained through its adjoint capability. Responses and derivatives are matched at the reference point to obtain a surrogate model. The model is compared with the surrogate obtained through the star distribution (13 points) PE [8] in the region of interest.

Table I shows a statistical analysis of  $\mathbf{R}_s$  modeling error for 20 random test points for implicit space-mapping-based modeling using a single point with derivatives and a star distribution in PE. The model built with the single point plus its derivative reaches the same level of error as the star

distribution. It takes 252 minutes to simulate the responses for the star distribution and 42 minutes for the single reference point and its derivatives. The star distribution requires that all 13 response vectors are matched, while matching the reference point and its derivatives, the vector cardinality is 7. Fig. 3 shows a plot of the modeling errors for the test points.

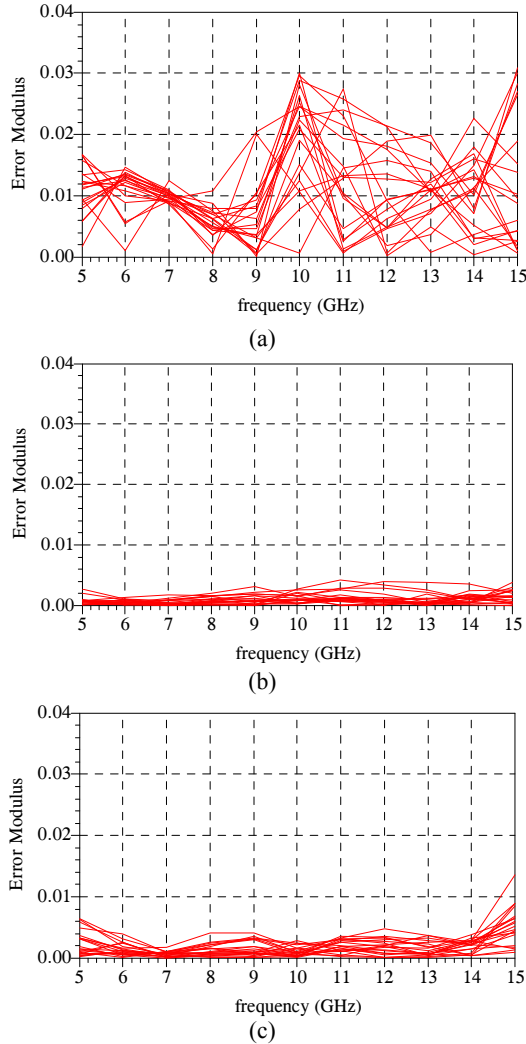


Fig. 3. ADS error plots for the three-section microstrip transformer (20 test points) between the response of the Sonnet fine model  $R_f$  and (a) the coarse model  $R_c$ ; (b) the surrogate  $R_s$  extracted using a star distribution (13 points); (c) the surrogate  $R_s$  extracted using a single point and corresponding derivatives.

### B. Six-Section H-plane Waveguide Filter

The six-section H-plane waveguide filter [11] is shown in Fig. 4(a). In [11], the design parameters are the lengths and widths:  $\mathbf{x}_f = [L_1 \ L_2 \ L_3 \ W_1 \ W_2 \ W_3 \ W_4]^T$ . The original design specifications are  $|S_{11}| \leq 0.16$  for frequency range 5.4-9.0 GHz,  $|S_{11}| \geq 0.85$  for frequency  $\omega \leq 5.2$  GHz, and  $|S_{11}| \geq 0.5$  for frequency  $\omega \geq 9.5$  GHz. The reference point is  $\mathbf{x}^0 = [0.4998 \ 0.4638 \ 0.4454 \ 0.4417 \ 0.6308 \ 0.645 \ 0.6654]^T$  inch. The region of interest under consideration is defined by the  $\pm 5\%$  deviation from the reference point.

TABLE I  
MODELING STATISTICS FOR THE THREE-SECTION TRANSFORMER  
USING 20 RANDOM TEST POINTS

model	relative error w.r.t. fine model ( $\ R_f - R_s\  / \ R_f\ $ )		fine model simulation time (HFSS)	matching vectors
	Sonnet	HFSS		
$R_c$	14.8%	13.1%		
$R_s$ with 13-point PE	1.4%	5.7%	252 min (21 min/point)	13
$R_s$ with single point PE plus derivatives	2.9%	6.4%	42 min	7

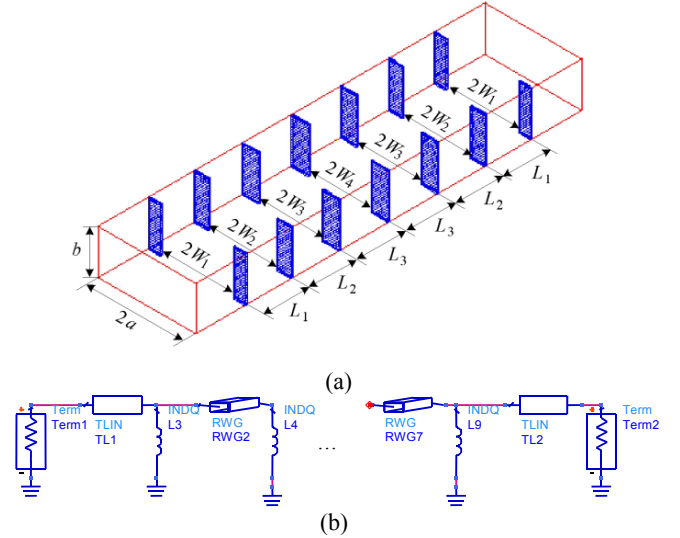


Fig. 4. (a) Six-section H-plane waveguide filter [11], (b) ADS coarse model.

A waveguide with a cross-section of 1.372×0.622 inches (3.485×1.58 cm) is used. The six sections are separated by seven H-plane septa, which have a finite thickness of 0.02 inches (0.508 mm). The coarse model consists of lumped inductances and waveguide sections. A simplified version of a formula due to Marcuvitz [12] is utilized to calculate the equivalent inductive susceptance corresponding to an H-plane septum. The coarse model is simulated using ADS as shown in Fig. 4(b).

The widths of the waveguide sections are selected as the preassigned parameters to calibrate the coarse model (surrogate). The frequency coefficient of each inductor, for convenience PI (equivalent to tuning a multiplying factor of  $k=1$ ), is harnessed as a preassigned parameter to compensate for the susceptance change. The fine model exploits Ansoft HFSS. The resulting surrogate is tested using a 24-point frequency sweep in a wider band (5.2-9.8 GHz). Table II compares a statistical analysis of the  $R_s$  modeling error for 50 random test points for surrogate modeling using a single point with derivatives with the star distribution (15 points) in PE. It takes approximately 150 minutes to obtain the Ansoft HFSS responses for the star distribution and 17 for one reference point and its corresponding adjoint derivatives. The star distribution requires that all 15 response vectors are matched, while matching the reference point and its derivatives, the

cardinality of the vector is 8. Figs. 5 and 6 show the error and test sample plots.

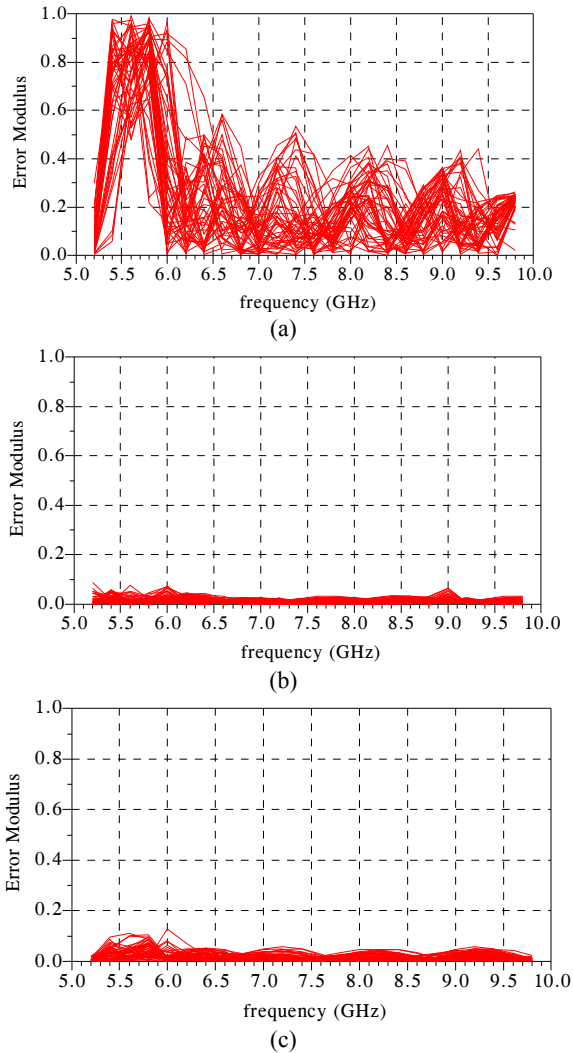


Fig. 5. ADS error plots for the H-plane filter (50 test points in  $\pm 5\%$  region) between the response of the fine model  $R_f$  and (a) the coarse model  $R_c$ ; (b) the surrogate  $R_s$  extracted using the star distribution (15 points); (c) the surrogate  $R_s$  extracted using a single point and corresponding derivatives.

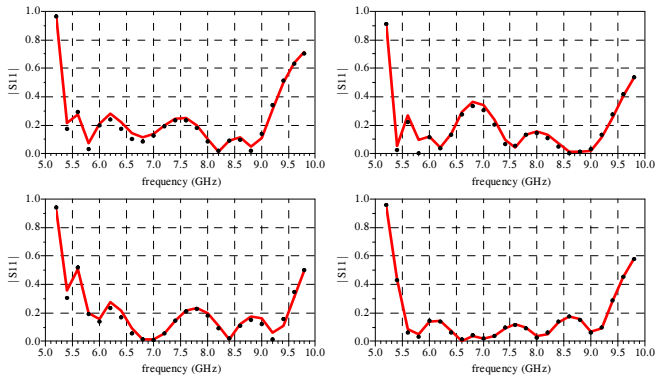


Fig. 6. Four test points for a  $\pm 5\%$  region: surrogate ( $\rightarrow$ ) and fine model ( $\bullet$ ) after parameter extraction using the response at a single point and corresponding derivatives.

TABLE II  
MODELING STATISTICS FOR THE H-PLANE FILTER  
USING 50 RANDOM TEST POINTS

model	relative error w.r.t. fine model ( $\ R_f - R_s\ /\ R_f\ $ ) $\pm 5\%$ region	fine model simulation time	matching vectors
$R_c$	114.9%		
$R_s$ with 15-point PE	5.9%	150 min (10 min/point)	15
$R_s$ with single point PE plus derivatives	8.1%	17 min	8

#### IV. CONCLUSIONS

This paper presents a space mapping modeling technique where a surrogate is calibrated using a fine model response at only a single design, and its corresponding adjoint sensitivity information. The technique is implemented entirely in an available simulation software package. It is demonstrated that the surrogate exhibits the same level of accuracy as a surrogate built using additional fine model data (star distribution). Other advantages include lower computational cost of acquiring the training data and a simpler extraction of the model parameters.

#### REFERENCES

- [1] V.K. Devabhaktuni, B. Chattaraj, M.C.E. Yagoub, and Q.J. Zhang, "Advanced microwave modeling framework exploiting automatic model generation, knowledge neural networks and space mapping," *IEEE Trans. Microwave Theory Tech.*, vol. 51, no. 7, pp. 1822-1833, July 2003.
- [2] J.W. Bandler, Q.S. Cheng, S.A. Dakroury, A.S. Mohamed, M.H. Bakr, K. Madsen, and J. Søndergaard, "Space mapping: the state of the art," *IEEE Trans. Microwave Theory Tech.*, vol. 52, no. 1, pp. 337-361, Jan. 2004.
- [3] Ansoft HFSS 12.1.1, Ansoft Corporation, Pittsburgh, PA, 2010.
- [4] CST Microwave Studio, CST AG, Darmstadt, Germany.
- [5] H. Akel and J.P. Webb, "Design sensitivities for scattering-matrix calculation with tetrahedral edge elements," *IEEE Trans. Magnetics*, vol. 36, no. 4, pp. 1043-1046, July 2000.
- [6] J. Xu, M.C.E. Yagoub, R. Ding, and Q.J. Zhang, "Exact adjoint sensitivity analysis for neural-based microwave modeling and design," *IEEE Trans. Microwave Theory Tech.*, vol. 51, no. 1, pp. 226-237, Jan. 2003.
- [7] J.W. Bandler, A.S. Mohamed, M.H. Bakr, K. Madsen, and J. Søndergaard, "EM-based optimization exploiting partial space mapping and exact sensitivities," *IEEE Trans. Microwave Theory Tech.*, vol. 50, no. 12, pp. 2741-2750, Dec. 2002.
- [8] Q.S. Cheng and J.W. Bandler, "An implicit space mapping technique for component modeling," *Proc. 36th European Microwave Conf.*, Manchester, UK, pp. 458-461, Sept. 2006.
- [9] Advanced Design System (ADS), Version 2009, Agilent Technologies, Santa Rosa, CA, 2009.
- [10] *em*<sup>TM</sup>, Version 12.52, Sonnet Software, Inc., 100 Elwood Davis Road, North Syracuse, NY 13212, USA.
- [11] J.W. Bandler, Q.S. Cheng, D.M. Hailu, and N.K. Nikolova, "A space mapping design framework," *IEEE Trans. Microwave Theory Tech.* vol. 52, no. 11, pp. 2601-2610, Nov. 2004.
- [12] N. Marcuvitz, *Waveguide Handbook*, 1st ed. New York, NY: McGraw-Hill, 1951, p. 221.