SYMPOSIUM ON NONLINEAR PROGRAMMING AT THE GEORGE WASHINGTON UNIVERSITY

A Symposium on Nonlinear Programming was held at The George Washington University on 14-16 March 1973. It was sponsored by the Mathematical Programming Society under the auspices of the Office of Naval Research and the university's Department of Operations Research. The conference was organized and conducted by Anthony V. Fiacco (Program Chairman), Garth P. McCormick (Arrangements Chairman) and James E. Falk (Publicity Chairman).

Over 100 people attended, including many well-known experts in both theory and applications on the one hand, and many novices on the other. This made for many interesting exchanges, fresh inquiries into established approaches, new twists on old ideas and an avante garde orientation. The regional "low pressure" flavor of the conference encouraged presentation of current research in progress, allowing reports on inconclusive, controversial and partially developed results. This was a happy outcome, generating a provocative and stimulating atmosphere and a very successful meeting.

Considerable activity was reported in the development of algorithms for problems having special structure, particularly unconstrained and linearly constrained optimization problems. Isolated results were presented in nonconvex programming - where theoretical breakthroughs are desperately needed - with additional results being reported for (the now fairly well understood area of) nonconvex quadratic programming. There were disappointedly few papers on sensitivity analysis and applications. There were no papers on control theory or on the solution of nonlinear equations, though these topics were explicitly mentioned in the call for papers.

A panel, consisting of E. Balas, K. O. Kortanek, G. P. McCormick, J. B. Rosen, P. Wolfe and W. I. Zangwill, was chaired by A. V. Fiacco and concluded the symposium with a discussion of the theme, "Directions of Research in NLP." Among the conclusions of the panel were that some key developments in NLP have been the results obtained in unconstrained minimization methodology and the merging of these with constrained methods, and complimentarity theory in conjunction with the recent development of techniques for calculating approximations of fixed points. As to future theoretical and computational trends, it was felt that the latter (fixed point) developments will lead to new algorithmic breakthroughs, and that activity in nonconvex programming will increase explosively in the near future. With regard to practice, it was concluded that: (1) Major applications will continue to rely on simple heuristics and loose approximations of optimal solutions, (2) applications are largely not being solved by the best available methods, and (3) NLP codes will be used in applications if they are simple and accessible (and not primarily because a given method is known to be most effective for a given structure).

Anthony V. Fiacco

ABSTRACTS FOR THE GEORGE WASHINGTON UNIVERSITY
NONLINEAR PROGRAMMING SYMPOSIUM, 14-16 MARCH 1973

AMOR, JEAN-PIERRE; ACCELERATION OF METHODS OF STEEPEST DESCENT VIA OPTIMAL SCALING. ABSTRACT: Some of the most widely used and most successful methods for unconstrained optimization and nonlinear programming are based on Cauchy's notion of steepest descent. All so-called methods of feasible directions, including the successful Gradient Projection Method of Rosen and Reduced Gradient Method of Wolfe are of this type. It is well known that the rate of convergence of methods of steepest descent is very sensitive to the scaling of a problem's decision variables. Yet, in practice, this matter is either ignored or a simple ad hoc rule of thumb is used. Thus there exists a need for studying how to scale a problem so as to maximize the rate of convergence. In an attempt to solve this "optimal scaling problem", we develop a method which is quite effective in improving the performance of methods of steepest descent in a simplified setting. The natural extensions of this scaling method to a more general setting are now being investigated.

ARMACOST, ROBERT L. and FIACCO, ANTHONY V.; COMPUTATIONAL EXPERIENCE IN SENSITIVITY ANALYSIS FOR NONLINEAR PROGRAMMING. ABSTRACT: A method for sensitivity analysis in nonlinear programming has recently been developed using the Sequential Unconstrained Minimization Technique. It is applied here to perform sensitivity analyses on four example problems to demonstrate the computational feasibility and characteristics of the approach. The sensitivity analysis is conducted along the minimizing trajectory for each problem. The convergence characteristics of the first partial derivatives of the variables and objective function with respect to the parameters in the sensitivity analysis are illustrated.

BALAS, EGON; NONCONVEX QUADRATIC PROGRAMMING VIA GENERALIZED POLARS. ABSTRACT: A generalized outer polar F^* is constructed for nonconvex, linearly constrained, quadratic programs, which can be used in essentially the same ways as outer polars are used in integer programming. First we discuss the derivation of valid cutting planes from F^*. Then we discuss an algorithm which does not generate cuts, but uses the properties of F^* in a different way. Starting with a subset of the Kuhn-Tucker constraints, some of the remaining constraints are successively activated so as to construct a polytope contained in F^*. When this is achieved, the best among the local optima found in the process is a global optimum.

BALINTFY, JOSEPH L.; FOOD PRICE INDEX COMPUTATION BY NONLINEAR PROGRAMMING. ABSTRACT: The fixed basket method of food price index measures, as well as recent improvements by linear programming techniques, seem to fall short in expressing and maintaining the consumer's true utility function subject to relative price changes. Experiments with the time-dependent character of food preferences indicate that measurable nonlinear utility functions can be associated with food consumption rates. This finding makes the Fisher definition of true price indexes computable by standard non-linear programming techniques. Gossen - type and more general quadratic form approximations for food utility functions subject to nutritional constraints were successfully tested under simulated time series of food prices to demonstrate the applicability of this new approach to index number theory.

BANSAL, P. P. and JACOBSEN, S. E.; CHARACTERIZATION OF LOCAL SOLUTIONS FOR A CLASS OF NONCONVEX PROGRAMMING PROBLEMS. ABSTRACT: A nonconvex programming problem (reverse-convex feasible region), which arises in the context of application of Bender's decomposition procedure to a class of network optimization problems, is considered. Necessary and sufficient conditions for a local maximum are derived. It is shown that it is sufficient to find a finite number of "basic local maxima" in order to locate an optimal solution. It is shown how this problem arises in the context of optimal flow capacity expansion when there are economies-of-scale; that is, incremental capacity cost functions are concave.

BEST, MICHAEL J.; FCD: A FEASIBLE CONJUGATE DIRECTION METHOD TO SOLVE LINEARLY CONSTRAINED OPTIMIZATION PROBLEMS. ABSTRACT: A first order method is presented which uses conjugate directions to solve linearly constrained optimization problems. At each subcycle the method chooses a direction, among those which are both feasible and conjugate, which optimizes a linear approximation to the objective function. Proof of convergence to a Kuhn-Tucker Point is given. Under strict second order sufficiency conditions a superlinear convergence rate is demonstrated and convergence of a sequence of dual vectors is shown. Computational results are presented and some comparisons are made with conjugate direction methods for unconstrained problems.

BURDET, CLAUDE-ALAIN; ON NON-CONVEX (INDEFINITE) QUADRATIC PROGRAMMING. ABSTRACT: A facial decomposition scheme can be used to solve the general quadratic programming problem (linearly constrained). This procedure decomposes the original problem into a finite number of convex subproblems (i.e., minimization of a positive semi-definite objective function); concave subproblems (i.e., minimization of a negative semi-definite objective function) and indefinite subproblems. This paper focuses on the indefinite case; simple polaroids are studied to generate cutting planes for the acceleration of the above algorithm.

CHARALAMBOUS, CHRISTAKIS; A FAMILY OF ALGORITHMS FOR UNCONSTRAINED OPTIMIZATION BASED ON HOMOGENEOUS MODELS. ABSTRACT: Most algorithms for unconstrained minimization of $F(x)$ are a combination of steepest descent and the Newton method based on the quadratic model $F(x) = \frac{1}{2}(x-\overset{v}{x})\, Q(x-\overset{v}{x}) + \overset{v}{F}$, where $\overset{v}{x}$ is the location of the minimum of $F(x)$, Q is an nxn constant positive definite matrix, $\overset{v}{F}$ the minimum value. Several algorithms belonging to this family have been proposed by Broyden (1967), Davidon (1968), Murtagh and Sargent (1969), Fletcher (1970), Huang (1970), and Adachi (1971). In 1972, Jacobson and Oksman derived an algorithm based on the homogeneous model $F(x) = \frac{1}{\gamma}(x-\overset{v}{x})\, g(x) + \overset{v}{F}$, where γ is the degree of homogeneity, and $g(x)$ is the gradient vector. Their algorithm converges to the minimum of a homogeneous function in n+2 iterations. Since the former quadratic function is a special case of the latter with $\gamma=2$, it seems that the homogeneous function is a better model than the quadratic model for a general function. In this paper, a family of algorithms based on the homogeneous model will be presented, and the algorithm due to Jacobson and Oksman will be derived as a special case. Some important properties of these algorithms are: (a) they do not require finding the minimum along one-dimensional search directions, (b) they converge in n+2 iterations on homogeneous functions and (c) they do not require the Hessian matrix to be nonsingular.

CHARALAMBOUS, CHRISTAKIS and BANDLER, JOHN W.; NEW ALGORITHMS FOR MINIMAX OPTIMIZATION. ABSTRACT: Consider a system of m real nonlinear functions $f_i(\phi)$, $i \in I$ where $\phi \triangleq [\phi_1 \ \phi_2 \ \ldots \ \phi_k]^T$ is a k-dimensional column vector containing the k adjustable parameters and $I \triangleq \{ 1,2, \ldots, m\}$. Let $U_\infty(\phi) \triangleq \max_{i \in I} f_i(\phi)$. The problem of minimax optimization consists of finding a point $\overset{v}{\phi}$ such that $U_\infty(\overset{v}{\phi}) \leq U_\infty(\phi)$ for all points ϕ in a neighborhood of $\overset{v}{\phi}$. Various algorithms have been proposed for solving the above problem. Most of them either transform the nonlinear minimax problem into a nonlinear programming problem and solve it by well-established methods, or they use linear programming to predict the direction of search. Due to recent developments on nonlinear least pth optimization due to the authors it is possible to derive two new methods of nonlinear minimax optimization. Unlike the Polya algorithm in which a sequence of least pth optimizations as $p \to \infty$ is taken, our methods do not require the value of p to tend to infinity. Instead we construct a sequence of least pth optimization problems with a finite value of p. It is shown that this sequence will converge to a point where the necessary conditions for a minimax optimum are satisfied. Two interesting minimax problems were constructed which illustrate some of the theoretical ideas. Further numerical evidence is presented on the modeling of a fourth-order system by a second-order model with values of p varying between 2 and 10,000.

CULLUM, J., DONATH, W., and WOLFE, P.; MINIMIZATION OF A CONVEX BUT NOT EVERYWHERE DIFFERENTIABLE FUNCTION. ABSTRACT: A convex function, at any given point, has a directional derivative in each direction. Therefore, a generalization to convex functions of the method of steepest descent for differentiable functions is easily obtained. However, analogs of the convergence theorems associated with the method of steepest descent are not valid for this generalization. Convergence of the iterates generated to a nonminimizing point is possible. Convergent minimization algorithms either involve sets that are not easily characterizable, thereby hindering computer implementation; or the function values generated by the algorithm are not monotonically decreasing. In this talk, a convergent and implementable algorithm is presented for minimizing a particular convex but not everywhere differentiable function that arises in graph partitioning problems.

CULLUM, JANE K. and WOLFE, PHILIP; OPTIMIZING NONDIFFERENTIABLE FUNCTIONS. ABSTRACT: Many problems of mathematical programming can be posed as requiring the minimization of a convex, but not necessarily differentiable, function, and for some of these quite simple solution procedures have proved alarmingly effective. The theoretical approaches to algorithms for this problem (primarily Russian) and the successful computational approaches (primarily Western) will be analyzed.

DINKEL, JOHN J., KOCHENBERGER, GARY A., and McCARL, BRUCE; ON THE NUMERICAL SOLUTION OF GEOMETRIC PROGRAMS. ABSTRACT: An algorithm is developed for prototype geometric programs based on the reduced geometric program. The features of the algorithm include: (1) Solution of Dual geometric program rather than the Primal; (2) An efficient test of Canonicality of the original problem; (3) Flexibility in implementing the optimization procedure; and (4) Provisions for dealing with inactive constraints. The algorithm will be illustrated with examples from engineering design, management science and regional design.

ELZINGA, JACK and MARQUEZ, JAVIER; GENERALIZED CYCLIC ASCENT. ABSTRACT: We extend the easily implemented cyclic ascent method for unconstrained optimization to solve the linearly constrained nonlinear programming problem. The algorithm does not require the computation of derivatives and is shown to converge to an optimal solution under the assumptions that the objective function is pseudoconcave and that no degeneracy occurs in the constraint set. The algorithm is employed to generate constrained conjugate directions. The finite convergence of this algorithm for the quadratic programming problem suggests more rapid convergence for the algorithm applied to the general linearly constrained problem.

ELZINGA, JACK and MOORE, THOMAS; THE CENTRAL CUTTING PLANE ALGORITHM. ABSTRACT: We consider a general formulation of the convex programming problem, that of maximizing a linear function over a compact convex set defined by differentiable functions. An algorithm is developed which iteratively proceeds to the optimum by successively constructing a cutting plane through the center of a polyhedral approximation to the optimum. The centers of the polyhedral approximations are obtained by solving a linear program in the manner of Nemhauser and Widhelm. The algorithm generates a sequence of feasible points whose convergence properties are established. We also show the existence of sequences whose limits are the optimal Kuhn-Tucker multipliers. Since successive subproblems are generated by the addition of one constraint to the linear programming subproblem, the computational effort required in each iteration is small (directional searches are not required). We develop a simple, easily implemented, yet powerful rule for dropping prior cuts, thus keeping small the size of the linear programming subproblems.

FISHER, MARSHALL L. and SHAPIRO, JEREMY F.; CONSTRUCTIVE DUALITY IN DISCRETE OPTIMIZATION. ABSTRACT: The purpose of this paper is to consolidate and extend recent advances in the use of mathematical programming duality theory in discrete optimization. Meaningful dual problems have been identified for the integer programming problem, the traveling salesman problem, resource constrained network scheduling problems and others. A general branch and bound schema is given for solving primal discrete optimization problems of these types by exploiting the appropriate duals. Three related methods for solving the dual problems are given: a primal-dual ascent algorithm, a search algorithm based on simplicial approximation and a subgradient relaxation algorithm.

FROMOVITZ, STAN and MORTON, RODNEY L.; COMPUTATION OF RANDOMIZED SOLUTIONS IN NONLINEAR PROGRAMMING. ABSTRACT: In a previous paper one of the authors has shown that certain nonlinear programming problems - such as those arising in stochastic programming - may have randomized optimal solutions. By combining a very efficient random search procedure with the Dantzig-Wolfe decomposition algorithm, we obtain a method of calculating these randomized solutions.

FROMOVITZ, STAN and ROMIG, WILLIAM J.; A QUASI-NEWTON, NON-GRADIENT SEARCH PROCEDURE. ABSTRACT: We have devised a special procedure for optimizing an unconstrained function that can handle non-differentiable or even discontinuous functions and yet becomes a quasi-Newton method for quadratic functions. Our method tests if the function is a positive definite quadratic form and either iteratively estimates the matrix of second partial derivatives, resulting in a quasi-Newton iteration, or, if these tests fail, becomes a multi-dimensional, non-gradient search procedure.

GILL, P. E. and MURRAY, W.; LINEARLY CONSTRAINED MODIFIED NEWTON METHODS. ABSTRACT: A method is described to solve the general linearly constrained optimization problem. The relationship of the algorithm to those for unconstrained problems, linear programming and indefinite quadratic programming will be illustrated. The algorithm has been adapted to solve sparse and structured problems, an example of which arises when solving certain optimal control problems. The algorithm utilizes new methods for updating matrix factorization in a numerically stable form.

GOULD, F. J.; OPTIMIZATION WITH COMBINATORIAL ALGORITHMS. ABSTRACT: A unified development is presented for complementary pivoting on arbitrary sets with a pseudomanifold property. Three basic complementary pivoting algorithms are discussed and illustrated in the contexts of (1) the linear complementarity problem (2) approximation of Brouwer fixed points (3) unconstrained optimization of convex functions via tangential linearization (4) convex programming with not everywhere differentiable exact penalty functions. Numerical work in progress will be reported.

GUSTAFSON, S. A., KORTANEK, K. O., and SAMUELSSON, H. M.; ON DUAL PROGRAMS AND FINITE-DIMENSIONAL MOMENT CONES. ABSTRACT: In this paper we treat so-called finite dimensional moment problems, that is, tasks where one seeks the upper bound of a linear functional on a space of continuous functions subject to a finite number of linear constraints on these linear functionals. With this problem we associate a dual task which seeks the minimum of a linear form in a finite number of variables subject to a possibly infinite number of linear constraints in these variables. We relate properties of the so-called moment cone with those of a dual pair of problems in this non-reflexive Banach space setting. This is accomplished by means of several classification tables which describe permissible and forbidden "states" of the problem pair, thus obtaining generalizations of the duality theorem of linear programming. By placing different general assumptions on the associated moment cones we obtain different classification results. The moment problems studied in this paper have had recent application to mathematical models for optimizing air pollution abatement policies.

HATFIELD, GORDON; A FIRST ORDER, PRIMAL-DUAL METHOD FOR MINIMIZING A REAL-VALUED FUNCTION SUBJECT TO LINEAR CONSTRAINTS. ABSTRACT: The purpose of this paper is to develop a general algorithm for solving the class of nonlinear programming problems that have linear constraints. The constraints can be either equations or inequalities and the variables can be free or non-negative. The objective function is assumed to be continuously differentiable. The algorithm is an "effective" second-order method in that slow convergence is eliminated without requiring second partial derivatives. In addition it combines the desirable features of projection methods, conjugate gradient methods, and methods that solve LP problems to obtain feasible directions. Computational results on a wide variety of test problems are given. Some comments on the effieiency of the algorithm as compared to other algorithms is included.

HEARN, DONALD and RANDOLPH, W. D.; A DUAL METHOD FOR QUADRATICALLY CONSTRAINED QUADRATIC PROGRAMS. ABSTRACT: By making appropriate strict convexity assumptions the Wolfe dual of the quadratically constrained quadratic program can be transformed into a concave program involving dual variables only. Computational results on sample problems are presented which compare Rosen's gradient projection applied to the dual problem with SUMT - V4 applied to the primal problem.

HILLESTAD, RICHARD J.; TWO ALGORITHMS FOR A CLASS OF NONCONVEX PROBLEMS. ABSTRACT: Two algorithms are presented for solving the problem, min cx, $Ax \leq b, g(x) \leq 0$ where g is a vector of concave constraints. The feasible region is, in general, a nonconvex disconnected set. Necessary conditions for a local minimum, in terms of specific linearized problems, are developed. It is shown that a finite number of basic local solutions need be considered in order to find an optimal solution. An algorithm which finds, finitely, all "basic" solutions which satisfy the necessary conditions is developed for the case where g is a single concave constraint. Finally, a cutting plane algorithm for solving the general problem is developed. This method converges from the infeasible region directly to the global optimum by constructing a series of hyperplane constraints which build a convex hull of the feasible region in a neighborhood of the global optimum. Special methods for handling unbounded feasible regions and degeneracy are given. Geometric sufficient conditions are given for convergence of the procedure.

HSIA, WEI SHEN; DECOMPOSITION OF NONLINEAR PROGRAMMING. ABSTRACT: A decomposition method for large-scale mathematical programming is presented. The payoff function is convex separable and all constraints except non-negativity restrictions are linear equalities. After separating the coupling constraints by allocating scarce resources, Zangwill's Convex Simplex Method (CSM) is used to solve subprograms. By utilizing the last tableaus of the CSM, the master program, a linear approximation program, is transformed in terms of Lagrange multipliers and an adjusting vector. The advantages of this method are (1) the feasibility of the solution is maintained at each iteration, (2) the master program is linear and can be obtained from subprograms which are already in computer storage, (3) from the payoff function of the master program, the decision maker can tell which plants have shortage or surplus of each scarce resource. A convergence proof is given for this method and it will apply also to the Frank-Wolfe gradient method. The idea of allocation of the scarce resources to each plant is extended to stochastic programming.

JAGANNATHAN, R.; CHANCE CONSTRAINED PROGRAMMING WITH JOINT CONSTRAINTS. ABSTRACT: Miller and Wagner have shown that a deterministic equivalent of a joint chance constrained programming model with independent random right-hand side elements is a concave programming problem. This paper obtains similar equivalents for chance constrained programming models with coefficient matrices that are normally distributed and with dependent random right-hand side elements.

KHUMAWALA, BASHEER M. and STINSON, JOEL P.; A BRANCH AND BOUND ALGORITHM FOR THE RESOURCE CONSTRAINED PROJECT SCHEDULING PROBLEM. ABSTRACT: This paper deals with a deterministic resource constrained project scheduling problem in which each of the resource classes is assumed to have a constant level of availability throughout the project length. The method used to solve this problem is branch and bound. Improved upper and lower bounding procedures are discussed in the paper. The algorithm has been found to perform quite well; computational experience, gained in solving problems found in the literature, are also reported in the paper.

LOBERT, ANDRE, A.B. WHINSTON, & G. P. WRIGHT; RELAXATION METHODS IN MATHEMATICAL PROGRAMMING. ABSTRACT: This paper presents an algorithm which uses relaxation methods of linear numerical analysis to solve a specific class of linear programming problems as well as to solve certain programming problems with concave objective functions and linear constraints. The class of linear programming problems the algorithm is applicable to includes most finite state and finite action space Markovian programming problems as well as Leontief substitution systems with linear and concave objective functions. The algorithm is related to the work of Howard and MacQueen for solving dynamic programming problems. Computational results are presented in the paper. Also, an application to water reservoir control is presented.

McCANN, R. BRUCE and SOLAND, RICHARD M.; PRELIMINARY EXPERIENCE WITH A COMPUTER CODE FOR NONCONVEX PROGRAMMING. ABSTRACT: A computer code called GLOBAL has been written to implement an algorithm for nonconvex programming due to one of the authors. GLOBAL is designed to solve problems of the following form: Find $x=(x_1,\ldots,x_n)$ to minimize $\Sigma_{j=1}^{n}\phi_{oj}(x_j) + \theta_o(x)$ subject to $\Sigma_{j=1}^{n}\phi_{ij}(x_j) + \theta_i(x) \leq 0$, $i=1,\ldots,m$, and $\ell \leq x \leq L$, where all ϕ_{ij} are concave and all θ_i are convex. It uses a branch-and-bound scheme in which a sequence of convex programming problems are solved (by SUMT). Preliminary computational experience is reported on a variety of problems, some of which have disconnected constraint sets and/or discontinuous problem functions. Some of the problems had to be converted to the form appropriate for GLOBAL by (1) adding additional variables and constraints and/or (2) decomposing some of the problem functions.

MANGASARIAN, O. L.; SECOND AND HIGHER ORDER DUALITY IN NONLINEAR PROGRAMMING. ABSTRACT: A dual problem associated with a primal nonlinear programming problem is presented that involves second derivatives of the functions constituting the primal problem. Duality results are derived for this pair of problems. More general dual problems are also presented, and duality results for these problems are also given.

MEYER, GERARD G. L.; CHARACTERISTIC SET OF CANONICAL STRUCTURE. ABSTRACT: In this paper a canonical structure and its characteristic set are presented. It is shown that these two concepts considerably simplify the analysis of finite memory algorithms.

MIFFLIN, ROBERT; A SUPERLINEARLY CONVERGENT ALGORITHM FOR MINIMIZATION WITHOUT EVALUATING DERIVATIVES. ABSTRACT: An algorithm for unconstrained minimization of a function of n variables that does not require the evaluation of partial derivatives is presented. It is a second order extension of the method of local variations and retains the local variations property that accumulation points are stationary for a continuously differentiable function. This extension is an approximate Newton method and the algorithm convergence is superlinear for a twice continuously differentiable strongly convex function. If available, numerical results for test problems and algorithm modifications for the case of large n and the case of bounded variables will be presented.

MYLANDER, W. CHARLES; CANDLER-TOWNSLEY'S CUTTING PLANE METHOD FOR QUADRATIC PROGRAMMING REVISITED. ABSTRACT: In 1964, W. Candler and R. J. Townsley proposed an algorithm for solving a general quadratic program. It is composed of two integrated parts. The first part is a method for locating a local minimum. The second part is a procedure for the construction of a cutting plane to exclude the just found local minimum. Their approach to the construction of the cutting plane is reformulated so that it may be used with any algorithm producing a local minimum at which the Kuhn-Tucker conditions are satisfied. The whole procedure is put on a rigorous basis and a method for strengthening their cuts is given. The author uses the Candler-Townsley cut construction in a program he coded to process general quadratic programs. Keller's method (1969) is used to locate the local minima. The program was used to process a number of quadratic programs and the results of these tests are reported. The author's experience with this cutting plane procedure is similar to that reported by P. B. Zwart (1971) for H. Tui's cutting plane procedure (1964) when applied to concave quadratic programs.

O'NEILL, RICHARD P. and WIDHELM, WILLIAM B.; PENALTY FUNCTIONS IN DECOMPOSITION TECHNIQUES. ABSTRACT: Penalty functions are used in the Lagrangian formulation of the standard Dantzig-Wolfe decomposition for non-linear programs. It is demonstrated for a class of penalty functions that the generated column will enter the restricted master program if the current solution is non-optimal. Computational results are presented which demonstrate up to 80% faster convergence for the penalty technique. The use of the Lagrangian-penalty techniques to solve classes of non-convex problems is presented. Certain aspects of possible finite convergence are discussed through the use of exact penalty functions.

OPACIC, JASNA and McCORMICK, GARTH P.; SOME NEW RESULTS REGARDING RESETTING FOR THE DAVIDON-FLETCHER-POWELL METHOD. ABSTRACT: A computer based investigation was undertaken to explore several resetting criteria for the DFP method, as it is known that for some functions the method may either be converging very slowly or not converging at all. Criteria were tried on three carefully selected problems. The results obtained are encouraging in that they establish that an angle criterion, which we are proposing, is superior to other tried criteria.

PARK, STEPHEN K. and STRAETER, TERRY A.; NEAR TERMINAL AREA OPTIMAL SEQUENCING AND FLOW CONTROL AS A MATHEMATICAL PROGRAMMING PROBLEM. ABSTRACT: The construction of algorithms for the automatic optimal sequencing and flow control of aircraft in a near terminal area has become an area of active research. In this paper, the situation where all merging aircraft are segregated according to their terminal area performance capabilities is considered. Two topics will be discussed. First, the determination of optimal approach and delay trajectory parameters is posed and analyzed as a non-linear programming problem and the attendant optimal sequencing problem is formulated as a linear programming problem. The resulting algorithms corresponding to each of these problems will be discussed. Second, the results of recent studies using these algorithms will be presented. Specifically, the sensitivity of optimal flight times to changes in entry velocity, merging velocity, acceleration and deceleration capabilities, and sink rate and turn rate capabilities will be discussed. In addition, the effect on mean delay time of such things as different sequencing strategies, length of common approach, and separation standards is shown.

PRABHAKAR, T.; OPERATION OF A LARGE NONLINEAR PRODUCTION AND DISTRIBUTION PLANNING MODEL. ABSTRACT: The nonlinear programming model discussed here is a short-range planning model which determines the minimum cost strategy for production and distribution of a set of olefin and chemical products of Union Carbide Corporation. The model has about 7500 linear variables, 150 nonlinear variables, 2500 linear constraints, and 100 nonlinear constraints. The model is solved through a small step gradient method involving sectional linear programming (LP) approach. The IBM's Mathematical Programming System Extended is used to solve the LP model. Mathematical formulation of the model and its solution procedure are first briefly discussed, followed by a description of the computer system and its operation, some problems associated with convergence rates, non-convex restrictions, algorithmic developments and sensitivity analysis.

RITTER, KLAUS; A METHOD OF CONJUGATE DIRECTIONS FOR NONLINEAR PROGRAMMING PROBLEMS. ABSTRACT: An iterative method is described for the minimization of a continuously differentiable function $F(x)$ of n variables subject to linear inequality constraints. Without any assumptions on second order derivatives it is shown that every cluster point of the sequence $\{x_j\}$ generated by this method is a stationary point. If $\{x_j\}$ has a cluster point z such that $F(x)$ is twice continuously differentiable in some neighborhood of z and the Hessian matrix of $F(x)$ has certain properties then $\{x_j\}$ converges to z and the convergence is $(n-p)$ - step superlinear where p is the number of constraints which are active for z. Furthermore, a simple procedure is given for deriving a new sequence $\{y_j\}$ from the sequence $\{x_j\}$ which converges faster to z in the sense that $||y_j - z||\ ||x_j - z||^{-1} \to 0$ as $j \to \infty$.

ROBINSON, STEPHEN M.; COMPUTABLE ERROR BOUNDS FOR NONLINEAR PROGRAMMING. ABSTRACT: This paper presents a method for obtaining computable bounds for the error in an approximate Kuhn-Tucker point of a nonlinear program.

ROSEN, J. B. and KREUSER, J. L.; A QUADRATICALLY CONVERGENT LAGRANGIAN ALGORITHM FOR NONLINEAR CONSTRAINTS. ABSTRACT: An algorithm for the nonlinearly constrained optimization problem is presented. The algorithm consists of a sequence of major iterations generated by linearizing each nonlinear constraint about the current point, and adding to the objective function a linear penalty for each nonlinear constraint. The resulting function is essentially Lagrangian. A Kantorovich-type theorem is given, showing quadratic convergence in terms of major iterations. This theorem insures quadratic convergence if the starting point (or any subsequent point) satisfies a condition which can be tested using computable bounds on the objectives and constraint functions.

SHANNO, DAVID, BERG, ALLEN, and CHESTON, GRANT; RESTARTS AND ROTATIONS OF QUASI-NEWTON METHODS. ABSTRACT: Some recent literature has been concerned with the problem of improving the computational properties of quasi-Newton methods by either resetting the approximate Hessian matrix to the identity matrix periodically or, when indicated, rotating the search direction toward the gradient vector. In this paper, the authors consider several criteria, both heuristic and analytic, for restarting and rotating the search vector. These criteria are tested numerically on a fair sample of test problems from the literature using a particular implementation of the Broyden-Fletcher-Shanno algorithm. Results are presented to show that neither resetting nor rotating ever appear to be indicated for the particular criteria and test problems used.

STAHA, R. S. and HIMMELBLAU, D. M.; AN EVALUATION OF NONLINEAR PROGRAMMING TECHNIQUES FOR PROBLEMS WITH NONLINEAR CONSTRAINTS. ABSTRACT: Quite a few nonlinear programming algorithms have been proposed in the literature to solve NLP problems with nonlinear equality and inequality constraints. However, not much information is available except from the limited study of Colville as to the relative effectiveness of the various algorithms. Based on the data of Colville's study plus other scattered information, the following algorithms seemed to merit detailed numerical tests: (1) Generalized Reduced Gradient Method of Abadie and Guigou; (2) Moving Exterior Truncation Method of Staha and Himmelblau; (3) Generalized Projection Method for Nonlinear Constraints of Rosen; (4) Mixed Parametric Penalty Function of Lootsma; (5) SUMT by Fiacco and McCormick; and (6) Projection Method for Nonlinear Constraints by Murtagh and Sargent. The criteria used to evaluate the algorithms are discussed and the characteristics of the 25 test problems summarized. Data are provided as to computation times, number of function evaluations, precision, and user preparation time for the effective algorithms. Some general conclusions are given to serve as a guide to those who wish to apply the algorithms.

STRAETER, TERRY A.; A VARIABLE METRIC PARALLEL OPTIMIZATION ALGORITHM. ABSTRACT: This paper introduces an algorithm designed to exploit the parallel or pipeline computing capabilities of the next generation of computers (ILLIAC and STAR). If p is the degree of parallelism, then one cycle of the parallel variable metric algorithm is defined as follows: first, the function and its gradient are computed in parallel at p locations; then the metric is modified by p rank one corrections, and finally a single univariant minimization is carried out in the Newton-like direction. Several properties of this algorithm are established in the paper. In addition, the convergence of the iterates to the solution is proved for a quadratic functional on a real separable Hilbert Space; in fact, for a finite dimensional space the convergence is in one cycle. Results of numerical experiments indicate that the new algorithm will exploit the parallel or pipeline computing capabilities of the new computers to effect faster convergence than serial techniques currently in use. In fact, the experiments indicate that even when the computations are done serially, the new algorithm is very competitive with the widely used Davidon-Fletcher-Powell technique.

WENDELL, R. E. and THORSON, S. J.; A MATHEMATICAL STUDY OF DECISIONS IN A DICTATORSHIP. ABSTRACT: One of the problems that has been considered by political scientists is the determination of the position that an omniscient and beneficent dictator should take on the various issues that confront him (her). Besides countries, such a problem also arises in committee decision making under an omniscient and beneficent chairman. Using a vector space approach to characterize issues, the dictator situation can be modeled as a location problem and can be analyzed using the technique of Mathematical Programming. To do this we first identify each citizen's position on the issues as a point in the vector space. Then we assume that the utility loss of each citizen is an increasing function of the distance that the dictator locates from his (her) position. (The choice of the particular distance measures are particularly interesting since they imply behavioral properties about the individuals. A person who is characterized by the Tchebycheff norm, for example, has quite a different behavior pattern from a person who is characterized by the city block norm.) Finally we choose some reasonable objective function for the dictator, such as minimizing total utility loss, and study the properties of optimal dictator locations. Besides being a

direct application of location theory, this political decision problem motivates further theoretical research questions. In short, this approach not only gives a new and exciting application of existing methodology to the Social Sciences, but also generates new theoretical questions in the methodology.

ZANGWILL, WILLARD I.; HYDRAULIC ANALOGIES IN OPTIMIZATION (SUBTITLE: A PLUMBERS APPROACH TO LINEAR PROGRAMMING). ABSTRACT: Recently, Professor White proposed a hydraulic system for solving LP problems using pistons, cylinders, pipes, and water. Two questions arise: Is this approach all wet, and are there important implications of this approach to large scale systems, nonlinear programming, and duality? The answer to both questions is yes and will be discussed.

ADDENDUM:

ARMACOST, R. L. and MYLANDER, W. C.; A GUIDE TO SUMT-VERSION 4 COMPUTER SUBROUTINE FOR IMPLEMENING SENSITIVITY ANALYSIS IN NONLINEAR PROGRAMMING. ABSTRACT: A computer program is presented which implements a procedure for conducting a sensitivity analysis of a nonlinear programming problem in conjunction with the Sequential Unconstrained Minimization Technique for Nonlinear Programming. The algorithm used in the procedure is also presented. The method of incorporating the sensitivity analysis subroutines in the SUMT-Version 4 computer code is described. Two options for using the sensitivity analysis either along the minimizing trajectory or at the final subproblem only are incorporated.