AUTOMATED NETWORK DESIGN WITH OPTIMAL TOLERANCES

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A new approach to network design to obtain optimal parameter values simultaneously with an optimal set of component tolerances is proposed. An automated scheme would start from an arbitrary initial acceptable or unacceptable design and under appropriate restrictions stop at an acceptable design which is optimum in the worst case sense for the obtained tolerances.

Introduction

It is the purpose of this paper to present a new concept in the network design and tolerance selection problem. The concept of a "floating and expanding polytope" suggests the two procedures of finding an acceptable nominal point and an optimal set of tolerances be replaced by one automated scheme. Using a suitable nonlinear programming technique, any arbitrary initial acceptable or unacceptable design may be used as a starting point. The scheme would stop at an acceptable design which is optimal in the worst case sense of obtained tolerances. The most suitable objective function to be minimized would seem to be one that best describes the cost of fabrication of the circuit, as suggested by some authors[1-6]. Several objective functions have been investigated and the results are discussed.

Definitions

The Tolerance Region

A point $\underset{\sim}{x} \triangleq [x_1\ x_2\ \dots\ x_k]^T$ in the component space is a vector of k elements and corresponds to the component values (may be normalized) of the network. A nominal point $\underset{\sim}{x}^o \triangleq [x_1^o\ x_2^o\ \dots\ x_k^o]^T$ is a point in the component space associated with a set of nonnegative tolerances $\underset{\sim}{\varepsilon}$, where $\underset{\sim}{\varepsilon} \triangleq [\varepsilon_1\ \varepsilon_2\ \dots\ \varepsilon_k]^T \geq \underset{\sim}{0}$ such that the tolerance region R_t can be defined as

$$R_t \triangleq \{\underset{\sim}{x} \mid x_i^o - \varepsilon_i \leq x_i \leq x_i^o + \varepsilon_i\ ,\ i = 1,2,\dots,k\} \quad (1)$$

Obviously, R_t is a polytope of k dimensions with sides of length $2\varepsilon_i$, i = 1,2,...k, and centered at $\underset{\sim}{x}^o$. The polytope has 2^k vertices. Each vertex is indexed from an index set $H \triangleq \{1,2,\dots,2^k\}$ such that

$$\underset{\sim}{x}^1 \triangleq \begin{bmatrix} x_1^o - \varepsilon_1 \\ x_2^o - \varepsilon_2 \\ \vdots \\ x_k^o - \varepsilon_k \end{bmatrix},\ \underset{\sim}{x}^2 \triangleq \begin{bmatrix} x_1^o + \varepsilon_1 \\ x_2^o - \varepsilon_2 \\ \vdots \\ x_k^o - \varepsilon_k \end{bmatrix},$$

$$\underset{\sim}{x}^3 \triangleq \begin{bmatrix} x_1^o - \varepsilon_1 \\ x_2^o + \varepsilon_2 \\ \vdots \\ x_k^o - \varepsilon_k \end{bmatrix},\dots,\ \underset{\sim}{x}^{2^k} \triangleq \begin{bmatrix} x_1^o + \varepsilon_1 \\ x_2^o + \varepsilon_2 \\ \vdots \\ x_k^o + \varepsilon_k \end{bmatrix} \quad (2)$$

A possible outcome of a circuit with a nominal design $\underset{\sim}{x}^o$ and tolerance $\underset{\sim}{\varepsilon}$ falls somewhere in or on the polytope. Depending on the location of $\underset{\sim}{x}^o$ and the size of $\underset{\sim}{\varepsilon}$, a circuit with parameters $\underset{\sim}{x}$ may or may not be acceptable.

The Acceptable Region

The following discussion refers to the frequency domain design of linear, time invariant circuits but the results can be applied to the time domain as well. Let the set of frequency points under consideration be $W = \{\omega_1, \omega_2, \dots, \omega_u, \omega_{u+1}, \dots, \omega_{u+\ell}\}$. Upper specifications $S_u(\omega_i)$, i = 1,2, ..., u are assigned to the first u frequency points and lower specifications $S_\ell(\omega_i)$, i = u+1, ..., u+ℓ to the rest. Frequency points that have both upper and lower specifications may appear twice in the set. Let the response of the network at frequency ω_i be $f(\underset{\sim}{x},\omega_i)$.

A network is acceptable if

$$S_u(\omega_i) - f(\underset{\sim}{x},\omega_i) \geq 0 \quad i = 1,2,\dots,u \quad (3)$$

and

$$f(\underset{\sim}{x},\omega_j) - S_\ell(\omega_j) \geq 0 \quad j = u+1,\dots,u+\ell \quad (4)$$

Let (3) and (4) be denoted by

$$g_i(\underset{\sim}{x},\omega_i) \geq 0 \quad i = 1,2,\dots,\ell+u \quad (5)$$

and assembled into a column vector $\underset{\sim}{g}(\underset{\sim}{x}) \geq \underset{\sim}{0}$. An acceptable region R_a is then defined as

$$R_a \triangleq \{\underset{\sim}{x} \mid \underset{\sim}{g}(\underset{\sim}{x}) \geq \underset{\sim}{0}\}. \quad (6)$$

Obviously, a design $\{\underset{\sim}{x}^o, \underset{\sim}{\varepsilon}\}$ is an acceptable design only if $R_t \subseteq R_a$.

A Theorem

It is impossible to test all the points in R_t to see whether they are in the acceptable region R_a. In order to make the problem tractable a number of simplifying assumptions could be made to obtain a solution to the problem with reasonable computational effort. Obviously, if R_a is convex and if all the vertices of R_t are interior or boundary points of R_a, then $R_t \subseteq R_a$. It can be shown that the assumption of convexity is unnecessarily restrictive.

Theorem[1]: If the vertices of R_t are in R_a, then $R_t \subseteq R_a$ if, for all j = 1,2,...,k, the assumption that

This work was supported by the National Research Council of Canada under grants A7239 and C154.

$\underset{\sim}{x}^a, \underset{\sim}{x}^{b(j)} = \underset{\sim}{x}^a + \alpha \underset{\sim}{u}_j \in R_a$, where α is a scalar and

$$\underset{\sim}{u}_1 \triangleq \begin{bmatrix}1\\0\\ \vdots\\0\end{bmatrix}, \underset{\sim}{u}_2 \triangleq \begin{bmatrix}0\\1\\ \vdots\\0\end{bmatrix}, \ldots, \underset{\sim}{u}_k \triangleq \begin{bmatrix}0\\0\\ \vdots\\1\end{bmatrix}$$

implies that $\underset{\sim}{x} = \underset{\sim}{x}^a + \lambda(\underset{\sim}{x}^{b(j)} - \underset{\sim}{x}^a) \in R_a$ for all λ satisfying $0 \le \lambda \le 1$. Under such assumptions, only the vertices of the polytope need be tested to ensure that $R_t \subseteq R_a$. Other constraints such as parameter constraints can be considered. These constraints define a feasible region R_f. Then it is required that $R_t \subseteq (R_a \cap R_f) = R_c$.

The Nonlinear Programming Problem

A new vector $\underset{\sim}{\phi}$ of dimension 2k is defined as

$$\underset{\sim}{\phi} \triangleq \begin{bmatrix}\phi_1\\ \vdots\\ \phi_k\\ \phi_{k+1}\\ \vdots\\ \phi_{2k}\end{bmatrix} = \begin{bmatrix}x_1^o\\ \vdots\\ x_k^o\\ \epsilon_1\\ \vdots\\ \epsilon_k\end{bmatrix}. \tag{7}$$

A function $U_1(\underset{\sim}{\phi})$ to be minimized may be

$$U_1 = \sum_{i=1}^{k} \frac{\alpha_i \phi_i}{\phi_{i+k}} = \sum_{i=1}^{k} \frac{\alpha_i x_i^o}{\epsilon_i} \tag{8}$$

where α_i is a weighting factor. U_1 is an approximation to the total cost of the components. Typically[4], $C_i = \beta_i + \alpha_i/t_i$ where $t_i = 100\ \epsilon_i/x_i^o$ %, the percentage tolerance, and C_i is the cost of the ith component, and where α_i and β_i are coefficients determined by curve-fitting to some known cost data.

Other possibilities are

$$U_2 = \sum_{i=1}^{k} \frac{\alpha_i}{\phi_{i+k}} = \sum_{i=1}^{k} \frac{\alpha_i}{\epsilon_i} \tag{9}$$

and

$$U_3 = \sum_{i=1}^{k} \alpha_i \log_e \frac{\phi_i}{\phi_{i+k}} = \sum_{i=1}^{k} \alpha_i \log_e \frac{x_i^o}{\epsilon_i}. \tag{10}$$

In (10) we would be minimizing the ratio of the volume of the polytope defined by the space diagonal $\underset{\sim}{x}^o$ and the volume of the polytope defined by $\underset{\sim}{\epsilon}$ if the $\alpha_i = 1$.

The constraints are $g_{ij}(\underset{\sim}{x}^i, \omega_j) \ge 0$ for all $i \epsilon H$ and $\omega_j \epsilon W$. That is, at each vertex $\underset{\sim}{x}^i$, there are $\ell + u$ frequency constraints. There are 2^k vertices for a polytope of k dimensions. A total of $2^k(\ell+u)$ constraints have to be considered. Other constraints can be added.

A suitable method for solving the nonlinear programming problem is to define[8]

$$F(r, \underset{\sim}{\phi}) = U(\underset{\sim}{\phi}) + \sum_{j=1}^{u+\ell} \sum_{i=1}^{2^k} \frac{r}{g_{ij}(\underset{\sim}{\phi})} \tag{11}$$

and minimize F with respect to $\underset{\sim}{\phi}$ for appropriately decreasing values of r. The adjoint network technique[9] and the Fletcher method[10] are used in this work in the optimization process.

Examples

Two-section Lossless Transmission-line Transformer[7]

A two-section transformer with a load-to-source impedance ratio of 10:1 and 100% relative bandwidth is studied to illustrate the concepts and procedures. The lengths of the lines are assumed to be fixed at the quarter-wave value at center frequency. The variables x_1 and x_2 are the normalized characteristic impedances of the transmission lines. Fig. 1 shows the contours of $M = \max|\rho(\underset{\sim}{x}, \omega_i)|$, $\omega_i \epsilon W$ where ρ is the reflection coefficient. The chosen set of normalized frequency points is $W = \{0.5, 0.6, \ldots, 1.5\}$.

A minimax solution without taking the tolerances into account is $\underset{\sim}{x}_a^o$ (see Fig. 1) at which M = 0.4286. However, suppose we have an upper specification of $|\rho(\underset{\sim}{x}, \omega_i)| \le 0.55$. The problem now is to find an optimal nominal point x^o and an optimal set of tolerances. Table 1 and Fig. 1 show some results. Note that some of the relative tolerances are equal, which can be proved analytically. A reduction of 37% of the original cost is achieved by moving the nominal point to an optimal position when using U_1.

A Bandpass Filter

The bandpass filter shown in Fig. 2 was studied by Butler[2], Karafin[3] and Pinel and Roberts[4]. An upper specification of 3 dB for the passband and a lower specification of 35 dB for the stopband relative to 0 dB at a central frequency at 420 Hz are assigned. See Fig. 3. W = {360,420,490,170,240,700,1000} in which the first three frequencies are assigned to the upper specification and the last 4 to the lower specification. Parameters x_3 and x_4 are assumed equal to x_1 and x_2, respectively, from physical considerations. A constant Q is assumed for the three inductors and therefore the three corresponding resistances are dependent variables. Nominal values used by Pinel and Roberts and a half-percent tolerance for each component were used as a starting point. Parameter values are scaled by normalizing to avoid ill-conditioning. Each cycle of the SUMT method takes about 50 function evaluations (about 1 minute on a CDC 6400) except for the first cycle. Initially, r=1 . r was reduced successively by a factor of 10. See Table 2 and Fig. 3 for some results.

Monte Carlo simulations were made for a number of tolerance sets with a nominal point obtained by minimizing U_1. The same assumptions were made as Pinel and Roberts[4] that the component distribution is uniformly concentrated within .05t of the extreme values. There were no failures for 1000 simulations with the tolerances shown under U_1 in Table 2, as well as with the percentage tolerances {5,5,5,5,5,5,5,5} and {5,7.5,5,7.5,5,5,5,5}. There were 90 failures for {5,10,5,10,5,5,5,5}.

Conclusions

It has been shown that, by moving the nominal point, a set of larger tolerances is obtained. For a network with six components (12 parameters) less than 5 minutes of computation time (for CDC 6400) is required for most practical cases.

A complete solution to the problem is not claimed, however, it may be concluded that this is a promising approach to network design, specially in the area of growing and removing components on the basis of reduction of cost.

Acknowledgements

The authors thank Dr. B.J. Karafin and Dr. E.M. Butler of Bell Telephone Laboratories, Holmdel, N.J., and Dr. J.F. Pinel of Bell-Northern Research, Ottawa, Canada, for valuable discussions and for providing some unpublished results of their work. The authors are grateful to V.K. Jha who obtained some preliminary results, and to C. Charalambous for discussions.

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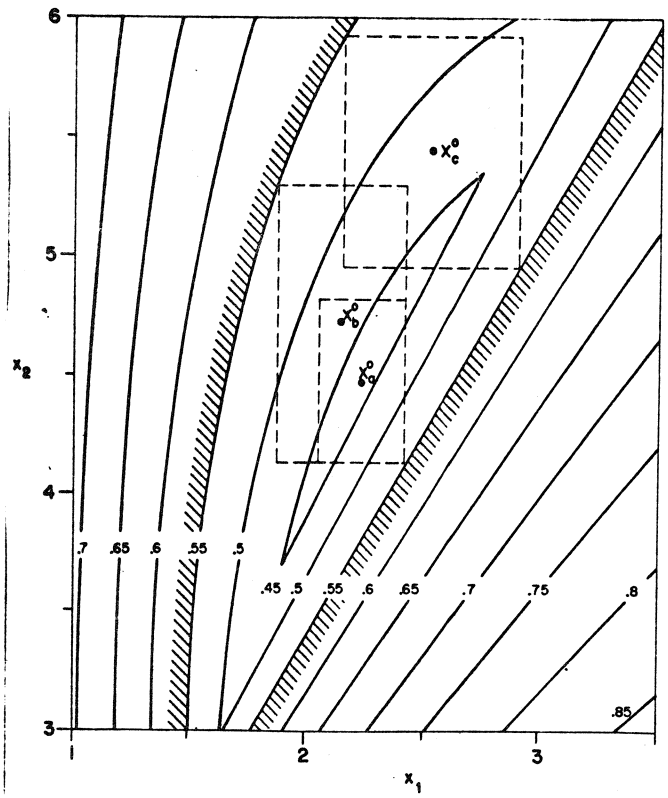

Fig. 1. Contours of $M = \max|\rho(\underset{\sim}{x},\omega_i)|$, $\omega_i \epsilon W$.

$$\underset{\sim}{x}_a^o = \begin{bmatrix} 2.2361 \\ 4.4721 \end{bmatrix} \quad \underset{\sim}{x}_b^o = \begin{bmatrix} 2.1487 \\ 4.7308 \end{bmatrix} \quad \underset{\sim}{x}_c^o = \begin{bmatrix} 2.5244 \\ 5.4395 \end{bmatrix}$$

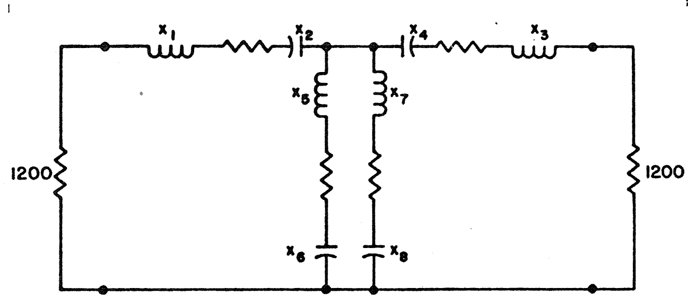

Fig. 2. Bandpass filter example.

$r = 10^{-10}$	Fixed Nominal U_1	U_2	U_3	Variable Nominal U_1	U_2	U_3
x_1^o		2.2361		2.1487	2.5244	2.1487
x_2^o		4.4721		4.7308	5.4395	4.7308
ε_1	0.1865	0.2200	0.1943	0.2739	0.3783	0.2739
ε_2	0.3443	0.2872	0.3310	0.6030	0.4937	0.6030
t_1	3.34	9.84	8.69	12.75	14.98	12.75
t_2	7.70	6.42	7.40	12.75	9.08	12.75
Cost $= \frac{1}{t_1} + \frac{1}{t_2}$	0.250	0.257	0.250	0.157	0.177	0.157
Area $= \varepsilon_1 \varepsilon_2$	0.0642	0.0632	0.0643	0.165	0.187	0.165
$\frac{\varepsilon_1 \varepsilon_2}{x_1^o x_2^o}$	0.00642	0.00632	0.00643	0.0163	0.0136	0.0163

Table 1. Comparison of the results for fixed and variable nominal points on the 2-section transformer.

	Karafin[3], Pinel and Roberts[4]	U_1	U_2	U_3
x_1^o	1.824×10^{0}	2.4822×10^{0}	2.3630×10^{0}	2.5926×10^{0}
x_2^o	7.870×10^{-8}	6.0408×10^{-8}	6.4491×10^{-8}	5.8278×10^{-8}
x_3^o	1.824×10^{0}	2.4822×10^{0}	2.3630×10^{0}	2.5926×10^{0}
x_4^o	7.870×10^{-8}	6.0408×10^{-8}	6.4491×10^{-8}	5.8278×10^{-8}
x_5^o	4.272×10^{-1}	7.7065×10^{-1}	6.5797×10^{-1}	7.9646×10^{-1}
x_6^o	9.800×10^{-7}	5.7990×10^{-7}	7.0295×10^{-7}	5.6244×10^{-7}
x_7^o	1.437×10^{-1}	2.7920×10^{-1}	2.6777×10^{-1}	2.8529×10^{-1}
x_8^o	3.400×10^{-7}	9.9596×10^{-8}	1.8840×10^{-7}	1.7161×10^{-7}
t_1	3 , 3.31	6.70	2.59	7.71
t_2	5 , 2.41	7.12	12.56	8.04
t_3	5 , 3.30	6.70	2.59	7.71
t_4	3 , 2.41	7.12	12.56	8.04
t_5	2 , 1.15	5.19	3.43	4.26
t_6	2 , 1.90	6.98	3.37	7.29
t_7	3 , 7.82	5.55	6.63	5.64
t_8	5 , 2.06	4.25	5.17	3.19
Cost	2.60,3.44	1.33	1.36	1.37

Table 2. Results for bandpass filter example.

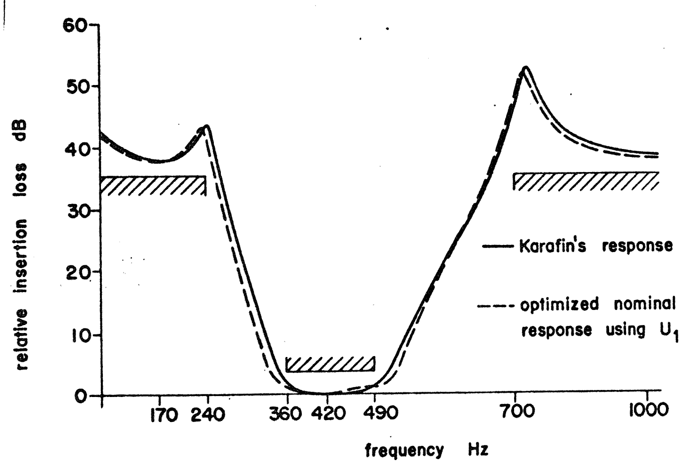

Fig. 3. Bandpass filter response.

Biographies

John W. Bandler received the Ph.D. degree from Imperial College, University of London, in 1967. He is currently Associate Professor with the Department of Electrical Engineering at McMaster University.

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Proceedings of the 1973 IEEE International Symposium on Circuit Theory

CORRECTIONS TO "AUTOMATED NETWORK DESIGN WITH OPTIMAL TOLERANCES"

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The authors regret that all the results relating to the bandpass filter example are incorrect. The numbers in the text, Table 2 and the curves in Fig. 3 are affected.

A corrected Table 2 is shown here with notes to explain how the new results were arrived at.

	Karafin,[3] Pinel and Roberts[4]	U_1^*	$U_2^\dagger$	$U_3^\dagger$
x_1^o	1.824×10^{0}	3.0142×10^{0}	2.3206×10^{0}	2.7682×10^{0}
x_2^o	7.870×10^{-8}	4.9750×10^{-8}	6.3694×10^{-8}	5.2611×10^{-8}
x_3^o	1.824×10^{0}	2.9020×10^{0}	2.3206×10^{0}	2.7682×10^{0}
x_4^o	7.870×10^{-8}	5.0729×10^{-8}	6.3694×10^{-8}	5.2611×10^{-8}
x_5^o	4.272×10^{-1}	8.2836×10^{-1}	6.0517×10^{-1}	7.7895×10^{-1}
x_6^o	9.880×10^{-7}	5.5531×10^{-7}	7.7708×10^{-7}	5.8726×10^{-7}
x_7^o	1.437×10^{-1}	3.0319×10^{-1}	2.1677×10^{-1}	2.5438×10^{-1}
x_8^o	3.400×10^{-7}	1.6377×10^{-7}	2.2630×10^{-7}	1.8981×10^{-7}
t_1	3 , 3.32	6.99	2.29	7.67
t_2	5 , 2.41	6.52	11.26	6.53
t_3	5 , 3.30	6.97	2.29	7.67
t_4	3 , 2.41	6.55	11.26	6.53
t_5	2 , 1.14	4.36	3.30	4.33
t_6	2 , 1.89	5.69	3.02	8.10
t_7	3 , 7.80	6.80	6.61	5.85
t_8	5 , 2.07	5.25	4.40	2.71
Cost	2.60 3.45	1.34	2.06	1.46

Table 2. Results for bandpass filter example (corrected).

* Symmetry not assumed; a selection of 2^8 vertices taken; no violations.

† Symmetry assumed; 2^6 vertices taken; no violations.

(1) Relative insertion loss in the passband becomes negative in some instances

(2) Both worst case and Monte Carlo simulations revealed no violations at the specified test frequencies.

The authors are indebted to Dr. J.F. Pinel of Bell-Northern Research, Ottawa, whose observations led them to these corrected results.