and/or the use of sigma–delta modulation (Reference 6) approximates in signal to quantisation noise ratio with $5{\cdot}6 \times 10^4$ bit binary-coded p.c.m. It is considered that, if an interchange between this system and standard p.c.m. is required, information will be carried on the standard p.c.m. system in the form of 6bit of delayed delta plus an additional bit to indicate to the receivers that the information is in the form of delta modulation.

Conclusions: This method provides an obvious means of access for telephone subscribers to an exchange where extensions are more limited than exchange capacity. Because of the great saving in copper (2 pairs replace 64 pairs), it could provide a possible economic saving for new installations.

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The work is being carried out at the Lanchester College of Technology, Coventry.

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OPTIMUM NONCOMMENSURATE STEPPED TRANSMISSION-LINE TRANSFORMERS

Computer-aided pattern search has been applied to the optimisation of resistively terminated impedance transformers consisting of cascaded noncommensurate transmission-line sections. For prescribed impedance ratios and fixed numbers of sections, optimum transformer designs were found to be of the quarter-wave Chebyshev type, improvement over which does not seem likely.

The exact synthesis of transmission-line impedance transformers employing homogeneous sections of equal length is now well established. Tables for Chebyshev responses are available for quarter-wave transformers[1] and for ones having sections shorter than quarter-wave.[2] For a prescribed impedance ratio and a fixed number of sections, the tables provide maximum bandwidth for a specified reflection coefficient or minimum reflection coefficient for a specified bandwidth.

Since interest is currently turning to the synthesis of cascades of noncommensurate transmission lines, one question which naturally arises is: can the commensurate transformer responses be improved on if the restriction of commensurate length sections is removed? The author is unaware of any previous investigation of this question with reference to resistively terminated transformers. Riblet, who has indicated certain limitations on any attempt to improve on quarter-wave transformers,[3] is also unaware of results on this question.*

A numerical investigation of this problem by computer-aided pattern search[4, 5] was conducted. The objective function required to be minimised is

$$U = \max_i |\rho(\phi, f_i)| \qquad i = 1, 2, \ldots, n \qquad (1)$$

*RIBLET, H. J.: Microwave Development Laboratories, Inc., Needham Heights, Mass., USA, private communication

where $\phi = [l_1, Z_1, l_2\ Z_2, \ldots, l_m, Z_m]^T$ . . . . . . (2)

subject to

$$\left.\begin{array}{l} l_k \geqslant 0 \\ Z_k > 0 \end{array}\right\} k = 1, 2, \ldots, m \qquad \begin{array}{r} (3) \\ (4) \end{array}$$

l_k and Z_k are the length and characteristic impedance of the kth section in the cascade (Fig. 1a), ρ is the transformer-

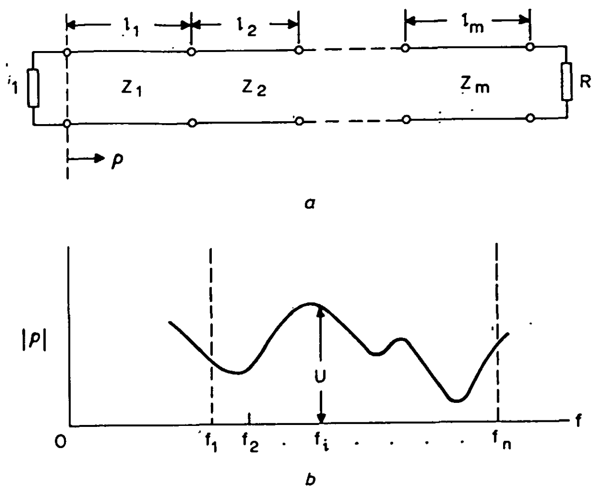

Fig. 1 *2mdimensional optimisation problem*
*a m*section resistively terminated noncommensurate transmission-line transformer
b Objective function *U* selected from the input reflection coefficient against frequency

input reflection coefficient, and f_i is the ith test frequency in the band of interest (Fig. 1b). The junctions were assumed to be ideal.

The computer program was provided with the number of sections m, the impedance ratio R, f_1 and f_n, which define the frequency band, the number of test frequencies n (variable), and the initial set of parameter values ϕ_0.

A wide range of starting points ϕ_0 was selected without regard to the monotonicity or symmetry of the transformer. It was thought, however, that optimum solutions would certainly lie within

$$l_k < \frac{\lambda_n}{2} \qquad k = 1, 2, \ldots, m \qquad (5)$$

where λ is wavelength $(= c/f)$, because the matching characteristics of a section are lost when its length approaches $\lambda/2$ at any frequency in the band. Consequently, the transformation

$$Q_k = \cot \frac{2\pi l_k}{\lambda_n} \qquad (6)$$

was employed in the search rather than l_k, in order to ensure that

$$0 < l_k < \frac{\lambda_n}{2} \qquad (7)$$

a combination of exprs. 3 and 5. Thus, ϕ (eqn. 2) was redefined as

$$\phi = [Q_1, Z_1, Q_2, Z_2, \ldots, Q_m, Z_m]^T \qquad (8)$$

and $\rho\ (\phi, f)$ was calculated as follows. First

$$Z_{I_{m+1}} = R \qquad (9)$$

The input impedance to the transformer was given at f_i by the iterative expression

$$Z_{I_k} = Z_k\left(\frac{Z_{I_{k+1}} + Z_k P_k}{Z_k + Z_{I_{k+1}} P_k}\right) \quad k = m, m-1, \ldots, 1 \quad (10)$$

where $P_k = j \tan\left(\frac{\lambda_n}{\lambda_i} \cot^{-1} Q_k\right)$ . . . . . . (11)

with the proviso that only solutions

$$0 < \cot^{-1} Q_k < \pi \qquad (12)$$

are allowed for any $-\infty < Q_k < \infty$.

Finally,

$$\rho(\phi, f_i) = \frac{Z_{I_1} - 1}{Z_{I_1} + 1} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (13)$$

The use of eqn. 8 rather than eqn. 2 greatly reduced the computing times required to find optima, indicating that a favourable parameter scaling had been introduced through eqn. 6.

The pattern-search strategy for selecting the next set of parameter values for ϕ to minimise the U of eqn. 1 is explained by Wilde and Beightler.[5] The original flow diagrams from which the present computer program was written are due to Hooke and Jeeves.[4] Some precautions on computing functions such as eqns. 10 and 11 have been pointed out by Bandler.*

Table 1 gives details for the transformers investigated. A representative range of starting points is listed in Table 2.

Table 1 TRANSMISSION-LINE TRANSFORMERS INVESTIGATED

Number of variables = $2m$

Number of sections m	Impedance ratio R	Percentage bandwidth $B = 2\left(\frac{f_n - f_1}{f_n + f_1}\right) \times 100$
1	4 10	100 40, 60, 80, 100
2	4, 25 10	100 40, 60, 80, 100
3	10	80, 100
4	10, 25	100

Table 2 TYPICAL STARTING POINTS IN THE OPTIMISATION

l_k normalised to $\frac{\lambda_1 \lambda_n}{2(\lambda_1 + \lambda_n)}$; Z_k normalised to source resistance

m	R	B	Parameter values			
1	10	40	$l_1 = \frac{2}{15}$		$Z_1 = 1$	
			l_1	Z_1	l_2	Z_2
2	4	100	2/3	4	2/3	1
	10	60	6/5	9	6/5	9
	25	100	6/5	15	2/3	2
			l_k		Z_k	
3	10	80	4/15		3	
			2/5		3	
4	10	100	2/3		3	
	25		4/5		1	

Pattern search yielded equal-ripple responses having maximum v.s.w.r.s from 0·1% to 2% above, and section lengths and impedances within a few percent of, the tabulated quarter-wave Chebÿshev values.[1] Pattern search terminated within regions which had already been shown by direct exploration to contain only the quarter-wave Chebÿshev designs as optima. None of the many examples tested led to lower maximum v.s.w.r.s. Thus, further improvement of these homogeneous resistively terminated transformers, by lifting the restriction of commensurate length sections for prescribed impedance ratios and fixed numbers of sections, does not seem likely.

The approach outlined in this letter can be readily extended to cascades of noncommensurate transmission lines terminated at one or both ends by frequency-dependent impedances. Nonideal junction capacitances can also be included. A further extension would be the design of minimum-overall-length transformers,[6] the section characteristic impedances being constrained in some required manner.

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† All programs were written in Fortran IV for the IBM 360/65

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LOW-VOLTAGE LIGHT-AMPLITUDE MODULATION

A 45° *y*cut low-voltage light-amplitude modulator is described, overcoming previously reported difficulties. The thermal dependence of retardation is discussed, and a feedback system is outlined to control the d.c. light-intensity drift to ±0·5% of its preset value.

The 45° *y*cut light-amplitude modulator can take four composite crystal arrangements.[1,2] In the simplest form, using two 45° *y*cut crystals, the electric field E_y is applied in opposite directions to produce an additive electro-optic effect, the retardation produced between the ordinary and extraordinary waves being

$$\Gamma = \frac{2\pi l}{\lambda}\left\{ n_0 - \sqrt{2}\left(\frac{1}{n_0^2} + \frac{1}{n_e^2}\right)^{-1/2} - \frac{\sqrt{2}\, r_{41} E_y}{\left(\frac{1}{n_0^2} + \frac{1}{n_e^2}\right)^{3/2}} \right\} \quad . \; . \; . \; . \quad (1)$$

where l = composite crystal length.

Variation of the natural retardation, due to the temperature dependence of l, n_0 and n_e, will produce oscillations in the output intensity of a frequency dependent on the thermal relaxation time and temperature difference between the crystal and the environment. With composite crystal arrangements of this type, using only *x* or *y*cut bars, optical equivalence can be obtained by taking crystal pairs cut side by side from the raw material. These crystal pairs will, after careful polishing, produce extinction ratios of better than 100:1.

In an attempt to produce a continuous stripline arrangement, 45° *x*–*y*cut modulators have been constructed by the authors, but difficulty in producing optically equivalent bars, owing to the orthogonal nature of the cuts, has prevented extinction ratios greater than 100:1 from being obtained to date. However, the electrical and constructional advantages gained from this design can override the disadvantage of the reduced extinction ratio.

Both modulator designs can include a halfwave plate, which, on reversing the ordinary and extraordinary ray paths and