NEW ALGORITHMS FOR NETWORK OPTIMIZATION

Christakis Charalambous and John W. Bandler
Department of Electrical Engineering
McMaster University
Hamilton, Ontario, Canada

Abstract

Two new algorithms suitable for optimization of networks are presented. Least pth approximation is used in such a way that any finite value of $p > 1$ can be used to produce minimax results very efficiently.

Introduction

The authors have already presented a justification of the use of least pth approximation techniques with large values of p for computer-aided network design[1]. They showed that the use of a fairly well-conditioned objective function with efficient gradient minimization methods such as the method by Fletcher[2], and the adjoint network method for gradient evaluation[3], yields very near minimax designs with little computational effort.

The present paper exploits all the advantages of that approach in presenting two new algorithms for practical minimax approximation. A basic difference in these algorithms is that, instead of requiring very large values of p, any finite value of p greater than one can be used to produce minimax solutions.

The paper discusses a six variable example (namely, a three-section transmission-line transformer) where values of p equal to 2, 4, 6, 10, 100, 1000, and 10,000 have all been used to obtain substantially the same solution. A comparison with other methods already known to microwave engineers is made. The Fletcher minimization method is used throughout. The advantage of the new algorithms is a combination of efficiency and flexibility which, it is believed, has not been previously enjoyed by computer-aided circuit designers.

The New Algorithms

Space in this summary does not permit elaboration of the details of the algorithms, so brief descriptions will be given.

As in previous work[1], error functions with respect to upper and lower response specifications are defined and sampled at a sufficient number of points. Norm-like functions are formed with nonnegative upper errors and/or nonpositive lower errors if some of the specifications are violated or with negative upper errors and positive lower errors if all the specifications are satisfied. All the errors are scaled by a number equal to the modulus of the maximum violation of the specifications or the minimum amount by which the specifications are exceeded, whichever is appropriate. This ensures that no number greater than unity is raised to an extremely large power, if such a value is used.

The specifications with respect to which the errors are defined are not the actual ones. They are artificial ones obtained by introducing an artificial margin of errors ξ. A sequence of optimization problems is initiated. If the given specifications are violated at the start then both algorithms set $\xi=0$ and optimize with a finite value of p.

In Algorithm 1, the next optimization sets ξ to the actual maximum error obtained plus a very small number of ε to avoid possible discontinuous derivatives. Algorithm 2 does the same if, after any optimization, the artificial maximum error becomes negative. Otherwise, Algorithm 2 sets ξ between the previous value and the actual maximum error. Both algorithms terminate when the absolute value of the difference between successive ξ values falls below some small specified number.

Examples

The algorithms have been applied to a wide range of design problems. Here, we will compare their performance using the Fletcher method on a 3-section transmission-line transformer problem which has already received attention in the literature[4,5]. See Table I. In Table II, ε was 10^{-8}. In Tables III and IV λ is the fraction of the actual maximum error that is added to $1-\lambda$ times the previous ξ to obtain the new value of ξ. In Table IV M is the actual maximum after the first optimization and m the value of the smallest ripple.

Conclusions

The results show that there is usually an optimum value of p for any given problem minimizing the effort required to reach a specific solution. The sensitivity with respect to p (for moderate values) does not seem to be great. Nor is the effort very sensitive to the value of λ. The algorithms are easy to implement. Their efficiency in comparison with other methods has also been demonstrated.

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References

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TABLE I

THE STARTING POINTS IN THE OPTIMIZATION OF A 3-SECTION 10:1 TRANSFORMER OVER 100 PERCENT RELATIVE BANDWIDTH

Parameters ϕ_i	Problem 1	Problem 2
ℓ_1/ℓ_q	1.0	0.8
Z_1	1.0	1.5
ℓ_2/ℓ_q	1.0	1.2
Z_2	3.16228	3.0
ℓ_3/ℓ_q	1.0	0.8
Z_3	10.0	6.0
Maximum reflection coefficient	0.70930	0.38813

TABLE II

OPTIMIZATION OF A 3-SECTION 10:1 TRANSFORMER OVER 100 PERCENT RELATIVE BANDWIDTH USING ALGORITHM 1

Value of p	Number of function evaluations to reach a reflection coefficient of 0.19729	
	Problem 1	Problem 2
2	178	160
4	143	128
6	142	116
10	112	89
100	136	69
1000	193	66
10000	249	104
Average number of function evaluations	165	105
Grazor search[4]	696	498
Osborne & Watson[4,*]	860(0.20831)	237(0.19788)
Razor search[5,*]	1300(0.19733)	1250(0.19731)

* Number of function evaluations to reach the value shown in brackets.

TABLE III

OPTIMIZATION OF A 3-SECTION 10:1 TRANSFORMER OVER 100 PERCENT RELATIVE BANDWIDTH USING ALGORITHM 2

Value of p	Number of function evaluations to reach a reflection coefficient of 0.19729					
	Problem 1			Problem 2		
	λ=0.5	λ=0.6	λ=0.7	λ=0.5	λ=0.6	λ=0.7
2	183	146	151	165	128	133
4	193	162	122	151	149	109
6	199	182	138	150	135	112
10	191	159	146	168	136	114
100	185	171	165	126	104	99
1000	211	211	202	83	91	83
10000	248	248	248	103	103	103
Average number of function evaluations	202	182	168	135	121	108

TABLE IV

OPTIMIZATION OF A 3-SECTION 10:1 TRANSFORMER

OVER 100 PERCENT RELATIVE BANDWIDTH USING

ALGORITHM 2 WITH $\lambda = \frac{M+m}{2M}$

Value of p	M	m	Number of function evaluations to reach a reflection coefficient of 0.19729	
			Problem 1	Problem 2
2	0.30819	0.11626	161	141
4	0.23680	0.16278	135	123
6	0.21759	0.17612	115	89
10	0.20636	0.18280	95	72
100	0.19781	0.19562	131	65
1000	0.19734	0.19712	193	66
10000	0.19730	0.19727	249	104
Average number of function evaluations			155	95