Computer Program Descriptions

A Special Program for Least *p*th Approximation Including Interpolation

PURPOSE: Minimization of a least pth objective function of k variables using gradient methods. Interpolation brings the discrete problem closer to the continuous minimax approximation problem.

LANGUAGE: Fortran IV; 1023 cards, including comments.

AUTHORS: J. R. Popović and J. W. Bandler, Department of Electrical Engineering, McMaster University, Hamilton, Ont., Canada.

AVAILABILITY: ASIS/NAPS Document No. NAPS 02273. Listing and user's manual also available from J. W. Bandler at $15.00.

DESCRIPTION: The program, called FMCLP, can be used for fitting a continuous approximating function to another single specified function or data on a closed interval, and thus is relevant in optimization used in computer-aided circuit and system design, and modelling [1].

FMCLP utilizes the practical least pth approximation approach with extremely large values of p proposed by Bandler and Charalambous [2] in conjunction with efficient gradient minimization algorithms such as Fletcher–Powell [3] and the Fletcher method [4]. Discrete least pth approximation with $p = 2$ is the well-known discrete least squares approximation and with extremely large values of p the corresponding optimal approximations tend to become discrete minimax (or Chebyshev) approximations. Proper scaling is used to alleviate the ill-conditioning resulting from very large values of p, such as 10^6. Quadratic interpolation is employed to bring the discrete problem closer to the continuous minimax approximation problem. Using quadratic interpolation the sampling for the objective function takes fewer points.

The user has to write the subprograms by which the weighting function, specified function, approximating function and its derivatives with respect to the parameters are explicitly available. The information about the number of sample points forming the discrete point set, the starting point for the design parameters, and the values of p should be supplied as data. Also the choice about quadratic interpolation, which optimization method is to be used, checking the gradients, the stopping criteria, and the form of the results may be made. The optimal point, the value of the objective function, the weighted errors, and execution time are printed out, and the intermediate results in the optimization procedure if desired.

There is no restriction on the number of design parameters and the sample points.

A recent publication [5] contains the background theory for the optimization algorithm, detailed organization of the program FMCLP, and instructions on how to use it. This includes a block diagram of the package and a description of the algorithm for quadratic interpolation with a flowchart of the corresponding subroutine. The examples which demonstrate FMCLP were taken in numerical analysis and system modelling. Document NAPS 02273 contains a complete listing and detailed user's manual for the given package fully illustrated with an example [6].

A few seconds of CDC 6400 computer time and a core requirement of about $14K_{10}$ is sufficient to optimize a five parameter design problem.

Acknowledgment

The authors wish to thank Dr. C. Charalambous, who is now with the Department of Combinatorics and Optimization, University of Waterloo, Waterloo, Ont., Canada, and some of whose recent work is embodied in the package.

Manuscript received April 17, 1973; revised August 24, 1973. This work was supported by the National Research Council of Canada under Grant A7239. This work was presented at the 16th Midwest Symposium on Circuit Theory, Waterloo, Ont., Canada, April 12–13, 1973.

See NAPS Document No. 02273 for 32 pages of supplementary material. Order from ASIS/NAPS, c/o Microfiche Publications, 305 E. 46th St., New York, N. Y. 10017. Remit in advance for each NAPS accession number $1.50 for microfiche or $5.30 for photocopies. Make checks payable to Microfiche Publications.

References

[1] G. C. Temes and D. Y. F. Zai, "Least pth approximation," *IEEE Trans. Circuit Theory* (Corresp.), vol. CT-16, pp. 235–237, May 1969.
[2] J. W. Bandler and C. Charalambous, "Practical least pth approximation with extremely large values of p," *Conf. Rec. of the 5th Asilomar Conf. Circuits and Systems* (Pacific Grove, Calif.), Nov. 1971, pp. 66–70.
[3] R. Fletcher and M. J. D. Powell, "A rapidly convergent descent method for minimization," *Comput. J.*, vol. 6, pp. 163–168, June 1963.
[4] R. Fletcher, "A new approach to variable metric algorithms," *Comput. J.*, vol. 13, pp. 317–322, Aug. 1970.
[5] J. R. Popović, J. W. Bandler, and C. Charalambous, "General programs for least pth and near minimax approximation," *Int. J. Systems Science*, to be published.
[6] J. R. Popović and J. W. Bandler, "A special program for least pth approximation including interpolation," McMaster Univ., Hamilton, Ont., Canada, Internal Rep. in Simulation, Optimization and Control, SOC-19, Oct. 1973.

A General Program for Discrete Least *p*th Approximation

PURPOSE: To minimize an objective function of k variables defined as the generalized discrete least pth objective using gradient methods.

LANGUAGE: Fortran IV; 1005 cards, including comments.

AUTHORS: J. R. Popović and J. W. Bandler, Department of Electrical Engineering, McMaster University, Hamilton, Ont., Canada.

AVAILABILITY: ASIS/NAPS Document No. NAPS 02274. Listing and user's manual also available from J. W. Bandler at $15.00.

DESCRIPTION: The program, called FMLPO, is applicable to problems of meeting and/or exceeding design specifications on several disjoint closed intervals, and thus is relevant to a wide range of specifications and a wide variety of network and system design problems, especially in filter design.

The program utilizes the approach of the practical generalized least pth approximation proposed by Bandler and Charalambous [1]. Gradient minimization algorithms due to Fletcher and Powell [2] and, more recently, to Fletcher [3] are used. Least pth approximation with $p = 2$ gives a discrete least squares approximation. With sufficiently large values of p an optimal solution very close to the optimal minimax solution can be obtained. Values of p up to 10^6 have been successfully employed. Proper scaling alleviates the ill-conditioning when large values of p are used and automatically defines both problems, meeting or exceeding design specifications, into one optimization problem.

The program can be used in a less general least pth approximation problem for fitting a continuous function to another one or to data on a closed interval. Although the program is not written for nonlinear programming, we found that it is also applicable to problems with parameter constraints.

The user has to write all the required specifications in each interval, the approximating functions with partial derivatives, and weighting functions for different specifications in a straightforward way. The number of intervals and discrete point sets are user specified as well as the values of p, the parameter constraints, and the initial parameter values. Also the choice about which optimization method is to be used, checking the gradients, and the stopping criteria may be

Manuscript received April 17, 1973; revised August 24, 1973. This work was supported by the National Research Council of Canada under Grant A7239. This work was presented at the 16th Midwest Symposium on Circuit Theory, Waterloo, Ont., Canada, April 12–13, 1973.

See NAPS Document No. 02274 for 38 pages of supplementary material. Order from ASIS/NAPS, c/o Microfiche Publications, 305 E. 46th St., New York, N. Y. 10017. Remit in advance for each NAPS accession number $1.50 for microfiche or $6.20 for photocopies. Make checks payable to Microfiche Publications.