

# Computer Program Descriptions

## A Special Program for Least $p$ th Approximation Including Interpolation

**PURPOSE:** Minimization of a least  $p$ th objective function of  $k$  variables using gradient methods. Interpolation brings the discrete problem closer to the continuous minimax approximation problem.

**LANGUAGE:** Fortran IV; 1023 cards, including comments.

**AUTHORS:** J. R. Popović and J. W. Bandler, Department of Electrical Engineering, McMaster University, Hamilton, Ont., Canada.

**AVAILABILITY:** ASIS/NAPS Document No. NAPS 02273. Listing and user's manual also available from J. W. Bandler at \$15.00.

**DESCRIPTION:** The program, called FMCLP, can be used for fitting a continuous approximating function to another single specified function or data on a closed interval, and thus is relevant in optimization used in computer-aided circuit and system design, and modelling [1].

FMCLP utilizes the practical least  $p$ th approximation approach with extremely large values of  $p$  proposed by Bandler and Charalambous [2] in conjunction with efficient gradient minimization algorithms such as Fletcher-Powell [3] and the Fletcher method [4]. Discrete least  $p$ th approximation with  $p = 2$  is the well-known discrete least squares approximation and with extremely large values of  $p$  the corresponding optimal approximations tend to become discrete minimax (or Chebyshev) approximations. Proper scaling is used to alleviate the ill-conditioning resulting from very large values of  $p$ , such as  $10^6$ . Quadratic interpolation is employed to bring the discrete problem closer to the continuous minimax approximation problem. Using quadratic interpolation the sampling for the objective function takes fewer points.

The user has to write the subprograms by which the weighting function, specified function, approximating function and its derivatives with respect to the parameters are explicitly available. The information about the number of sample points forming the discrete point set, the starting point for the design parameters, and the values of  $p$  should be supplied as data. Also the choice about quadratic interpolation, which optimization method is to be used, checking the gradients, the stopping criteria, and the form of the results may be made. The optimal point, the value of the objective function, the weighted errors, and execution time are printed out, and the intermediate results in the optimization procedure if desired.

There is no restriction on the number of design parameters and the sample points.

A recent publication [5] contains the background theory for the optimization algorithm, detailed organization of the program FMCLP, and instructions on how to use it. This includes a block diagram of the package and a description of the algorithm for quadratic interpolation with a flowchart of the corresponding subroutine. The examples which demonstrate FMCLP were taken in numerical analysis and system modelling. Document NAPS 02273 contains a complete listing and detailed user's manual for the given package fully illustrated with an example [6].

A few seconds of CDC 6400 computer time and a core requirement of about  $14K_{10}$  is sufficient to optimize a five parameter design problem.

### ACKNOWLEDGMENT

The authors wish to thank Dr. C. Charalambous, who is now with the Department of Combinatorics and Optimization, Univer-

sity of Waterloo, Waterloo, Ont., Canada, and some of whose recent work is embodied in the package.

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## A General Program for Discrete Least $p$ th Approximation

**PURPOSE:** To minimize an objective function of  $k$  variables defined as the generalized discrete least  $p$ th objective using gradient methods.

**LANGUAGE:** Fortran IV; 1005 cards, including comments.

**AUTHORS:** J. R. Popović and J. W. Bandler, Department of Electrical Engineering, McMaster University, Hamilton, Ont., Canada.

**AVAILABILITY:** ASIS/NAPS Document No. NAPS 02274. Listing and user's manual also available from J. W. Bandler at \$15.00.

**DESCRIPTION:** The program, called FMLPO, is applicable to problems of meeting and/or exceeding design specifications on several disjoint closed intervals, and thus is relevant to a wide range of specifications and a wide variety of network and system design problems, especially in filter design.

The program utilizes the approach of the practical generalized least  $p$ th approximation proposed by Bandler and Charalambous [1]. Gradient minimization algorithms due to Fletcher and Powell [2] and, more recently, to Fletcher [3] are used. Least  $p$ th approximation with  $p = 2$  gives a discrete least squares approximation. With sufficiently large values of  $p$  an optimal solution very close to the optimal minimax solution can be obtained. Values of  $p$  up to  $10^6$  have been successfully employed. Proper scaling alleviates the ill-conditioning when large values of  $p$  are used and automatically defines both problems, meeting or exceeding design specifications, into one optimization problem.

The program can be used in a less general least  $p$ th approximation problem for fitting a continuous function to another one or to data on a closed interval. Although the program is not written for non-linear programming, we found that it is also applicable to problems with parameter constraints.

The user has to write all the required specifications in each interval, the approximating functions with partial derivatives, and weighting functions for different specifications in a straightforward way. The number of intervals and discrete point sets are user specified as well as the values of  $p$ , the parameter constraints, and the initial parameter values. Also the choice about which optimization method is to be used, checking the gradients, and the stopping criteria may be

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made. The optimal point, weighted errors from various intervals, and execution time are printed out, and the intermediate results in the optimization procedure if desired.

There is no restriction on the number of design parameters, number of intervals, or discrete point sets.

A recent publication [4] contains the background theory for the optimization algorithm, detailed organization of the program FMLPO, and instructions on how to use it for both unconstrained and constrained optimization problems. This includes a block diagram of the package and flowcharts of its subroutines. The examples demonstrating FMLPO were taken in system modelling and multi-section transmission-line filter design. Document NAPS 02274 contains a complete listing and detailed user's manual for the given package fully illustrated with examples [5].

Typically less than a minute of CDC 6400 computer time and a core requirement of about  $15K_{10}$  is sufficient to optimize a constrained problem with five parameters and fifty-two sample points.

#### ACKNOWLEDGMENT

The authors wish to thank Dr. C. Charalambous, who is now with the Department of Combinatorics and Optimization, University of Waterloo, Waterloo, Canada, and some of whose recent work is embodied in the package.

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### DICOSY (Directional Coupler Synthesis)

- PURPOSE:** This program performs a synthesis procedure for  $N$ -stepped asymmetric TEM-transmission-line directional couplers of any desired mean voltage coupling and coupling tolerance.
- LANGUAGE:** Fortran IV. Program deck length about 210 cards.
- AUTHORS:** M. Hoffmann, Institut für Hochfrequenztechnik, Ruhruniversität D-463 Bochum, Germany. G. Mörlner, Messerschmitt-Bölkow, D-8012 Ottobrunn, Germany.
- AVAILABILITY:** ASIS/NAPS Document No. NAPS 02231.
- DESCRIPTION:** Since the program DICOSY is based on an exact synthesis procedure published by Levy

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See NAPS Document No. 02231 for 4 pages of supplementary material. Order from ASIS/NAPS, c/o Microfiche Publications, 305 E. 46th St., New York, N. Y. 10017. Remit in advance for each NAPS accession number \$1.50 for microfiche or \$5.00 for photocopies up to 30 pages, 15¢ for each additional page. Make checks payable to Microfiche Publications.

[1]-[3], no details of the theory itself will be given in this computer program description.

The input variables are specified as follows.

- $N$  Number of coupling elements.  
 $CM$  Mean coupling.  
 $R$  Voltage ripple in percent (i.e., coupling varies between  $CM(1 - R/100)$  and  $CM(1 + R/100)$ ; if coupling is wanted to vary between  $CM - R$  and  $CM + R$  (in decibels), only three cards have to be replaced).  
 $SCAT$  Logical variable: `IF.TRUE` an analysis of the synthesized coupler is carried out.  
 The output data are as follows.  
 $N, CM, R$  See above.  
 $BW$  Bandwidth of coupler.  
 $TETA$  Lower bound  $(l/\lambda)_c$  of the operating frequency band ( $l$  being the overall length of the coupler).  
 $Z$  Even-mode impedances  $Z_{oei}$  of the  $N$  coupling sections normalized to the impedance of the input lines.  
 $KOP$  Coupling coefficients  $k_i = (Z_{oei}^2 - 1)/(Z_{oei}^2 + 1)$  of the  $N$  coupling sections.

If  $SCAT = \text{TRUE}$ , the real part, the imaginary part, and the modulus of the scattering coefficients  $S_{14}$  and  $S_{12}$  of the coupler are computed and printed for  $0 \leq l/\lambda \leq 4.5$ , thus allowing a check of the synthesis procedure.

The program has no limitations in principle and the highest value of  $N$  that can be reached by DICOSY only depends upon the number of significant digits the computer processes. For this reason the program is written in double precision.

With 18 significant decimal digits [IBM 7040]  $N_{\max}$  is about 40, while 25 significant decimal digits [Telefunken TR 440] give  $N_{\max} \approx 75$ . These high values of  $N$  allow the construction of directional couplers with high-pass Chebyshev equal-ripple coupling response [4], [5].

The part of the program performing the reduction process of the overall transfer matrix can also be used in connection with the synthesis procedure of multielement symmetrical directional couplers [6].

The running time only depends on the number of elements  $N$ . Some typical values for a run on a Telefunken TR 440 computer are as follows:

$N$	Synthesis (s)	Synthesis and Analysis (s)
10	0.3	2.8
40	2.1	9.7
75	6.3	19.5

On this computer the word-storage requirements for the program are 24 K of the core and 49 K of the drum storage.

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