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Cascaded Network Optimization Program

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Abstract—A user-oriented computer program package is presented that will analyze and optimize certain cascaded linear time-invariant electrical networks such as microwave filters and all-pass networks. The program is organized in such a way that future additions or deletions of performance specifications, constraints, optimization methods, and circuit elements are readily implemented. Presently, a variety of two-port lumped and distributed elements, all-pass C-type sections and all-pass D-type sections can be treated as fixed or variable between upper and lower bounds on the parameters. Adjoint network sensitivity formulas are incorporated. The Fletcher–Powell or Fletcher optimization methods can be called to optimize in the least pth sense of Bandler and Charalambous an objective function incorporating simultaneously, at the user's discretion, input reflection coefficient, insertion loss, group delay, and the parameter constraints (if any). The program is particularly flexible in the way in which response specifications are handled at any number of, in general, overlapping frequency bands. The package, which is written in Fortran IV, has been tested on a CDC 6400 digital computer.

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I. Introduction

A USER-ORIENTED computer program package is presented that will analyze and optimize certain cascaded linear time-invariant networks such as microwave filters and all-pass networks in the frequency domain.

State-of-the-art techniques in gradient minimization of functions of many variables such as the Fletcher–Powell [1] and Fletcher [2] methods are available to the user. The adjoint network method of gradient evaluation for circuit elements in the frequency domain [3]–[5] is largely employed. State-of-the-art techniques in least pth approximation developed by Bandler and Charalambous and generalized for such tasks as filter design are incorporated [6]. The present work and computer program associated with it represent a significant advance in efficiency and organization over previous similar work [7].

The program is organized in such a way that future additions or deletions of performance specifications, constraints, optimization methods, and circuit elements are readily implemented. Presently, a variety of two-port lumped elements, including resistors, inductors, and capacitors as well as lumped resonant and antiresonant circuits, and distributed elements such as lossless transmission lines including open and shorted stubs, all-pass C-type sections, and all-pass D-type sections can be handled. Upper and lower bounds on all relevant parameters can be specified by the user. At the user's discretion, a least pth objective function or a sequence of least pth objective functions incorporating simultaneously input reflection coefficient, insertion loss, relative group delay, and parameter constraints (if any) are automatically created. Finite values of p greater than 1 can be used. It is felt that the program is particularly flexible in the way in which response specifications are handled at any number of, in general, overlapping frequency bands.

The package, which is written in Fortran IV, has been tested on a CDC 6400 digital computer. Some of the many test examples will be presented here to illustrate the approach. Examples of input and output as well as actual execution times will be given.

II. Theory

The Problem

The discrete approximation problem which the package solves can be stated, in general, as follows. A point $\boldsymbol{\phi}$ is sought which results in the "best" solution of the set of design inequalities

$$S_{rl}(\psi_i) \le F_r(\boldsymbol{\phi},\psi_i) \le S_{ru}(\psi_i) \tag{1}$$

$$C_{lj} \le C_j(\boldsymbol{\phi}) \le C_{uj} \tag{2}$$

where

- $F_r(\boldsymbol{\phi},\psi_i)$ rth actual response function evaluated at ψ_i;
- $S_{ru}(\psi_i)$ rth upper specified response bound evaluated at ψ_i;
- $S_{rl}(\psi_i)$ rth lower specified response bound evaluated at ψ_i;
- $C_j(\boldsymbol{\phi})$ jth constraint function;
- C_{uj} jth upper constraint bound;
- C_{lj} jth lower constraint bound;
- $\boldsymbol{\phi}$ vector containing the k independent design parameters;
- ψ_i ith sample point of the independent variable ψ.

Some of the upper bounds may be $+\infty$ and some of the lower bounds may be $-\infty$, in which case they are ignored. Some of the upper and lower bounds may be the same (single specification/constraint). An acceptable and feasible design is one for which the inequalities are satisfied. It is the job of the designer to ensure that his design problem is specified in a physically meaningful way.

For notational simplicity we define a specification s_i, which may be an upper or lower response bound or constraint bound, and a corresponding weight x_i such that

$$x_i = \begin{cases} +1.0 \text{ if } s_i \text{ is an upper bound} \\ -1.0 \text{ if } s_i \text{ is a lower bound.} \end{cases} \tag{3}$$

Then the problem essentially becomes one of satisfying a number of inequalities of the form

$$x(y - s) \le 0 \tag{4}$$

where all subscripts are dropped to avoid confusion, y is F_r or C. y will be called the approximating function. It is understood that (4) must include all design specifications and constraints implied by (1) and (2).

The Objective Function

The objective function to be minimized is computed as [6]

$$U \leftarrow M\left(\sum\left(\frac{e}{M}\right)^q\right)^{1/q} \tag{5}$$

and the gradient vector as [6]

$$\nabla U \leftarrow \left(\sum\left(\frac{e}{M}\right)^q\right)^{(1/q)-1}\left(\sum\left(\frac{e}{M}\right)^{q-1}\nabla e\right) \tag{6}$$

where

$$e \leftarrow wx(y - s) - \xi \tag{7}$$

$$M \leftarrow \max e \tag{8}$$

$$q \leftarrow p \operatorname{sgn} M \tag{9}$$

and

- $\sum$ summation over $\begin{cases}\text{all } e, & \text{if } M < 0 \\ \text{all } e > 0, & \text{if } M > 0;\end{cases}$
- p any finite real number greater than 1;
- w positive weighting factor;
- ξ artificial margin.

The designer exercises his own discretion as to the values of p, the weighting factors w and the artificial margin ξ. Discussion of these parameters is available in the literature [5], [6], [8]–[10] and so will not be repeated here. An important point to remember, however, is that the first optimization with a particular value of p will determine whether the specifications and constraints can be satisfied for any other value [9].

Performance specifications and parameter constraints are clearly treated in essentially the same way by the objective function. Fig. 1(a) shows possible contours of the least pth objective function without parameter constraints, and Fig. 1(b) shows possible contours for the same problem when a single upper bound on one parameter is desired (see also Charalambous [10]).

Interval Translation and Artificial Points

To distinguish conveniently between the various responses or constraint functions, particularly because of the different ways in which the corresponding gradients are calculated we have employed the following scheme. We assume that all responses are to be considered on the interval $[0,z_u)$ or subintervals or points contained in that interval of the independent variable z. Let the total number of response functions F_r be n_r. Then we let

$$z' \leftarrow \begin{cases} z + n_r z_u, & \text{if } r = 0 \\ z + (r-1)z_u, & \text{if } r \in \{1,2,\cdots,n_r\} \end{cases} \tag{10}$$

where $r = 0$ denotes that n_c constraint functions C_z are to be considered for $z = 1,2,\cdots,n_c$.

Thus we can identify any response function to be considered and the point at which it is to be calculated as well as any constraint function by examining the value of z' as follows:

$$\text{if } \begin{cases} z' > n_r z_u \text{ then } y \leftarrow C,\ \nabla y \leftarrow \nabla C \\ (r-1)z_u \le z' < r z_u \text{ for any } r \in \{1,2,\cdots,n_r\} \end{cases} \tag{11}$$

then $y \leftarrow F_r$, $\nabla y \leftarrow \nabla F_r$.

III. Implementation

The Subprograms

Fig. 2 shows a block diagram of the subprograms comprising the network optimization program. A brief description of these subprograms is given in this section.

CANOPT (CAscaded Network OPTimization program) is the subroutine called by the user. It reads and organizes input data, determines z' as in (10), controls the other subprograms, and prints out results. It also enables the user to enter, conveniently, single specifications (upper equals lower) by setting the parameter x to 0. The program splits these into the upper and lower specifications which it is designed to handle.

Subroutine OBJECT computes the objective function (5) and the gradients (6). Calculation of e as in (7) is performed through function subprogram ERROX. Subroutine APPROX is responsible for calculating y and ∇y as in (11). OPTIM1 performs optimization by the Fletcher method and OPTIM2 by the Fletcher–Powell method. See Table I for a summary of the features and options currently programmed and the parameters which must be specified by the user. Tables II and III expand some of the items of Table I to show the circuit elements presently incorporated.

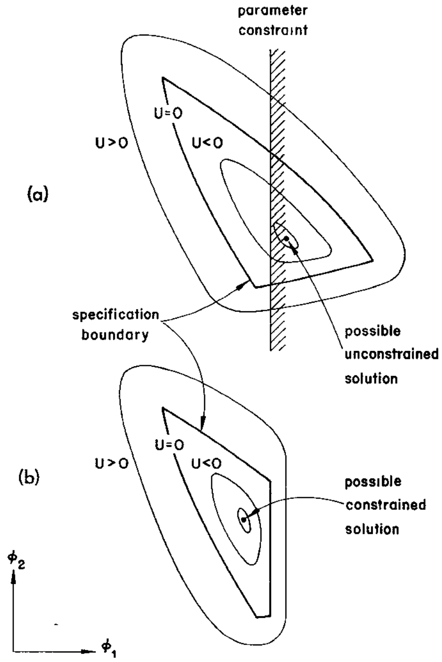

Fig. 1. (a) Example of contours of objective function (5) without constraints. (b) Example of contours of objective function (5) with one parameter constraint.

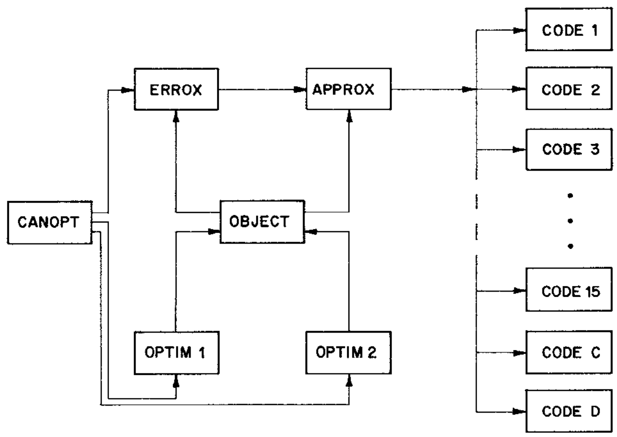

Fig. 2. The subprograms.

The Circuit Configuration

The package will optimize a cascade connection of the two-port elements listed in Tables II and III. Elements 1–15 may be connected in any order (sequentially from the source to the load) using as many as required or as many as the computer being used can accommodate.

The first six elements are one-parameter lumped elements. Their parameter values should be normalized by the user to his center frequency and source resistance, appropriately, as outlined in the Appendix.

The next four elements are three-parameter tuned circuits. They are characterized by resonant or antiresonant

TABLE I
SUMMARY OF FEATURES, OPTIONS, AND PARAMETERS REQUIRED

Features	Type	Options	Parameters
Objective Functions	Least pth	$1 < p < \infty$	Value of p for each of a specified number of optimizations Artificial margin Difference in objective functions for termination
Performance Specifications and Parameter Constraints	Upper (+1.) Lower (-1.) Single (0.)	Reflection coefficient (1) Insertion loss (2) Group delay (3) Parameter value (0)	Normalization frequency Number of points and constraints / Number of bands or intervals For each: Specification/constraint Weighting factor Type Option Frequency (sample point) or parameter / Lower and upper frequencies (band edges) Number of subintervals
Optimization Methods	Gradient	Fletcher (1) Fletcher-Powell (2)	Option Number of iterations allowed Estimate of lower bound on objective function Test quantities for termination
Circuit Elements	Cascaded Two-port	See Tables II and III	Number of elements Sequence of code numbers Parameter values Indicator for fixed or variable parameters Load resistance See Table III for C- and D-sections

TABLE II
ELEMENTS AND CODE NUMBERS

Element	Connection	Code	Parameters
inductor	series	1	inductance
	shunt	4	
capacitor	series	3	capacitance
	shunt	2	
resistor	series	5	resistance
	shunt	6	
resonant circuit	series	7	resonant frequency quality factor slope reactance
	shunt	10	
antiresonant circuit	series	9	antiresonant frequency quality factor slope susceptance
	shunt	8	
lossless transmission line	series shorted	11	length characteristic impedance
	shunt shorted	14	
	series open	13	
	shunt open	12	
	cascade	15	

TABLE III
ALL-PASS SECTIONS

Parameters	
All fixed or all variable (determined by one indicator)	Fixed
location of real zeros of C-sections	number of C-sections
location of real parts of zeros of D-sections location of imaginary parts of zeros of D-sections	number of D-sections
delay level	cutoff frequency

frequency, quality factor, and slope reactance or susceptance, as appropriate. Normalization as before must again be carried out by the user.

Elements 11–15 are two-parameter lossless transmission-line components. All are characterized by normalized length and characteristic impedance (see Appendix).

The all-pass sections (Table III) are treated in the same way as, for example, Kudsia [11]. Group delay relative to the delay level in nanoseconds is calculated.

Presently, the source and load are real constant resist-

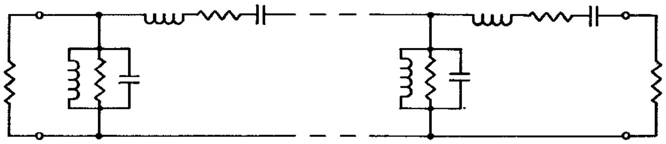

Fig. 3. Filter design example.

ances, the source being assumed to be unity. Frequency-dependent complex source and load impedances are readily accommodated or can be constructed or modeled, where appropriate, by defining suitable fixed components.

Additional Elements

The simplest way of handling two-port sections not in the present list is to replace an existing element, frequent use of which is not anticipated, by the desired element, preferably with the same number of parameters. In this case, only a few Fortran lines dealing with the *ABCD* matrix of the element and its sensitivites need be changed. If the parameters of the new element are not to be changed then sensitivity formulas are not necessary.

Adding elements is slightly more complicated in that more Fortran lines need adjustment. The procedure used for the existing elements can be readily followed. Distributed *RC* lines, nonuniform lines, and transistor amplifier stages are examples of two-ports that can be added.

Calculation of Functions

Subroutine CANOPT specifically reads actual frequencies (which are automatically normalized) for the response functions and actual parameter number for the upper and lower parameter bounds. The normalized frequency and parameter number become the values of the variable z. Presently, $z_u = 10$ and $n_r = 3$. Subroutine OBJECT ensures that the y and $\nabla y(z')$ are calculated only once for each distinct value of z'. When bands overlap or there are upper and lower specifications/constraints at any z' the objective function U may require the appropriate information but this need not be calculated twice.

Subroutine APPROX is organized in such a way that the first approximating function, namely, reflection coefficient (see Table I) and its derivatives ($r = 1$) are calculated by only one analysis of the original network at each frequency. One analysis of the original network and one analysis of the suitably terminated adjoint network (see Bandler and Seviora [4]) yields all the information needed for the evaluation of the second approximating function, namely, insertion loss in decibels (see Table I) and its derivatives ($r = 2$).

When $r = 3$ the group delay in nanoseconds is calculated for elements 1–15. Sensitivities are calculated by perturbation techniques since the small savings in computing time realized by the adjoint network method [5] did not seem to be worth the additional programming complexity. The group delay and sensitivities for the *C*-type and *D*-type sections are calculated from analytic expressions [11].

Additional response functions and constraints are readily accommodated in APPROX since these are identified by z'.

IV. Examples

Example 1

To illustrate the input data and output results we optimized the high-power output filter considered previously [7]. Fig. 3 shows the circuit diagram, and Fig. 4 shows actual user-specified data printed out by the package. This data defines the problem to be solved. Note that only the slope reactances and susceptances at 11 885.5 MHz are varied. The specifications to be met are 0.85 dB on the interval 11 843–11 928 MHz, which is the passband, and at least 66 dB at 11 700 MHz, at least 31 dB at 12 038 MHz, and at least 41 dB at 12 080 MHz. Observe that the response at the starting point is printed out (Fig. 5), results for $p = 2$ (Fig. 6), and results for $p = 1000$ (Fig. 7), where the starting point for $p = 1000$ is the best solution reached using $p = 2$.

Note the large number of significant figures to which the structure has symmetrical parameter values. Since the corresponding parameters were not forced to be symmetrical we feel this is a good indication of the efficiency of the package along with the very small execution times.

The results differ slightly from those presented previously [7] since an upper (passband) specification of 0.85 dB was not explicitly demanded before. The execution times are also significantly improved. The parameters were unconstrained here.

Example 2

This example concerns the design of an optimum group delay equalizer using one microwave *C*-section [7]. The given delay and the starting and optimized values of the parameters as well as the corresponding total relative group delay is shown in Table IV.

To give an indication of increased efficiency, the execution times obtained previously [7] for $p = 2$, 10, and 10 000 were about $\frac{1}{2}$ s, $1\frac{1}{4}$ s, and 10 s, as compared with the corresponding figures shown in Table IV.

V. Conclusions

An efficient user-oriented cascaded network optimization package suitable for microwave circuit design has been presented. It has been extensively tested on a wide range of problems of interest, in particular, to microwave engineers. Some of these are presented in this paper. Other tests using transmission-line filters including constraints reproduce efficiently the results presented elsewhere [6], [12]. The availability of the complete program and documentation is indicated in a computer program description in this issue [13]. It is hoped that the release of this work

INPUT DATA

NUMBER OF ELEMENTS 6

THE CALCULATED NUMBER OF PARAMETERS 18

CODE NUMBER	PARAMETER NUMBER	PARAMETER VALUE	PARAMETER CONDITION
8	1	1.00000000E+00	FIXED
8	2	6.00000000E+03	FIXED
8	3	2.40000000E+02	VARIABLE
7	4	1.00000000E+00	FIXED
7	5	6.00000000E+03	FIXED
7	6	4.20000000E+02	VARIABLE
8	7	1.00000000E+00	FIXED
8	8	6.00000000E+03	FIXED
8	9	5.70000000E+02	VARIABLE
7	10	1.00000000E+00	FIXED
7	11	6.00000000E+03	FIXED
7	12	4.60000000E+02	VARIABLE
8	13	1.00000000E+00	FIXED
8	14	6.00000000E+03	FIXED
8	15	4.50000000E+02	VARIABLE
7	16	1.00000000E+00	FIXED
7	17	6.00000000E+03	FIXED
7	18	2.10000000E+02	VARIABLE

NUMBER OF C SECTIONS 0

NUMBER OF D SECTIONS 0

LOAD RESISTANCE 1.00000000E+00

NUMBER OF FREQUENCY INTERVALS 1

NUMBER OF FREQUENCY POINTS 3

LOWER FREQUENCY	UPPER FREQUENCY	NUMBER OF SUBINTERVALS	SPECIFICATION		TYPE	WEIGHTING FACTOR
1.18430000E+04	1.19280000E+04	20	8.50000000E-01	INSERTION LOSS	UPPER	1.00000000E+00

FREQUENCY	SPECIFICATION		TYPE	WEIGHTING FACTOR
1.17000000E+04	6.60000000E+01	INSERTION LOSS	LOWER	1.00000000E+00
1.20380000E+04	3.10000000E+01	INSERTION LOSS	LOWER	1.00000000E+00
1.20800000E+04	4.10000000E+01	INSERTION LOSS	LOWER	1.00000000E+00

THE CALCULATED TOTAL NUMBER OF INTERVALS 4

CENTER FREQUENCY 1.18855000E+04

CUT-OFF FREQUENCY -0.

FLETCHER METHOD WILL BE USED

TEST QUANTITIES TO BE USED IN FLETCHER METHOD
1.00000000E-06
1.00000000E-06
1.00000000E-06
1.00000000E-06
1.00000000E-06
1.00000000E-06

ESTIMATE OF LOWER BOUND ON FUNCTION TO BE MINIMIZED -8.50000000E-01

DIFFERENCE IN THE OBJECTIVE FUNCTION IN SUCCESSIVE OPTIMIZATIONS -0.

ARTIFICIAL MARGIN -0.

NUMBER OF COMPLETE OPTIMIZATIONS 2

VALUES OF P
2
1000

MAXIMUM NUMBER OF ALLOWABLE ITERATIONS 500

INTERMEDIATE OUTPUT TO BE PRINTED EVERY 100 ITERATIONS

Fig. 4. Printout of data supplied by user for the filter design example.

RESPONSE AT THE STARTING POINT

FREQUENCY	INSERTION LOSS
1.18430000E+04	2.80927880E+00
1.18472500E+04	1.51942685E+00
1.18515000E+04	1.20840581E+00
1.18557500E+04	1.08791719E+00
1.18600000E+04	1.01489617E+00
1.18642500E+04	9.70925293E-01
1.18685000E+04	9.37555058E-01
1.18727500E+04	9.04943534E-01
1.18770000E+04	8.75963788E-01
1.18812500E+04	8.56762537E-01
1.18855000E+04	8.50229740E-01
1.18897500E+04	8.56757876E-01
1.18940000E+04	8.75928190E-01
1.18982500E+04	9.04840986E-01
1.19025000E+04	9.37368626E-01
1.19067500E+04	9.70606237E-01
1.19110000E+04	1.01418410E+00
1.19152500E+04	1.08630990E+00
1.19195000E+04	1.20471409E+00
1.19237500E+04	1.50305680E+00
1.19280000E+04	2.72971555E+00
1.17000000E+04	8.50941585E+01
1.20380000E+04	7.39043389E+01
1.20800000E+04	8.67580142E+01

Fig. 5. Printout of starting response for the filter design example.

OPTIMIZATION BY FLETCHER METHOD

ITERATION NUMBER	FUNCTION EVALUATIONS	TIME ELAPSED (SECONDS)	OBJECTIVE FUNCTION	VARIABLE VECTOR	GRADIENT VECTOR
0	1	2.13000000E-01	2.95288376E+00	2.40000000E+02	-5.32943688E-03
				4.20000000E+02	1.64374414E-02
				5.70000000E+02	2.45410552E-02
				4.60000000E+02	2.71682250E-02
				4.50000000E+02	9.69313766E-03
				2.10000000E+02	-6.08806447E-03
100	129	2.57450000E+01	-3.90208981E-02	1.95979158E+02	-8.37816041E-05
				2.74751101E+02	-7.08907354E-05
				3.68221455E+02	4.95481559E-05
				3.17541956E+02	-8.87958811E-05
				3.23166020E+02	2.95129547E-06
				1.60383670E+02	-5.53247502E-05
200	229	4.62640000E+01	-3.90354639E-02	1.93429470E+02	-1.78888931E-04
				2.68505001E+02	-2.47973744E-04
				3.61031344E+02	-6.69251700E-05
				3.16209775E+02	-1.42958306E-04
				3.23694142E+02	-1.11094646E-04
				1.61779570E+02	-2.43869076E-04
300	329	6.68600000E+01	-3.90527943E-02	2.00540368E+02	-1.18238968E-04
				2.66835435E+02	-2.84260846E-04
				3.49815658E+02	-1.11070774E-04
				3.18212895E+02	-7.91521478E-05
				3.18684312E+02	-1.29238318E-04
				1.64807744E+02	-2.68698235E-04

IEXIT= 1CRITERION FOR OPTIMUM HAS BEEN SATISFIED

OPTIMUM SOLUTION

ITERATION NUMBER	FUNCTION EVALUATIONS	EXECUTION TIME (SECONDS)	OBJECTIVE FUNCTION	VARIABLE VECTOR	GRADIENT VECTOR
348	377	7.67540000E+01	-4.17714294E-02	1.83517813E+02	-1.54111883E-11
				2.94466089E+02	-1.22887240E-11
				3.29204539E+02	-9.32953065E-12
				3.29204540E+02	-9.19923989E-12
				2.94466090E+02	-1.00157384E-11
				1.83517814E+02	-1.42720548E-11

VALUE OF Q -2

(a)

FINAL RESPONSE OF THE CIRCUIT

FREQUENCY	INSERTION LOSS
1.18430000E+04	7.25264483E-01
1.18472500E+04	6.82511513E-01
1.18515000E+04	6.70141364E-01
1.18557500E+04	6.66440677E-01
1.18600000E+04	6.56996345E-01
1.18642500E+04	6.39311402E-01
1.18685000E+04	6.18584912E-01
1.18727500E+04	6.01208443E-01
1.18770000E+04	5.90386800E-01
1.18812500E+04	5.85488731E-01
1.18855000E+04	5.84241531E-01
1.18897500E+04	5.85487763E-01
1.18940000E+04	5.90376062E-01
1.18982500E+04	6.01152591E-01
1.19025000E+04	6.18471073E-01
1.19067500E+04	6.39129468E-01
1.19110000E+04	6.56815650E-01
1.19152500E+04	6.66348608E-01
1.19195000E+04	6.70037096E-01
1.19237500E+04	6.81831092E-01
1.19280000E+04	7.23019460E-01
1.17000000E+04	6.62673744E+01
1.20380000E+04	5.45704943E+01
1.20800000E+04	6.79881381E+01

(b)

Fig. 6. Printout of results for $p = 2$ for the filter design example.

```
OPTIMIZATION BY FLETCHER METHOD
--------------------------------
ITERATION  FUNCTION     TIME ELAPSED     OBJECTIVE          VARIABLE VECTOR      GRADIENT VECTOR
NUMBER     EVALUATIONS  (SECONDS)        FUNCTION

   0          1         2.05000000E-01   -1.24735517E-01    1.83517813E+02       5.57082060E-04
                                                            2.94466089E+02       1.76834715E-03
                                                            3.29204539E+02       1.55160336E-04
                                                            3.29204540E+02       1.55160353E-04
                                                            2.94466090E+02       1.76834716E-03
                                                            1.83517814E+02       5.57082059E-04

 100        133         2.72990000E+01   -1.45592932E-01    1.94484866E+02      -5.39064065E-03
                                                            2.83370971E+02      -4.78752641E-03
                                                            3.23462701E+02      -3.69865653E-03
                                                            3.23462701E+02      -3.69865653E-03
                                                            2.83370971E+02      -4.78752640E-03
                                                            1.94484867E+02      -5.39064063E-03

IEXIT= 4EITHER OF THE FOLLOWING THINGS HAS HAPPENED
        1. EPS CHOSEN IS TOO SMALL
        2. GRADIENTS ARE NOT CORRECT
        3. MATRIX H GOES SINGULAR


RESULTS AT LAST ITERATION
--------------------------
ITERATION  FUNCTION     EXECUTION TIME   OBJECTIVE          VARIABLE VECTOR      GRADIENT VECTOR
NUMBER     EVALUATIONS  (SECONDS)        FUNCTION

 118        152         3.12790000E+01   -1.45706223E-01    1.95673764E+02      -3.30610069E-08
                                                            2.83558905E+02      -3.08131929E-08
                                                            3.21527739E+02      -2.32158352E-08
                                                            3.21527739E+02      -2.32166530E-08
                                                            2.83558905E+02      -3.08092901E-08
                                                            1.95673764E+02      -3.30569529E-08


VALUE OF Q        -1000
```

(a)

```
           FINAL RESPONSE OF THE CIRCUIT
           ------------------------------

   FREQUENCY                  INSERTION LOSS
1.18430000E+04                7.04264805E-01
1.18472500E+04                6.88663687E-01
1.18515000E+04                6.98915381E-01
1.18557500E+04                7.03956914E-01
1.18600000E+04                6.90308582E-01
1.18642500E+04                6.61381501E-01
1.18685000E+04                6.28680163E-01
1.18727500E+04                6.02619865E-01
1.18770000E+04                5.87370419E-01
1.18812500E+04                5.81044943E-01
1.18855000E+04                5.79586006E-01
1.18897500E+04                5.81043773E-01
1.18940000E+04                5.87355949E-01
1.18982500E+04                6.02553197E-01
1.19025000E+04                6.28505020E-01
1.19067500E+04                6.61087769E-01
1.19110000E+04                6.90013463E-01
1.19152500E+04                7.03889327E-01
1.19195000E+04                6.99177454E-01
1.19237500E+04                6.88774947E-01
1.19280000E+04                7.02855331E-01
1.17000000E+04                6.61462898E+01
1.20380000E+04                5.43471449E+01
1.20800000E+04                6.78782997E+01

        EXECUTION TIME IN SECONDS= 110.53300
```

(b)

Fig. 7. Printout of results for $p = 1000$ for the filter design example.

will stimulate practical application of our ideas and their incorporation into future general design programs [14], [15]. It should be noted that interactive versions of CANOPT can be readily written by very straightforward modifications to the present program.

Appendix

Normalization of Parameter Values

To illustrate the normalization process we may consider the following examples. For element 1, a series inductance, we consider a parameter L_n such that $\omega_n L_n$, where ω_n is the normalized frequency, yields the desired reactance in ohms. Thus, if the normalization frequency is 3 GHz, the inductance 2 nH, then $L_n = 12\pi$. For elements 11–15, for example, we consider a length l_n such that $\tan(\pi/2)\omega_n l_n$ yields the desired value of the frequency variable for lossless transmission lines.

Sensitivity Expressions

Sensitivity expressions for elements 1–15 have either been published [4], [5] or are readily obtainable using a procedure similar to the example which follows. For element 7, for example, the quantity $\mathbf{I}^T \Delta \mathbf{Z}^T \hat{\mathbf{I}}$ (see Bandler and Seviora [4, table I]) is given by

$$I\hat{I}\Delta Z = \left[\frac{-I\hat{I}X'\omega_R}{2Q^2}\right]\Delta Q + \left[I\hat{I}\left(\frac{X'}{2Q} - j\,\frac{\omega_R X'}{\omega}\right)\right]\Delta\omega_R + \left[I\hat{I}\,\frac{Z}{X'}\right]\Delta X' + \left[I\hat{I}j\,\frac{X'}{2}\left(1 + \frac{\omega_R^2}{\omega^2}\right)\right]\Delta\omega$$

where Z is the impedance of the element, I is the original network current and $\hat{I}$ is the adjoint network current through it, ω_R is the resonant frequency, Q is the quality factor, and X' the slope reactance at $\omega = \omega_R$. The expressions in square brackets are appropriate sensitivity expressions with respect to Q, ω_R, X', and ω, respectively.

TABLE IV
GROUP DELAY EQUALIZER DESIGN USING THE FLETCHER METHOD

			Parameters		
	Value of p	Starting point	2	10†	10,000
	σ [11]	340	349.27	365.94	368.77
	d [11]	86	86.64	87.68	87.75
Frequency (MHz)	Given delay (nsec)		Total relative group delay (nsec)		
7,976	69.03	4.11	3.53	2.56*	2.49*
7,977	62.61	0.30	-0.19	-0.99	-1.04
7,978	58.03	-1.48	-1.84	-2.42*	-2.43*
7,979	54.79	-1.83	-2.03	-2.33	-2.29
7,980	52.52	-1.28	-1.31	-1.29	-1.19
7,981	50.79	-0.52	-0.37	-0.02	0.13
7,982	49.98	0.56	0.85	1.48	1.69
7,983	49.49	1.09	1.47	2.26*	2.49*
7,984	49.49	1.08	1.46	2.26*	2.49*
7,985	49.97	0.54	0.83	1.46	1.66
7,986	50.95	-0.37	-0.23	0.12	0.28
7,987	52.50	-1.32	-1.35	-1.33	-1.23
7,988	54.75	-1.89	-2.09	-2.39	-2.35
7,989	57.99	-1.54	-1.90	-2.48*	-2.49*
7,990	62.55	0.22	-0.27	-1.07	-1.12
7,991	68.94	4.01	3.43	2.46*	2.39*
	Maximum error	4.11	3.53	2.56	2.49
	Execution time (sec)	0	0.4	0.8	6

† Optimization for $p = 10$ was started at the optimum for $p = 2$.
* Extrema in the response.

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