Computer Optimization of a Stabilizing Network for a Tunnel-Diode Amplifier

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Abstract—**Computer-aided direct search is a useful and flexible method of optimizing noncommensurate networks or networks for which exact synthesis theories culminating in some particular response are not available. It can accommodate network parameter constraints and unconventional performance specifications and is not accompanied by problems of component realizability.**

A simple form of direct search is applied in this paper to the design of a microwave network whose performance is optimized within certain specifications. The network is a stabilizing and biasing arrangement for a tunnel-diode amplifier operating in a reduced height S-band rectangular waveguide, and takes the form of a coaxial-line band-stop filter. Parameter constraints are inherent to the problem so they are taken into account. The requirements of stability and low noise broadband amplification in conjunction with the external circuitry impose nonsymmetrical response restrictions on the input resistance and reactance of the network. At the same time it is required to minimize the square of the input reactance at selected frequencies. No available exact synthesis of band-stop filters can solve this problem as presented here.

I. Introduction

Swept frequency techniques are often applied to improving the performance of a broadband microwave network designed by an approximate method. The procedure involves monitoring the response pictorially, possibly adjusting those parameters that are variable in an arbitrary manner to obtain an initial or base setting, followed by some systematic adjustment procedure to converge on the most desirable response. It may, however, not be a practical proposition to fabricate an experimental prototype with all the key parameters variable; alternatively the response under consideration might not lend itself experimentally to direct display. Possibly the components of the network may not be commensurate, or the response specifications and the parameter constraints may be severe or unconventional, or experimental data must be taken into account. In such situations an exact synthesis is almost certainly out of the question.

This paper describes the design of a microwave network, the specifications for which are such that a simple direct search solution[1]–[8] by digital computer, analogous to a swept frequency technique, appears most attractive. The network is a stabilizing and biasing arrangement for a tunnel-diode amplifier operating in a rectangular waveguide, and takes the form of a coaxial-line band-stop filter. The exact insertion loss synthesis of a band-stop filter[9]–[11] requires a performance specification expressed in terms of attenuation versus frequency. Available theories are restricted to commensurate transmission-line (or waveguide) lengths and result in responses that are symmetrical about the center frequency. No independent control can be exercised over input resistance and reactance as a function of frequency, and there is no guarantee that the synthesized structure will have practically realizable element values. In the present problem critical restrictions are imposed nonsymmetrically both on the input resistance and on the input reactance at discrete, widely spaced frequencies. Insertion loss is not the primary concern.

The example described here is particular but the method of solution is very general and should be of wider interest. Almost no background in programming is necessary to follow the main arguments of this paper.

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II. Optimization by Direct Search

One may specify the synthesis of a microwave filter or matching network in the form of minimizing a function of several variables subject to certain parameter constraints and performance specifications. This objective function to be minimized will be

$$U = U(\boldsymbol{\phi}, \boldsymbol{\psi}) = \sum_{j=1}^{n} g_k(\boldsymbol{\phi}, \psi_j) \tag{1}$$

where

$$\boldsymbol{\phi} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_m \end{bmatrix} \tag{2}$$

and

$$\boldsymbol{\psi} = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_n \end{bmatrix} \tag{3}$$

where $\boldsymbol{\phi}$ represents the variable parameters of the network and $\boldsymbol{\psi}$ represents the test points, e.g., particular frequencies. g_k is some chosen kth function of $\boldsymbol{\phi}$ and ψ evaluated at ψ_j, and which involves any desired response or error criterion. The parameter constraints might be given as

$$\phi_{jl} \leqq \phi_j \leqq \phi_{ju}, \qquad j = 1,2, \cdots, m \tag{4}$$

where ϕ_{jl} and ϕ_{ju} are the lower and upper bounds, respectively, of the jth parameter. Possibly the value of one parameter may be influenced by one or more of the rest, i.e.,

$$\phi_j = \phi_j(\phi). \tag{5}$$

Performance specifications could be stated in the form

$$g_{kjl} \leqq g_k(\phi, \psi_j) \leqq g_{kju} \begin{cases} j = 1, 2, \cdots, n \\ k = 1, 2, 3, \cdots \end{cases} \tag{6}$$

where g_{kjl} and g_{kju} are the lower and upper bounds, respectively, of $g_k(\phi, \psi_j)$. k may or may not be the same as in (1).

"Direct search" describes the sequential examination of trial solutions in which each trial solution is compared with the "best" obtained up to that time, with a strategy for deciding what the next trial solution will be, based on the experience gained from the previous results. It relies only on the success or failure of a "move," i.e., a change in parameter values, and not on partial derivatives which are involved in steepest descent—these depend in any case on the scaling of the parameters. It was proposed formally by Hooke and Jeeves[1] who have successfully applied it to a variety of problems including constrained maximization or minimization of functions.

Many variations of direct search are possible, from the examination of trial and error solutions to the more sophisticated "pattern search" devised by Hooke and Jeeves. It has been found remarkably efficient[1],[2],[4] and has solved problems unassailable by classical methods, particularly pattern search, possibly due to its ability to follow along sharp "valleys" in the response hypersurface. The search employed would clearly depend on the complexity of the problem with regard, for example, to the dimensionality and the presence of pitfalls such as local optima or sharp valleys.

A successful move in the present context is one in which the current value of U is less than any previous value, while (4) through (6) are not violated. For transmission-line networks, the range of practical values given by (4) can be quite small, e.g., reasonable characteristic impedances usually extend from 15 to 150 ohms.[10] Thus, a relatively flexible optimization process such as direct search is called for, particularly because its simple repetitive nature makes it well suited for use on digital computers.

III. The Present Problem

It is required to design a coaxial-line stabilizing and biasing network for an S-band rectangular waveguide tunnel-diode amplifier (TDA). The tunnel-diode (TD) is to be mounted symmetrically in a reduced height waveguide section, its terminals effectively connected to the broad faces of the waveguide. A block diagram of the TDA is shown in Fig. 1. The stabilizing network is to render the TD as far as possible inactive from the point of view of the external circuitry above the amplification band. Within this band, 2.6 to 3.3 GHz, it should present ideally zero impedance to the TD, and below 2.6 GHz it must assist in stabilization. Henoch and Kvaerna[12] and Bandler[13] have described the investigation of TD stability through real frequency immittance plots from 0 to f_R, the resistive cutoff frequency.

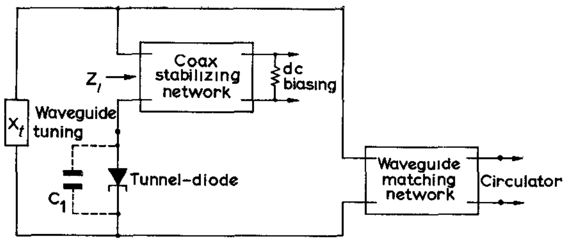

Fig. 1. Schematic of proposed rectangular waveguide tunnel-diode amplifier.

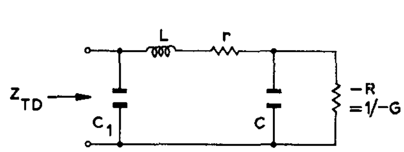

Fig. 2. Tunnel-diode equivalent circuit.

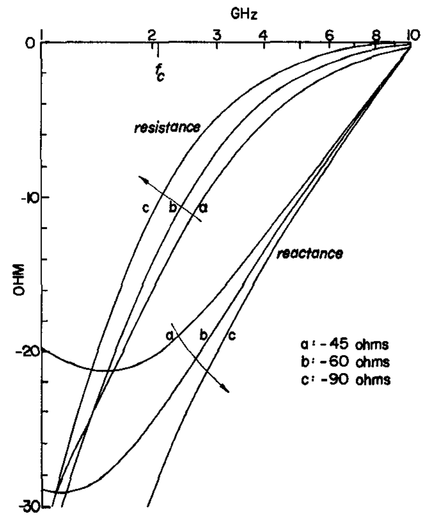

Fig. 3. $Z_{\mathrm{TD}}(j\omega)$ for three values of R when $C=2$ pF, $r=1$ ohm, $L=0.12$ nH and $C_1=0.3$ pF. Curves (a) $R=45$ ohms, $f_R=11.7$ GHz, $f_X=10.1$ GHz; curves (b) $R=60$ ohms, $f_R=10.2$ GHz, $f_X=10.2$ GHz; curves (c) $R=90$ ohms, $f_R=8.3$ GHz, $f_X=10.2$ GHz. f_R and f_X are the resistive cutoff and self-resonant frequencies, respectively. C_1 is not included in calculating f_X. But the inequality given by (1) of Bandler[13] concerning the potential stability of TD's is satisfied by this TD, thus simplifying the stability analysis. Note that the equivalent circuit parameters are assumed to be frequency independent.

General Considerations

The parameters of the TD for small signals (Fig. 2) are:

$$R_{\min} = 45\ \Omega, \qquad C = 2\ \mathrm{pF}, \qquad r = 1\ \Omega,$$
$$L = 0.12\ \mathrm{nH}, \qquad C_1 = 0.3\ \mathrm{pF},$$

where $R_{\min}$ is the minimum value of R, the negative resistance. A plot of $Z_{\mathrm{TD}}(j\omega)$, the TD impedance, for three values of R is shown in Fig. 3. Because $|R_{\mathrm{TD}}(\omega)|$ decreases with frequency, $R_i(\omega)$, the positive stabilizing resistance, should be less at the high-frequency end than at the low-frequency end of the amplification band so as not to deteriorate the gain and noise figure. For stability, less $R_i(\omega)$ is required anyway to cancel $R_{\mathrm{TD}}(\omega)$. $|X_i(\omega)|$ must also be restricted, particularly at the high-frequency (inductive) end before $R_i(\omega)$ cuts the TD off.

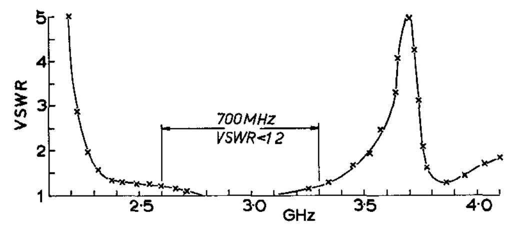

Fig. 4. The circulator VSWR.

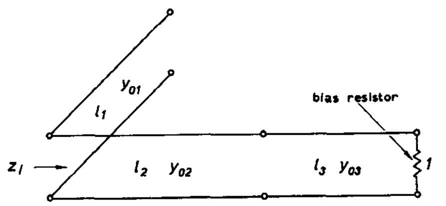

Fig. 5. Prototype of the stabilizing network for the TDA of Fig. 1.

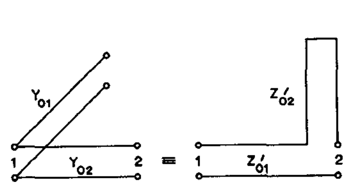

Fig. 6. Kuroda identity for the OC shunt line to SC series line transformation. $Z_{01}'=1/(Y_{01}+Y_{02})$, $Z_{02}'=Y_{01}Z_{01}'/Y_{02}$.

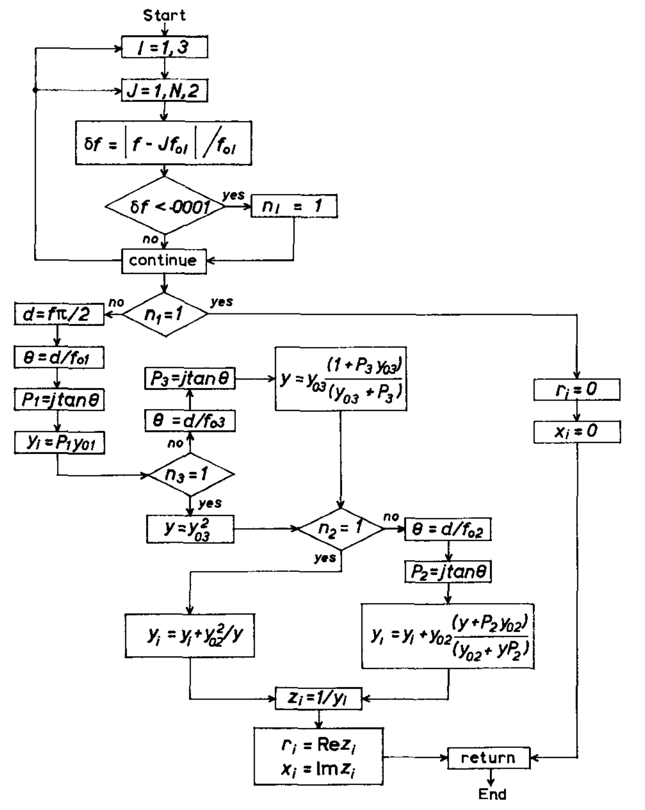

Fig. 7. A flow diagram for subroutine STAG[2] (see the Appendix) written to calculate $z_i(=Z_i/R_b)$ for Fig. 5 at any specified frequency f.

The circulator response (Fig. 4) must also be taken into account. High circulator VSWR, e.g., at 3.7 GHz, could under certain phase conditions cause instability,[12],[13] but not if $R_i(\omega)+R_{\mathrm{TD}}(\omega)>0$. Also important is f_c, the waveguide cutoff frequency, here 2.078 GHz. An infinitely long ideal waveguide has a predictable response around f_c but a practical waveguide test bench can produce unpredictable spurious reflections in that region.

A suitable prototype stabilizing network is shown in Fig. 5. If it had commensurate length sections, $R_i(\omega)$ and $X_i(\omega)$ would be symmetrical and antisymmetrical, respectively, about f_o, the center frequency.[9]–[11] This is not desirable, indeed some control over both $R_i(\omega)$ and $X_i(\omega)$ is necessary[1] in accordance with the design and performance specifications already outlined. Thus, the line lengths l_1, l_2, and l_3 (consequently f_{o1}, f_{o2}, and f_{o3}) as well as the characteristic admittances (normalized to the bias conductance) y_{01}, y_{02}, and y_{03} are taken as variable. Since it is not physically feasible here to have an open circuit (OC) shunt line very close to the TD it must be transformed into a short circuit (SC) series line distance l_1 from the TD by Kuroda's identity[10] shown in Fig. 6 so that

$$l_2 \geqq l_1. \tag{7}$$

[1] Young has discussed the interrelation of input resistance and input reactance of minimum reactance networks.[14] The reactance of such networks is determined by the resistance.

Fig. 7 is a flow diagram for a subroutine called STAG for calculating $Z_i(j\omega)$.[2] It defines the variables used in the computation and summarizes the transmission-line transformations involved. (If further cascaded sections are required, it is convenient to use the iterative transformation shown in the Appendix.)

Specific Requirements

The specific requirements of the problem are given in Tables I and II in terms of (1) to (6). Fig. 8 illustrates the performance specifications S_1 to S_9 and the test frequencies f_1 to f_6. The rest of this section is devoted to explaining these tables and Fig. 8.

A suitable objective is to minimize the square of the input reactance at the edges and just above the amplification band, viz.,

$$U=\sum_{j=1}^{6} w_j\{X_i^2(\omega)\}\,\big|_{f=f_j} \qquad w_j=\begin{cases}0 & j=1,5\\ 1 & j=2,3,4,6\end{cases} \tag{8}$$

Column 2 of Table I illustrates this. Referring to Fig. 8 we observe that $f_2 \leqq f_{01} \leqq f_3$ automatically ensures that $Z_i(j\omega)=0$ somewhere in the band; while minimizing X_i^2 at the band edges is to reduce TDA bandwidth degradation and mini-

[2] The part between "start" and "continue" determines when $f\pi/2f_{oI}$ is sufficiently close to $(2J-1)\pi/2$, where J is an integer, so that the Ith line can be taken as an odd number of quarter wavelengths long at f. This avoids computation of excessive values of $\tan\theta$.

TABLE I

THE SPECIFIC REQUIREMENTS OF THE PRESENT CONSTRAINED OPTIMIZATION PROBLEM

Eq.	1	2	3	4		5	6					
j	g_1	ϕ_j	ψ_j	ϕ_{jl}	ϕ_{ju}	ϕ_j	g_2	g_{2jl}	g_{2ju}	g_3	g_{3jl}	g_{3ju}
1	0	y_{01}	f_1	$\frac{1}{6}$	$\frac{10}{6}$	$5 \geqq y_{01}+y_{02} \geqq .5$		S_1	—		—	—
2	X_i^2	y_{02}	f_2	$\frac{1}{3}$	$\frac{10}{3}$	$10 \geqq \dfrac{y_{02}(y_{01}+y_{02})}{y_{01}} \geqq 1$		—	S_2		—	S_3
3	X_i^2	y_{03}	f_3	$\frac{1}{3}$	$\frac{10}{3}$	—		—	S_4		—	S_5
							R_i			$\lvert X_i \rvert$		
4	X_i^2	f_{01}	f_4	f_2	f_3	$f_{01} \geqq f_{02}$		S_6	—		—	S_7
5	0	f_{02}	f_5	$\simeq \frac{1}{2} f_2$	f_3	$f_{02} \leqq f_{01}$		S_8	—		—	—
6	X_i^2	f_{03}	f_6	$\simeq \frac{1}{2} f_2$	—	—		—	—		—	S_9

TABLE II

VALUES ASSIGNED TO THE TEST FREQUENCIES AND PERFORMANCE SPECIFICATIONS

j	f_j (GHz)	Specifications: Values (Ω)		Specifications: Requirements
1	$f_1=f_c=2.078$	$S_1=16$		$S_1+R_{TD}>0$
2	$f_2=2.6$	$S_2=1.6$	$S_3=8$	S_2 to S_5 as small as possible; actual values found from trial run to avoid unrealizable specifications
3	$f_3=3.3$	$S_4=.32$	$S_5=3.2$	
4	$f_4=3.7$	$S_6=6.8$	$S_7=8.8$	$S_6+R_{TD}>0$ $S_7+X_{TD}<0$
5	$f_5=4.2$	$S_8=7.05$		$S_8+R_{TD}>0$
6	$f_6=3.5$		$S_9=6.8$	$S_9+X_{TD}<0$

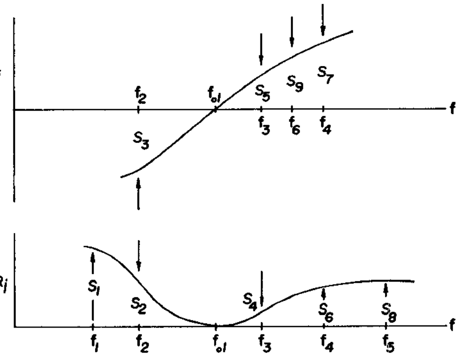

Fig. 8. Specifications to shape the input resistance and reactance response of the stabilizing network.

mizing X_i^2 above the band will improve conditional stability: (8) is a compromise to maintain the latter at some expense to the former.

Columns 3 and 4 list the variables and test frequencies, respectively.[3] The admittance constraints of columns 5 to 7 restrict (see Fig. 6 for definitions) Z_{01}', Z_{02}', Z_{02}, and Z_{03} in ohms to[4]

$$8 \leqq Z_{01}' \leqq 80$$
$$4 \leqq Z_{02}' \leqq 40$$
$$12 \leqq Z_{02} \leqq 120$$
$$12 \leqq Z_{03} \leqq 120,$$

the bias resistance R_b being 40 ohms. The Kuroda transformation determines column 7. The resonant frequencies are also constrained, in particular f_{o1}, since $Z_i(j\omega)$ is to be zero between f_2 and f_3. Lower limits for f_{o2} and f_{o3} are determined by the maximum tolerable overall length of the network; the upper limits are unimportant except for f_{o2}, since $f_{o2} \leqq f_{o1}$ and hence, ultimately, $f_{o2} \leqq f_3$.

Values assigned to f_1 to f_6 and S_1 to S_9 that fulfill the aims set out in this section[5] are shown in Table II. That a potentially stable broad-band waveguide TDA can result from a combination of the responses of Figs. 3 and 8 and proper inductive tuning (see Fig. 1) can be checked by application of the theory of a previous paper by Bandler.[13] (Fig. 10 of that paper illustrates to what the present situation should correspond.)

[3] It is coincidental that the number of parameters equals the number of test frequencies.

[4] The SC line was to be dielectric filled in order to shorten its physical length, hence a lower characteristic impedance Z_{02}' was considered tolerable. Note also that Kuroda's identity dictates that $Z_{02}=Z_{01}'+Z_{02}'$.

[5] The values of S_2 to S_5 could be related to TDA noise figure and bandwidth deterioration if required, but this was not explicitly done here.

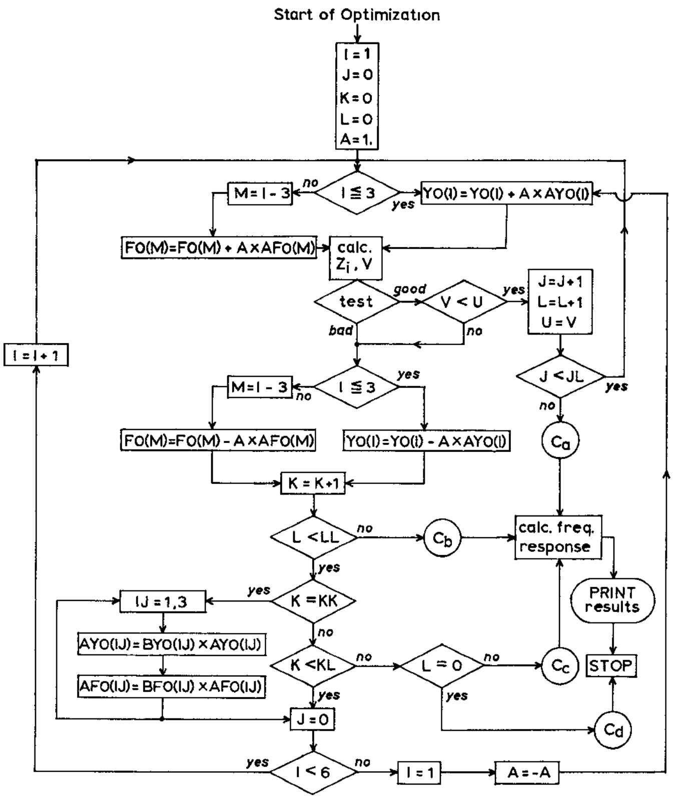

Fig. 9. A flow diagram of the optimization loops of the stabilizing network program. YO and FO correspond to y_0 and f_o, respectively. Z_i is calculated by STAG. Calculation of V and "test" is done by TEST. FREQ instructs the calculation of the frequency response of Z_i by STAG, and the printing out of the results in tabular form, and graphically through XYPLOT. Refer to the Appendix for nomenclature and further details.

IV. Computer Solution and Experimental Realization

The Appendix provides a brief description of the five constituent parts of the computer program written to carry out the direct search to solve the problem formulated in Section III. Also given is an algorithm which describes the trial and error portion, for which all the parameters of the network are arbitrarily changed within the constraints listed in Table I. When the loops are exhausted the best solution obtained, i.e., the one having the minimum U, within the specifications of Table II is used as the initial solution to the optimization process of Fig. 9. This figure is explained and the nomenclature used is clarified in the Appendix.

No pitfalls were encountered in the running of the program; therefore, a more sophisticated direct search technique was not required. The computed results are

$$\begin{aligned} y_{01} &= 1.417001 \quad & f_{o1} &= 3.100200 \text{ GHz} \quad & l_1 &= 2.417526 \text{ cm} \\ y_{02} &= 2.111000 \quad & f_{o2} &= 2.699709 \text{ GHz} \quad & l_2 &= 2.776155 \text{ cm} \\ y_{03} &= 2.099800 \quad & f_{o3} &= 2.899991 \text{ GHz} \quad & l_3 &= 2.584426 \text{ cm.} \end{aligned}$$

Fig. 10 is a plot of $Z_i(j\omega)$ using these values. In the final stages of the optimization when the parameter increments were reduced the sixth decimal place was allowed to vary. None of the limits imposed on the number of iterations around the optimization loops (Fig. 9 and Appendix) was, however, reached. Note that $l_2 > l_1$. It is further readily verified that all specifications and constraints have been met.

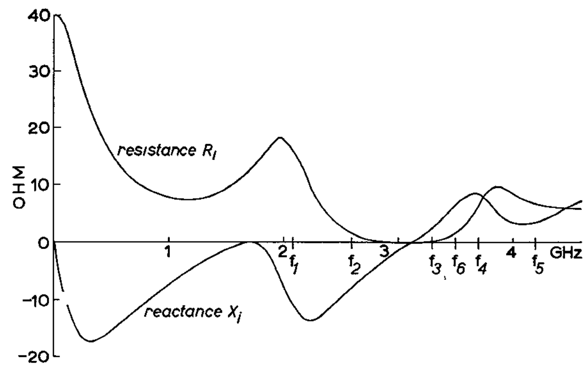

Fig. 10. Input impedance $Z_i(j\omega)$ to the stabilizing network for the design parameter values obtained by computer.

It is no coincidence that y_{03}, f_{o1}, f_{o2}, and f_{o3} differ only slightly from 2.1, 3.1, 2.7, and 2.9, respectively. Two principal reasons for this are proposed. First, the OC shunt line at the input (Fig. 5) is the predominating element in the vicinity of resonance, since

$$Z_i \simeq \frac{1}{jY_{01} \tan \dfrac{\pi f}{2f_{o1}}} \simeq 0. \tag{9}$$

Secondly, once trial and error values assigned to the parameters satisfy the specifications, y_{01} the first parameter incremented (as seen from Fig. 9) tries as far as possible to make up for the others before they too are incremented.

These arguments suggest the possibility of obtaining a performance fulfilling the specifications with, for example, $l_3 = 0$. This can be effected by setting $y_{03} = 1$ and not permitting y_{03} or f_{o3} to be incremented. The program itself is unaffected by this change. In the present case, however, y_{02} differs from y_{03} by about 0.5 percent so that y_{02}, for practical purposes, can be taken as equal to y_{03}.

A printout of $Z_i(j\omega) + Z_{TD}(j\omega)$ instructed by a subroutine called CHART 1 described in a previous paper[15] is shown in Fig. 11. $Z_i(j\omega)$ is taken from Fig. 10; $Z_{TD}(j\omega)$ is taken from Fig. 3 for $R = R_{min} = 45$ ohms. A number of observations should be made. The impedance has a capacitive component while its net resistance is negative; the net resistance is positive around f_c. (Compare this with Fig. 7(b) of Bandler.[13]) At the upper end of the amplification band the plot leaves the negative resistance chart perpendicularly to the unit circle, but the implications of this phenomenon are not at present understood.

A photograph of the assembled stabilizing network constructed with coaxial-lines[6] is reproduced in Fig. 12. The realization has

[6] The actual TDA failed at first to operate in a stable manner. The fault was traced to a substantial series inductance associated with the TD and its waveguide environment[16] in accordance with Getsinger,[17] and which was overlooked. A stability analysis of the whole system was made, and a slightly modified stabilizing network was fabricated which led to stable operation.

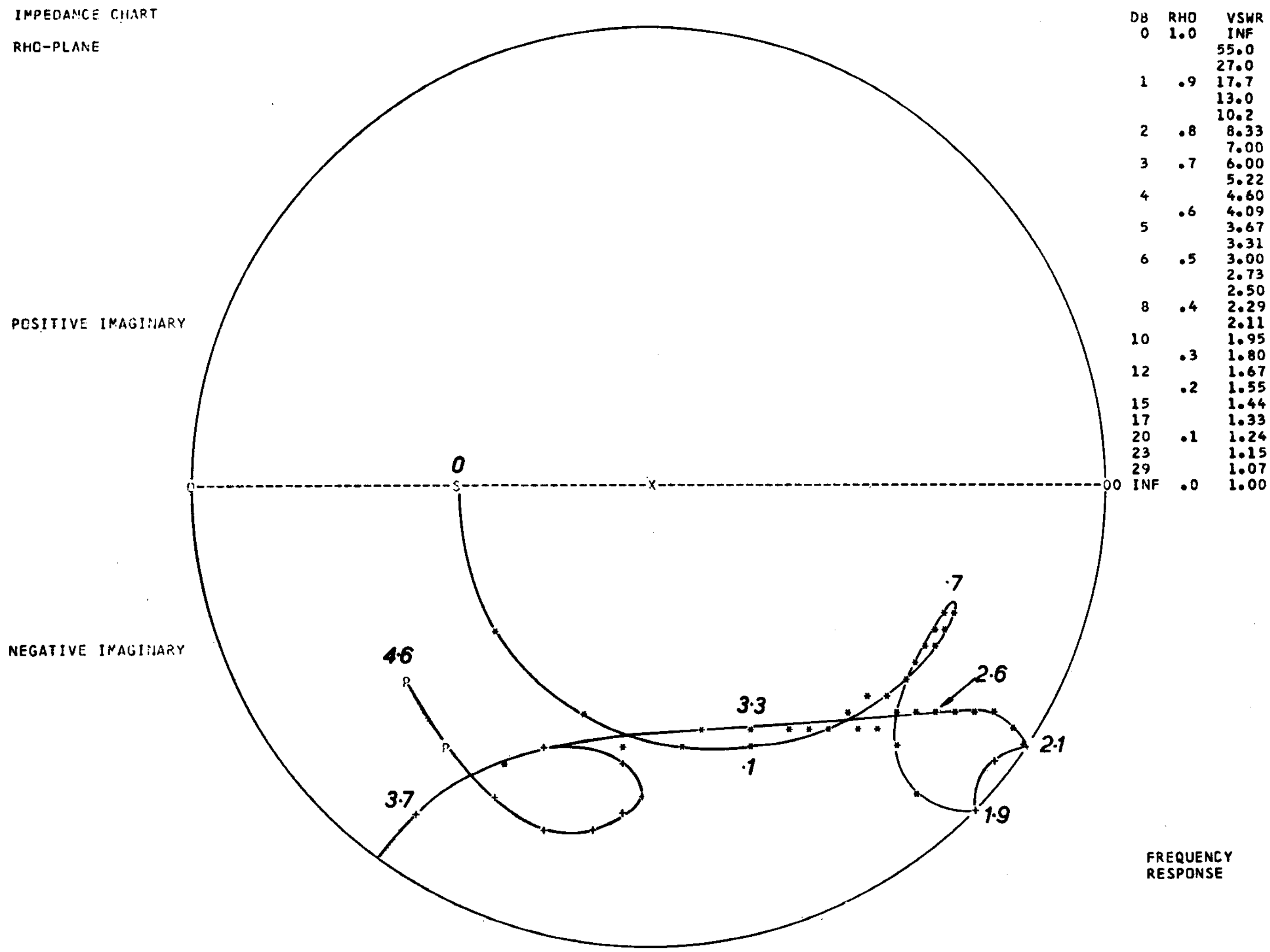

Fig. 11. Plot of $Z_i(j\omega)+Z_{TD}(j\omega)$ normalized to 10 ohms. The frequencies are: 0 to 175 MHz in steps of 25 MHz; 200 MHz to 4.6 MHz in steps of 100 MHz. Quantization inevitably caused some overwritten points, hence they are missing. The * indicates negative resistance and the + indicates postive resistance regions.

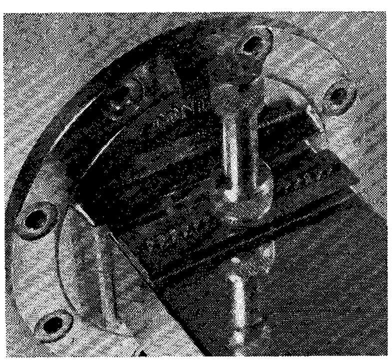

Fig. 12. The assembled stabilizing network shown connected to the waveguide amplifier.

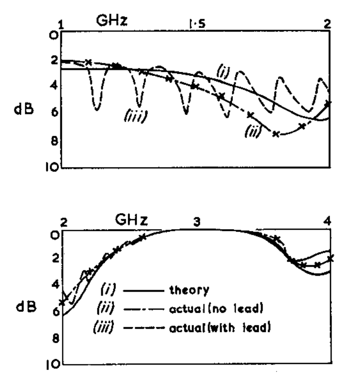

Fig. 13. The experimental response of the stabilizing network. Curve (i) is derived from theory, curve (ii) is the experimental curve with the miniature coaxial cable removed, and curve (iii) the corresponding one with this cable attached (see text).

$Z_{01}' = 11.338$ ohms $\quad Z_{02}' = 7.611$ ohms

$Z_{02} = 18.948$ ohms $\quad Z_{03} = 19.049$ ohms.

The SC series line (Z_{02}') is physically shortened by filling it with a dielectric with $\epsilon_r = 5$. The terminating resistor is a disc-shaped resistive-film card of about 800 ohms per square. The center conductor of a miniature 50-ohm coaxial line connects the center rod of the stabilizing network to the dc biasing network.

A special coaxial transition[16] was made to connect the stabilizing network to a 50-ohm coaxial-line reflectometer test bench through an N-type connector. The return loss of the stabilizing network was measured in the usual way and is depicted in Fig. 13. The ripples in curve (iii) are 160 MHz apart and are due to the miniature coaxial line which was effectively $\lambda/2$ long at this frequency. When comparing the curves it should be remembered that discontinuity effects are not theoretically accounted for. Using a coaxial slotted line, the standing-wave minimum at 3.1 GHz was found, as required, to lie at the input which was also the reference plane. This information permits the measured return loss to be reconciled with the theoretical return loss derived from $Z_i(j\omega)$.

V. Conclusions

A rather straightforward direct search technique has been successfully applied to the constrained optimization of a coaxial-line microwave network. It resembles the experimental swept-frequency technique, but any desired parameter can be varied and any desired response can be observed and optimized. Unlike true network synthesis in which a mathematical expression describing the desired response is used to generate the configuration and its parameter values, this technique is not accompanied by problems of the physical realizability of the components, and is not limited by the presence of noncommensurate components. The rate of convergence towards a final solution and the avoidance of local optima (not studied here by the author) may, on the other hand, be significant. Both of these problems are, of course, inherent in experimental optimization. It is true to say, however, that program efficiency is no longer of prime concern to modern high-speed computers. Furthermore, a local optimum that meets all the parameter constraints and performance specifications is better than a global optimum that does not.

It is anticipated that the next few years will see the publication of many papers devoted to microwave network design by direct search methods of varying sophistication. Emery and O'Hagan's paper on matching networks for microwave transistor amplifiers[7] seems to be one of the first in this field. According to a recent book,[18] their "spider search" program has been successfully applied to other problems also.

It is hoped that the present paper will stimulate further interest in microwave engineers to solve some of their design problems by computer-aided direct search.

Appendix

Summary of Optimization Program

The five constituent parts of the complete program are as follows.

1) OPT, the main program. This reads all the data, and instructs the execution of the trial and error loops and optimization process.

2) STAG, a subroutine which calculates the input impedance of the stabilizing network (Fig. 5) at any specified frequency. For the flow diagram see Fig. 7.

3) TEST, a subroutine which compares the input resistance or input reactance of the network at frequencies f_1 to f_6 with the test values S_1 to S_9. See Table I. Control is returned to OPT if any specification is violated. V (Fig. 9) is calculated by TEST.

4) FREQ, a subroutine which instructs a printout of the frequency response along with the current values of y_{01} to y_{03} and f_{o1} to f_{o3}, and feeds the response into XYPLOT.

5) XYPLOT, a library subroutine, which produces a plot of the resistance and reactance as a function of frequency.

The program was written in FORTRAN IV and used on the IBM 7090.

Description of OPT

The essentials of this program are presented partly by an algorithm and partly by the flow diagram of Fig. 9. The algorithm follows.

1) Read 21 integers and 7 associated scale factors to determine the trial and error loops for y_0 and f_o parameters and the frequencies for the desired frequency response. Read f_1 to f_6. Read integers JL, LL, KK, and KL to limit the number of iterations around the optimization loops (Fig. 9). Read 3 values each for subscripted variables AYO, BYO, AFO, and BFO for adjusting the parameter increments. Read bias resistance R_b for normalization. Read S_1 to S_9 in ohms.

2) Normalize S_1 to S_9 with respect to R_b.

3) Calculate $U = S_3^2 + S_5^2 + S_7^2 + S_9^2$.

4) Store this U in W.

5) Set values for y_{01}, y_{02}, y_{03}, f_{o1}, f_{o2}, and f_{o3}.

6) Call TEST. TEST calls STAG.

7) If any specification violated go to 11).

8) If $V \geqq U$ go to 11).

9) Set $U = V$.

10) Overwrite any previously stored values of y_{01}, y_{02}, y_{03}, f_{o1}, f_{o2}, and f_{o3} with the present ones.

11) Continue with next iteration from 5) until loops are exhausted.

12) If $U < W$ go to 15).

13) Print out "The specifications cannot be met."

14) Stop.

15) Set y_{01}, y_{02}, y_{03}, f_{o1}, f_{o2}, and f_{o3} equal to the respective stored values.

16) Call FREQ. FREQ calls STAG and XYPLOT.

17) Start of optimization from top of Fig. 9.

Appropriate comments for Fig. 9 might be: C_a: THE LIMIT OF THE CYCLE HAS BEEN REACHED; C_b: THE TOTAL LIMIT OF THE CYCLE HAS BEEN REACHED; C_c: THE OPTIMUM SOLUTION FOLLOWS; C_d: THE TRIAL AND ERROR VALUES ARE BEST, RECOMMEND FINER INCREMENTS.

I (maximum value 6) determines which y_0 or f_o is incremented, e.g., $I=2$ refers to y_{02}; $I=6$ refers to f_{o3}. J is the consecutive number of times a successful change in one parameter value is made. If the limit JL is reached comment C_a is printed out. K is the number of times I has changed, limited to KL. When $K=KK$ (for instance 12) the AYO and AFO values are multiplied by the fractions BYO and BFO to reduce the increments. L is the total number of successful changes in the parameters. When L reaches the limit LL comment C_b is printed out. When K reaches KL and $L\neq 0$ comment C_c is printed out; if $L=0$ comment C_d is printed out instead. The current values of I, J, K, and L are all printed out following any of the comments. Subroutine FREQ is called except after C_d (because no change has taken place in parameter values). The choice of increments and limits for the loops depends on the past experience of the operator with regard to the behavior of the program and on the refinements required, e.g., the number of decimal places for the y_0 and f_o parameters.

Cascaded Transmission Lines

An iterative form of the well-known transformation for cascaded transmission-line sections is

$$H_{in} = H_{0n} \frac{H_{i(n+1)} + P_n H_{0n}}{H_{0n} + P_n H_{i(n+1)}},$$

where n signifies the nth line in the cascade, i signifies input, and $P=j\tan\beta l$. The termination of the cascade is represented as the input immittance to an additional transmission line (which it often is). The special cases of lines terminated by a SC and an OC are obtained for $H_{i(n+1)}=0$, giving

$$H_{in} = P_n H_{0n}$$

where $H=Z$ applies to the SC and $H=Y$ applies to the OC.

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References

[1] R. Hooke and T. A. Jeeves, " 'Direct search' solution of numerical and statistical problems," *J. ACM*, vol. 8, pp. 212–229, April 1961.

[2] D. J. Wilde, *Optimum Seeking Methods.* Englewood Cliffs, N. J.: Prentice-Hall, 1964, ch. 5.

[3] H. Glass and L. Cooper, "Sequential search: a method for solving constrained optimization problems," *J. ACM*, vol. 12, pp. 71–82, January 1965.

[4] C. F. Wood, "Review of design optimization techniques," *IEEE Trans. Systems Science and Cybernetics*, vol. SSC-1, pp. 14–20, November 1965.

[5] A. Lavi and T. P. Vogl, Eds., *Recent Advances in Optimization Techniques.* New York: Wiley, 1966.

[6] P. E. Fleischer, "Optimization techniques in system design," in *System Analysis by Digital Computer*, F. F. Kuo and J. F. Kaiser, Eds. New York: Wiley, 1966.

[7] F. E. Emery and M. O'Hagan, "Optimal design of matching networks for microwave transistor amplifiers," *IEEE Trans. Microwave Theory and Techniques* (*Correspondence*), vol. MTT-14, pp. 696–698, December 1966.

[8] G. C. Temes and D. A. Calahan, "Computer-aided network optimization the state-of-the-art," *Proc. IEEE*, vol. 55, pp. 1832–1863, November 1967. An up-to-date review with a comprehensive bibliography.

[9] B. M. Schiffman and G. L. Matthaei, "Exact design of band-stop microwave filters," *IEEE Trans. Microwave Theory and Techniques*, vol. MTT-12, pp. 6–15, January 1964.

[10] L. Young, "Microwave filters—1965," *IEEE Trans. Microwave Theory and Techniques*, vol. MTT-13, pp. 489–508, September 1965.

[11] B. M. Schiffman and L. Young, "Design tables for an elliptic-function band-stop filter ($N=5$)," *IEEE Trans. Microwave Theory and Techniques*, vol. MTT-14, pp. 474–482, October 1966.

[12] B. T. Henoch and Y. Kvaerna, "Stability criteria for tunnel-diode amplifiers," *IRE Trans. Microwave Theory and Techniques* (*Correspondence*), vol. MTT-10, pp. 397–398, September 1962.

[13] J. W. Bandler, "Stability and gain prediction of microwave tunnel-diode reflection amplifiers," *IEEE Trans. Microwave Theory and Techniques*, vol. MTT-13, pp. 814–819, November 1965.

[14] L. Young, "Diplexers and multiplexers," IEE Microwave Summer School, Leeds University, England, 1966. Lecture Notes for Lectures 12 and 13, Microwave Components Synthesis III and IV, pt. 5, pp. 88–117.

[15] J. W. Bandler, "Frequency responses in the reflection coefficient plane plotted by digital computer," *IEEE Trans. Microwave Theory and Techniques* (*Correspondence*), vol. MTT-14, pp. 399–400, August 1966.

[16] J. W. Bandler, "Stable broadband tunnel-diode amplifiers in rectangular waveguides," Ph.D. thesis, University of London, 1967, ch. 7.

[17] W. J. Getsinger, "The packaged and mounted diode as a microwave circuit," *IEEE Trans. Microwave Theory and Techniques*, vol. MTT-14, pp. 58–69, February 1966.

[18] D. J. Wilde and C. S. Beightler, *Foundations of Optimization.* Englewood Cliffs, N. J.: Prentice-Hall, 1967, pp. 312–313.