COMPUTER-AIDED TOLERANCE OPTIMIZATION APPLIED TO MICROWAVE CIRCUITS

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Abstract

A very important practical problem in microwave circuit design is the problem of optimal design subject to component tolerances. An approach which treats the component tolerance assignment as an integral part of computer-aided design is, to the authors' knowledge, new to microwave engineers. Using recent nonlinear programming techniques and Dakin's branch and bound technique in conjunction with Fletcher's unconstrained minimization program, a variety of continuous and discrete tolerance problems may be solved. It is planned to make the full program available.

Introduction

Previous design work has been concentrated on obtaining a best nominal design, disregarding the manufacturing tolerances and material uncertainties. Basically, the tolerance assignment problem is to ensure that a design when fabricated will meet performance or other specifications[1].

It is the purpose of this paper to draw the attention of microwave engineers to a design concept whereby the optimal nominal parameter values and tolerances would be determined simultaneously. A general purpose computer program for continuous and discrete nonlinear programming problems called DISOPT is used[2]. The user need only supply simple subroutines to set up the appropriate objective function and the constraint functions. With any arbitrary initial acceptable or unacceptable design as a starting point, the program would output the set of nominal component parameters together with a set of optimal tolerances satisfying all the specifications in the worst-case sense. The user may optionally choose a combination of many recent techniques and algorithms[3-5] and decide on a continuous solution and/or discrete solutions.

Another practical problem which is analogous to the tolerance assignment problem is to determine the optimum component values to a certain number of significant figures, which can also be done with DISOPT.

The optimization of a noncommensurate 5-section low-pass transmission-line filter[6] and a 2-section transformer[7] serve as illustrations of the ideas presented.

The Tolerance Problem

A design consists of design data of the nominal design point $\phi^o \triangleq [\phi_1^o \; \phi_2^o \; \dots \; \phi_k^o]^T$ and a set of associated tolerances $\epsilon \triangleq [\epsilon_1 \; \epsilon_2 \dots \epsilon_k]^T$, where k is the number of independent design parameters. An outcome of the circuit is any point $\phi = [\phi_1 \; \phi_2 \dots \phi_k]^T$ in the tolerance region R_t, where $R_t \triangleq \{\phi \mid \phi_i^o - \epsilon_i \le \phi_i \le \phi_i^o + \epsilon_i, \; i = 1,2, \dots, k\}$. The constraint region R_c is the region of points ϕ such that all performance specifications and constraints are satisfied. The worst-case design requires that $R_t \subseteq R_c$. The optimal worst-case design can, therefore, be stated as follows:

minimize some objective function $C(\phi^o, \epsilon)$

subject to $R_t \subseteq R_c$

This would, in general, mean that there need to be an infinite number of constraint functions which has led to different assumptions and different approaches to make the formulation manageable.

A theorem[8] was proved that if R_c is one dimensionally convex for each variable ϕ_i, $i = 1,2,\dots,k$, and if the vertices of R_t are in R_c, then $R_t \subseteq R_c$. Thus, a finite number (2^k) of points need be constrained. In many situations, the constraint functions are monotonic with respect to some of the variables in the interval of interest. A subroutine called VERTST will select the most critical vertices of the tolerance region, based on information obtained from the partial derivatives and magnitudes evaluated at the vertices. If these few critical vertices are in R_c, then $R_t \subseteq R_c$.

Depending on the situation, the objective function may be, for example, $C_1 = \sum_{i=1}^{k} \frac{c_i}{t_i}$, where $t_i = \frac{\epsilon_i}{\phi_i^o} \times 100\%$ or $C_2 = \sum_{i=1}^{k} \frac{c_i}{\epsilon_i}$ or a mixture of the terms, where the c_i are some suitable weighting factors. Minimizing C_1 will tend to maximize the relative tolerances and minimizing C_2 will tend to maximize the absolute tolerances. The latter one is applicable, for example, to machining components.

The program DISOPT may be used to minimize the constrained objective function $C(\phi^o, \epsilon)$ efficiently.

Design Examples

Consider a 5-section cascaded transmission line network with characteristic impedances fixed at

$$Z_1 = Z_3 = Z_5 = .2$$

$$Z_2 = Z_4 = 5$$

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and terminated in unity resistances[6]. Suppose the specifications are a maximum of .02 dB insertion loss in the passband 0-1 GHz and a minimum of 25 dB over the range 2.5 to 10 GHz. Let us consider two cases.

1. A uniform 1% relative tolerance is allowed for each characteristic impedance. Maximize the absolute tolerances on the section lengths ℓ_i, i = 1, ...,5.

2. A uniform absolute tolerance on the section lengths is given. Maximize the relative tolerances on the impedances.

It was decided that frequency points at .35, .4, .45, .75, .8, .85, 1.0, 2.5 and 10 GHz be taken. It was assumed that the nominal values and tolerances of components 1 and 2 are equal to the corresponding values of components 5 and 4, respectively. A total of 15 constraint functions were used throughout the optimization. The set of discrete impedance tolerances is {.5, 1, 1.5, 2, 3, 5}% and the discrete set of length tolerances is from .0005 to .005 with .0005 step-size. All the length units presented are normalized with respect to $\ell_q = \frac{c}{4f_o}$ and f_o = 1 GHz. See Table 1 for some results. There are actually many possible solutions, but space does not permit a discussion of this.

To illustrate that the discrete solution cannot always be obtained by rounding or truncating to the nearest discrete point, a lossless two-section transformer with quarter-wave length sections and source to load impedance of 10:1 and 100% relative bandwidth is optimized. An upper specification of reflection coefficient magnitude of .55 was assumed. The best solution obtained by rounding the tolerance value is 10% for both t_{Z_1} and t_{Z_2}. The set of discrete tolerances is {1,2,5,7.5, 10,15,20}. Some results are shown in Table 2. Note that the nominal points for the discrete cases differ from the continuous case.

Conclusions

An approach to computer-aided tolerance optimization of microwave circuits has been presented. A very efficient and general program for continuous and discrete nonlinear programming called DISOPT is used for the purpose. The program, written in FORTRAN IV for the CDC 6400 computer, is to be made available to the microwave community.

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Parameters	Continuous Solution	Discrete Solution
$\varepsilon_{\ell_1} = \varepsilon_{\ell_5}$	.00326	.00300
$\varepsilon_{\ell_2} = \varepsilon_{\ell_4}$	.00279	.00300
ε_{ℓ_3}	.00267	.00250
$\ell_1^o = \ell_5^o$	.07877	
$\ell_2^o = \ell_4^o$	.14143	
ℓ_3^o	.17383	

TABLE 1a. Problem 1: t_{Z_i} = 1%, $C = \sum_{i=1}^{5} \frac{1}{\varepsilon_{\ell_i}}$

Parameters	Continuous Solution	Discrete Solution
$t_{Z_1} = t_{Z_5}$	3.5617	3.0000
$t_{Z_2} = t_{Z_4}$	2.2698	2.0000
t_{Z_3}	1.9833	2.0000
$\ell_1^o = \ell_5^o$	.07857	
$\ell_2^o = \ell_4^o$	.14152	
ℓ_3^o	.17364	

TABLE 1b. Problem 2: ε_{ℓ_i} = .001, $C = \sum_{i=1}^{5} \frac{1}{t_{Z_i}}$

Parameters	Continuous Solution	Discrete Solution
$t_{Z_1} = t_{Z_5}$	2.4659	2.0000
$t_{Z_2} = t_{Z_4}$	1.5746	1.5000
t_{Z_3}	1.3703	1.5000
$\ell_1^o = \ell_5^o$	.07866	
$\ell_2^o = \ell_4^o$	.14147	
ℓ_3^o	.17375	

TABLE 1c. Problem 2: $\varepsilon_{\ell_i} = .002$, $C = \sum_{i=1}^{5} \frac{1}{t_{Z_i}}$

Parameters	Continuous Solution	Discrete Solution
$t_{Z_1} = t_{Z_5}$	1.3708	1.0000
$t_{Z_2} = t_{Z_4}$	0.8774	1.0000
t_{Z_3}	0.7602	0.5000
$\ell_1^o = \ell_5^o$	.07876	
$\ell_2^o = \ell_4^o$	.14143	
ℓ_3^o	.17385	

TABLE 1d. Problem 2: $\varepsilon_{\ell_i} = .003$; $C = \sum_{i=1}^{5} \frac{1}{t_{Z_i}}$

Parameters	Continuous Solution	Discrete Solution	Discrete Solution
t_{Z_1}	12.746	10.000	15.000
t_{Z_2}	12.746	15.000	10.000
Z_1^o	2.1487	2.0851	2.2130
Z_2^o	4.7308	4.6110	4.8334

TABLE 2. 2-section transformer example; $C = \sum_{i=1}^{2} \frac{1}{t_{Z_i}}$