4 Acknowledgments

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Correspondence

MICROWAVE ABSORPTION AND SERIES RESISTANCE OF SILICON-MESA PARAMETRIC-AMPLIFIER DIODES

The paper by Thompson [*Proc. IEE*, 1965, **112**, (11), pp. 2013–2018] presents a new equivalent circuit for parametric-amplifier diodes which takes into account microwave losses attributed to the *p–n* junction. His theory and experiments appear to agree that

(*a*) the microwave loss occurs within the *p–n* junction
(*b*) it is due to a dielectric type of loss mechanism
(*c*) a resistance in series with the junction capacitance results
(*d*) this resistance is independent of the junction capacitance, and vice versa.

The case for (*a*) and (*b*) seems rather qualitatively presented, but will be assumed for the purposes of this correspondence. In this context, it might be pointed out that eqn. 14 appears to suffer from some inconsistency in the units. Possibly it should read

$$P = 0{\cdot}555\epsilon_g \tan\delta f E^2 \text{ mW/cm}^3 \quad \ldots \ldots \quad \text{(A)}$$

where E is in volts per centimetre and f is in gigahertz.

The author attempts in Fig. 4(i) to indicate an absorption resistance within the *p–n* junction in series with the junction capacitance C_p. Electrically, however, the representation is a parallel one. Now, assuming the loss mechanism is given by eqn. A, it follows that the effective resistivity ρ, in ohm-centimetres, in the junction region is obtained from

$$P = \frac{10^3 E^2}{\rho} = 0{\cdot}555\epsilon_g \tan\delta f E^2$$

$$\text{hence } \rho = \frac{10^3}{0{\cdot}555\epsilon_g \tan\delta f}\,\Omega\text{cm} \quad \ldots \ldots \quad \text{(B)}$$

It should be observed that the electric field E is common to both resistance and capacitance. Thus the natural equivalent circuit in which both absorption resistance and junction capacitance appear, linked presumably by a common voltage, is a parallel one at first glance. Suppose we define

C_{pp} = parallel capacitance

R_{pp} = parallel resistance

C_{ss} = series capacitance

R_{ss} = series resistance

then it is well known and easily derived that

$$\tan\delta = \frac{1}{\omega C_{pp} R_{pp}} = \omega C_{ss} R_{ss} \quad \ldots \ldots \quad \text{(C)}$$

Assuming uniform material, we can see the analogy between eqn. B and the parallel version of eqn. C. Now

$$R_{ss} = \frac{R_{pp}}{1 + (\omega C_{pp} R_{pp})^2} \quad \ldots \ldots \quad \text{(D)}$$

$$\text{and} \quad C_{ss} = \frac{1 + (\omega C_{pp} R_{pp})^2}{\omega^2 C_{pp} R_{pp}^2} \quad \ldots \ldots \quad \text{(E)}$$

Two possibilities are

(i) $\tan\delta \gg 1$, $R_{ss} \simeq R_{pp}$, but $C_{ss} \simeq \dfrac{\tan\delta}{\omega R_{pp}}$

(ii) $\tan\delta \ll 1$, $C_{ss} \simeq C_{pp}$, and $R_{ss} \simeq R_{pp} \tan^2\delta$

The first case involves a frequency-dependent series capacitance. The second involves a capacitance independent of loss with a series resistance which depends on capacitance. Neither of these is consistent with Thompson's experiments; it would seem that the reason is that the loss is not uniform within the *p–n* junction. If this is so, both eqn. 14 and Fig. 4(i) are misleading. (Note that, because of its high value, the author's R_p does not affect these generalisations at high frequencies.) Could it be possible to combine cases (i) and (ii) for the

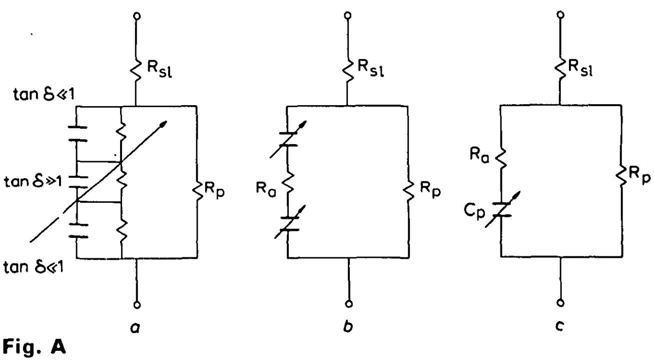

Fig. A

An equivalent circuit which could represent the experimental results on Thompson's parametric-amplifier diode

Circuit (*a*), intended to illustrate nonuniform loss within the *p–n* junction, is approximately equivalent to circuit (*b*), from which we obtain Thompson's proposed circuit (*c*)

parametric diode into the circuit of Fig. A(*a*) to a first approximation?

The author states that he found the series resistance of a parametric diode to be independent of frequency in the range 2–6GHz. This raises the question of how does the series resistance actually vary as a function of frequency from d.c. into the microwave region for a particular diode. Does the author have more recent data or any further speculation? The only data in the paper are for d.c. and 2GHz.

The author concludes that 'these considerations also apply to . . . Esaki diodes'. Would the author suggest that the proper equivalent circuit of the Esaki diode is like Fig. 4(ii) [Fig. A(c) here], in which R_p is replaced by the junction negative resistance of the diode? Then eqn. 8 could be used to predict the resistive cutoff frequency.

I would like to thank Prof. E. Bridges of the University of Manitoba for helpful discussions on dielectric-loss phenomena.

7th December 1967 J. W. BANDLER

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STABILITY OF PASSIVE TIME-VARIABLE CIRCUITS

In the paper by Klamm, Anderson and Newcomb [*Proc. IEE*, 1967, **114**, (1), pp. 71–75], the authors claim to have shown that 'circuits composed of a finite connection of linear passive time-variable elements are necessarily stable'. Quoting from the introduction: 'This is accomplished by making circuit replacements to obtain pseudostate variables as voltages across graphically independent capacitors. From such an equivalent circuit, a cutset analysis shows that the energy in the capacitors serves as a suitable Lyapunov function . . . .'

The equivalent circuit referred to is generated by replacing each time-variable element by their 'gyrator equivalents' illustrated in Figs. A, B and C. Since the equivalence is on an element basis, it seems obvious that the total stored energy at any instant in the equivalent circuit must be equal to that stored in the original network. This leads one to ask what is accomplished by the rather cumbersome gyrator equivalents.

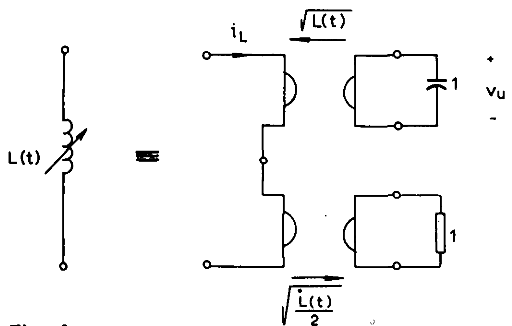

Fig. A
Gyrator equivalent of a passive time-variable inductor

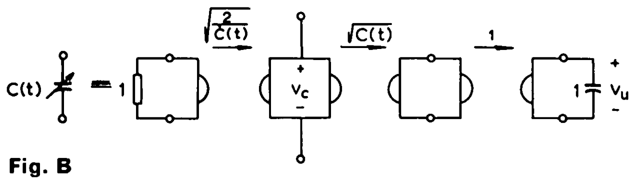

Fig. B
Gyrator equivalent of a passive time-variable capacitor

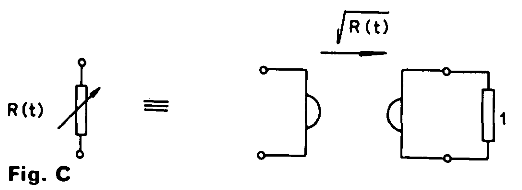

Fig. C
Gyrator equivalent of a passive time-variable resistor

To elaborate this viewpoint somewhat, it can be easily shown that the input current of the gyrator equivalent of a time-variable inductor and the input voltage of the gyrator equivalent of a time-variable capacitor are, respectively, related to the voltage across the unit capacitor by

$$i_L = \frac{v_u}{\sqrt{L(t)}} \qquad \text{and } v_C = \frac{v_u}{\sqrt{C(t)}}$$

For networks containing neither inductor cutsets nor capacitor loops, the gyrator equivalents can therefore be regarded as physical realisations of a transformation relating the inductor-current and capacitor-voltage state variables to the unit-capacitor state variables; i.e.

$$X(t) = M^{-1/2}(t)Y(t)$$

where

$X(t)$ = state vector of inductor currents and capacitor voltages

$Y(t)$ = state vector of unit-capacitor voltages

and

$M^{-1/2}(t)$ = diagonal matrix whose nonzero entries are reciprocals of the square roots of the time-variable inductor and capacitor values

Alternatively,

$$Y(t) = M^{1/2}(t)X(t),$$

and therefore

$$Y^T(t)Y(t) = X^T(t)M(t)X(t)$$

which shows that the stored-energy functions are indeed equal.

There seem to be two errors in the original paper. The first appears in the gyrator constants of the capacitor equivalent, which are given as the reciprocal of those shown in Fig. B of this correspondence. This error persists in the circuit example of Section 4.2, which suffers additionally from a missing gyrator and resistance. Since the analytical part of the example is carried out on the incorrect circuit, the result, which supports the theorem, is not valid.

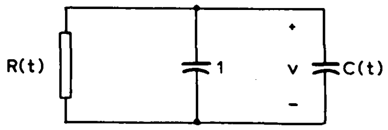

Fig. D
Passive-time-variable-circuit example

Looking at this example (Fig. D) without transforming the circuit, the state equation is

$$\dot{v}(t) = \frac{-v(t)}{1 + C(t)}\left(\frac{1}{R(t)} + \dot{C}(t)\right)$$

The stored energy function is

$$V(t) = \frac{[1 + C(t)]v^2(t)}{2}$$

and its derivative is therefore

$$\dot{V}(t) = -v^2(t)\left(\frac{1}{R(t)} - \frac{\dot{C}(t)}{2}\right)$$

While $V(t)$ is definite for nonnegative $C(t)$, this is not the case for $\dot{V}(t)$; therefore, nothing conclusive can be said about stability. It is interesting to note that if

$$1 + C(t) = e^{-\alpha t}$$

and $R(t) = e^{\alpha t}$

then $\dot{V}(t) = v(t)(\alpha - 1)$

which is unstable for $\alpha > 1$. The argument that this happens because $C(t)$ becomes negative can be refuted by removing the 1 F capacitor and making $C(t) = e^{-\alpha t}$.

18th March 1968 P. J. GRAHAM

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