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PRACTICAL DESIGN CENTERING, TOLERANCING AND TUNING

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This paper presents the results of a numerical investigation of simultaneous optimal design centering, tolerancing and tuning of circuits. Practical implementation requires a reasonable number of parameters and constraints to be identified to make the problem tractable. Two circuits are studied to illustrate the benefits and difficulties encountered.

Introduction

We present the results of a numerical investigation of simultaneous optimal design centering, tolerancing and tuning of circuits. The optimal worst-case tolerance problem has received much attention in the literature[1-4] and benefits in terms of increased tolerances by permitting the nominal point to move have been established[2,4]. This work brings in the tuning of one or more circuit components basically in order to further increase tolerances on all the components.[5] We have to minimize an objective function representing the cost of the circuit. There are, in general, an infinite number of variables and an infinite number of constraints even for a small circuit. To make the problem tractable we need a sufficient but reasonable number of variables and constraints to be identified. The present approach usually requires a few preliminary runs to determine relevant parameters and active constraints.

Basic Theory

The problem can be stated as[5]

$$\text{minimize } C(\underset{\sim}{\phi}^o, \underset{\sim}{\varepsilon}, \underset{\sim}{t})$$

where $\underset{\sim}{\phi}^o$ is the nominal point (nominal parameter vector), $\underset{\sim}{\varepsilon}$ is the tolerance vector and $\underset{\sim}{t}$ the tuning vector. It is required that

$$\underset{\sim}{\phi} \in R_c \tag{1}$$

where R_c is the constraint region (involving performance specifications and design constraints) given by

$$R_c \triangleq \{\underset{\sim}{\phi} \,|\, \underset{\sim}{g}(\underset{\sim}{\phi}) \geq \underset{\sim}{0}\} \tag{2}$$

and where, for k designable parameters,

$$\left.\begin{aligned} \phi_i &= \phi_i^o + \varepsilon_i\mu_i + t_i\rho_i \\ \phi_i^o&, \varepsilon_i, t_i \geq 0 \end{aligned}\right\} i = 1,2,\ldots,k \tag{3}$$

for all specified values of μ_i and some allowable values of ρ_i. We consider here

$$\mu_i, \rho_i \in [-1, 1], \; i = 1,2,\ldots,k \tag{4}$$

Intuitively, for each outcome $\{\underset{\sim}{\phi}^o, \underset{\sim}{\varepsilon}, \underset{\sim}{\mu}\}$ of a design $\{\underset{\sim}{\phi}^o, \underset{\sim}{\varepsilon}, \underset{\sim}{t}\}$ there must be a ρ such that $\underset{\sim}{\phi} \in R_c$, where $\underset{\sim}{\mu}$ and $\underset{\sim}{\rho}$ are k-element vectors.

We let the tolerance region R_ε be given by[5]

$$R_\varepsilon \triangleq \{\underset{\sim}{\phi} \,|\, \phi_i^o - \varepsilon_i \leq \phi_i \leq \phi_i^o + \varepsilon_i, \; i = 1,2,\ldots,k\} \tag{5}$$

and the tuning region $R_t(\underset{\sim}{\mu})$ be given by

$$R_t(\underset{\sim}{\mu}) \triangleq \{\underset{\sim}{\phi} \,|\, \phi_i^o + \varepsilon_i\mu_i - t_i \leq \phi_i \leq \phi_i^o + \varepsilon_i\mu_i + t_i, \; i = 1,2,\ldots,k\}. \tag{6}$$

Other essential concepts are

$$\varepsilon_i' \triangleq \varepsilon_i - t_i \text{ for } I_\varepsilon \triangleq \{i \,|\, \varepsilon_i > t_i\} \tag{7}$$

$$t_i' \triangleq t_i - \varepsilon_i \text{ for } I_t \triangleq \{i \,|\, t_i \geq \varepsilon_i\} \tag{8}$$

called the effective tolerance and effective tuning, respectively, and a diagonal matrix $\underset{\sim}{P}$ with diagonal element p_i given by

$$p_i = \begin{cases} 0 & \text{for } i \in I_t \\ 1 & \text{for } i \in I_\varepsilon \end{cases} \tag{9}$$

In using effective tuning or tolerancing we may replace (3) by

$$\phi_i = \phi_i^o + \begin{cases} \varepsilon_i' \mu_i, \; \varepsilon_i' \geq 0 \text{ for } i \in I_\varepsilon \\ t_i' \rho_i', \; t_i' \geq 0 \text{ for } i \in I_t \end{cases} \tag{10}$$

$$\phi_i^o \geq 0, \; i = 1,2,\ldots,k$$

where

$$\mu_i, \rho_i' \in [-1, 1]. \tag{11}$$

Instead of considering $\underset{\sim}{g}(\underset{\sim}{\phi}) \geq \underset{\sim}{0}$ as in (2) we take constraints of the form

$$\underset{\sim}{g}(\underset{\sim}{P}\underset{\sim}{\phi} + \sum_{i \in I_t} (\phi_i^o + t_i' \rho_i')\underset{\sim}{e}_i) \geq \underset{\sim}{0} \tag{12}$$

where $\underset{\sim}{e}_i$ is the ith unit vector, and $\underset{\sim}{\phi}$ describes an outcome or an effective outcome, whichever is appropriate.

Implementation

The constraints associated with response specifications are of the form $g = w(S - F) \geq 0$ with appropriate subscripts, where F is the circuit response function of $\underset{\sim}{\phi}$ and ψ, which is an independent parameter denoting frequency or any number to identify a particular function. S is a specification and w a weighting factor. Both are functions of ψ. We take $w_i = +1(-1)$ if S_i is an upper (lower) specification.

Data for a specific problem is contained in a vector $\underset{\sim}{a}$, which has the form $[i \; \underset{\sim}{\mu}^T \; \psi \; S \; w]^T \Rightarrow \{\underset{\sim}{\phi}^i, \psi\} \Rightarrow g$, where i is an integer indexing a distinct outcome to be considered in the subspace spanned by the effectively toleranced components.

If vertices of the tolerance region are considered, the rth vertex corresponds to $\underset{\sim}{\mu}^r$, where

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$$r = 1 + \sum_{j=1}^{k} \left(\frac{\mu_j^r + 1}{2} \right) 2^{j-1}, \quad \mu_j^r \in \{-1,1\}, \qquad (13)$$

and we assume, unless otherwise specified, that vertices provide active constraints. The number of variables is n and the number of constraints m.

Lowpass Filter

A 3-element LC ladder lowpass filter is considered.[2,4] The sample points used in optimization are $\omega_1 = 0.45$, $\omega_2 = 0.5$, $\omega_3 = 0.55$ and $\omega_4 = 1.0$ rad/s for the passband and $\omega_5 = 2.5$ rad/s in the stopband. 1.5 dB and 25 dB are the passband and stopband specifications, respectively. Both terminations are 1Ω.

The optimization program used is based on recent work in least pth approximation and nonlinear programming[6] and incorporates a quasi-Newton method of unconstrained optimization.[7,8]

Example 1: No Tuning ($\underset{\sim}{t} = \underset{\sim}{0}$)

For each frequency point $2^k = 8$ vertices for the tolerance region R_ε given by (5) can be obtained. The active vertices correspond to μ^6 at $\omega = \omega_1, \omega_2, \omega_3$; μ^8 at $\omega = \omega_4$; and μ^1 at $\omega = \omega_5$. Hence, we consider, for $\phi_1^o = L_1^o$, $\phi_2^o = C^o$, $\phi_3^o = L_2^o$, $\varepsilon_1 = \varepsilon_{L_1}$, $\varepsilon_2 = \varepsilon_C$ and $\varepsilon_3 = \varepsilon_{L_2}$,

$$\underset{\sim}{\phi}^1 = \underset{\sim}{\phi}(\underset{\sim}{\mu}^6) = \begin{bmatrix} \phi_1^o + \varepsilon_1 \\ \phi_2^o - \varepsilon_2 \\ \phi_3^o + \varepsilon_3 \end{bmatrix}, \quad \underset{\sim}{\phi}^2 = \underset{\sim}{\phi}(\underset{\sim}{\mu}^8) = \begin{bmatrix} \phi_1^o + \varepsilon_1 \\ \phi_2^o + \varepsilon_2 \\ \phi_3^o + \varepsilon_3 \end{bmatrix}, \quad \underset{\sim}{\phi}^3 = \underset{\sim}{\phi}(\underset{\sim}{\mu}^1) = \begin{bmatrix} \phi_1^o - \varepsilon_1 \\ \phi_2^o - \varepsilon_2 \\ \phi_3^o - \varepsilon_3 \end{bmatrix}. \qquad (14)$$

We confirm Bandler and Liu's results[2] for

$$C(\underset{\sim}{\phi}^o, \underset{\sim}{\varepsilon}) = \frac{L_1^o}{\varepsilon_{L_1}} + \frac{C^o}{\varepsilon_C} + \frac{L_2^o}{\varepsilon_{L_2}}. \qquad (15)$$

Example 2: Effective Tuning for One Component

(a) L_1 tuned, C and L_2 toleranced.

We consider an objective function similar to (15) but based on the relative tolerances of C and L_2. Five functions $g_1, g_2, \ldots, g_5$ are chosen as before, except that from (9), (12) and (14) $\underset{\sim}{\phi}^1 = \underset{\sim}{P}\underset{\sim}{\phi}(\underset{\sim}{\mu}^6) + (\phi_1^o + t_1\rho_1^1)\underset{\sim}{e}_1$, $\underset{\sim}{\phi}^2 = \underset{\sim}{P}\underset{\sim}{\phi}(\underset{\sim}{\mu}^8) + (\phi_1^o + t_1\rho_1^2)\underset{\sim}{e}_1$ and $\underset{\sim}{\phi}^3 = \underset{\sim}{P}\underset{\sim}{\phi}(\underset{\sim}{\mu}^1) + (\phi_1^o + t_1\rho_1^3)\underset{\sim}{e}_1$ where $t_1 = t_{L_1}$, $p_1 = 0$, $p_2 = 1$, $p_3 = 1$ and where ρ_1^1, ρ_1^2 and ρ_1^3 are new variables. The ρ variables are appropriately constrained, and a constraint to limit the tuning range to t_r is introduced. Table 1 shows results for three values of t_r. n=9, m=12. The same results are obtained by setting $t_1 = t_r$.

(b) C tuned, L_1 and L_2 toleranced.

We consider an objective function similar to (15) but based on the relative tolerances of L_1 and L_2. In this case $\underset{\sim}{\phi}^1 = \underset{\sim}{P}\underset{\sim}{\phi}(\underset{\sim}{\mu}^6) + (\phi_2^o + t_2\rho_2^1)\underset{\sim}{e}_2 = \underset{\sim}{P}\underset{\sim}{\phi}(\underset{\sim}{\mu}^8) + (\phi_2^o + t_2\rho_2^1)\underset{\sim}{e}_2$ and $\underset{\sim}{\phi}^2 = \underset{\sim}{P}\underset{\sim}{\phi}(\underset{\sim}{\mu}^1) + (\phi_2^o + t_2\rho_2^2)\underset{\sim}{e}_2$, where $t_2 = t_C$, $p_1 = 1$, $p_2 = 0$, $p_3 = 1$ and where ρ_2^1 and ρ_2^2 are new variables. Additional constraints on the ρ and tuning variable are imposed, as before. Table 2 shows results for three values of t_r. n = 8, m = 10. The same results are obtained by setting $t_2 = t_r$. Larger tolerances are obtained than before for corresponding tuning ranges.

Example 3: Tolerancing and Tuning for One Component

We consider C to be both toleranced and tuned and minimize (15). Here, $\underset{\sim}{\phi}^1 = \underset{\sim}{\phi}(\underset{\sim}{\mu}^6) + t_2\rho_2^1\underset{\sim}{e}_2$, $\underset{\sim}{\phi}^2 = \underset{\sim}{\phi}(\underset{\sim}{\mu}^8) + t_2\rho_2^2\underset{\sim}{e}_2$ and $\underset{\sim}{\phi}^3 = \underset{\sim}{\phi}(\underset{\sim}{\mu}^1) + t_2\rho_2^3\underset{\sim}{e}_2$, with $t_2 = t_r C^o$. ρ_2^1, ρ_2^2 and ρ_2^3 are new constrained variables. n = 9, m = 11.

The results are shown in Table 3 where we note that for 5% and 10% tuning we have an effective tolerance problem, whereas for 20% tuning we have an effective tuning problem.

Example 4: Optimal Tuning

(a) Tolerancing and tuning for one component.

To (15) we add ct_2/C^o, where c is a weighting factor. The constraints remain the same. n = 10, m = 11.

Table 4 shows results for different values of c. Note that a threshold value of c seems to occur somewhere between 10 and 20. Below that threshold, the solution in terms of an effective tuning and tolerance problem is unaffected. Note also the transition for c = 50 from effective tuning to effective tolerancing. When c is very large we obtain the tolerance solution of Example 1.

(b) Tolerancing and tuning for 3 components.

The objective function considered is of the form

$$C(\underset{\sim}{\phi}^o, \underset{\sim}{\varepsilon}, \underset{\sim}{t}) = \sum_{i=1}^{3} \left[\frac{\phi_i^o}{\varepsilon_i} + c\,\frac{t_i}{\phi_i^o} \right]. \qquad (16)$$

We consider one additional distinct vertex such that $\underset{\sim}{\phi}^1$, $\underset{\sim}{\phi}^2$ and $\underset{\sim}{\phi}^3$ are as in (14), and $\underset{\sim}{\phi}^4 = \underset{\sim}{\phi}(\underset{\sim}{\mu}^3)$ in order to bound the solution during optimization. n=21, m=36.

We omit details of the constraints, and summarize the final results in Table 5 for different c. The results are the same as in Table 4, but the computational effort has substantially increased. This formulation, however, has verified that ϕ_2 should be effectively tuned for c less than 50, and the other parameters effectively toleranced. The values of $\underset{\sim}{\rho}^1$, $\underset{\sim}{\rho}^2$, $\underset{\sim}{\rho}^3$ and $\underset{\sim}{\rho}^4$ confirm these observations.

Highpass Filter

This problem was suggested by Pinel and Roberts.[9,10] The circuit diagram is shown in Fig. 1 and the basic specifications for the design are listed in Table 6. The insertion loss relative to the loss at 990 Hz is to be constrained as indicated with resistances R_5 and R_7 related to L_5^o and L_7^o with constant Q. The terminations are fixed, the designable parameters being C_1, C_2, C_3, C_4, L_5, C_6 and L_7.

The objective function throughout was taken as

$$\sum_{i=1}^{7} \frac{\phi_i^o}{\varepsilon_i} \qquad (17)$$

where $\underset{\sim}{\phi}^o = [C_1^o\ C_2^o\ C_3^o\ C_4^o\ L_5^o\ C_6^o\ L_7^o]^T$ and $\underset{\sim}{\varepsilon} = [\varepsilon_{C_1}\ \varepsilon_{C_2}\ \varepsilon_{C_3}\ \varepsilon_{C_4}\ \varepsilon_{L_5}\ \varepsilon_{C_6}\ \varepsilon_{L_7}]^T$.

The optimization package used here is DISOPT[11], which has been previously employed in worst-case tolerance problems[4]. In most cases extrapolation[12] was chosen to accelerate convergence.

Verification of the designs was carried out using all 2^7 vertices plus the nominal point at 170, 360, 440, 630-680 and 680-1800 Hz. 42 logarithmically spaced points were taken for the latter interval, and 8 for the former interval.

Problem 1: No Tuning ($t = 0$)

The final tolerances are listed in Table 7. 14 variables and 45 constraints were used. Table 7 also lists the shifts in nominal parameter values with respect to those of an uncentered design[9,10].

Problem 2: 3% Tuning for L_5

Results corresponding to the ones for Problem 1 are tabulated in Table 7. 51 constraints were used. Note that all the tolerances have increased over the results of Problem 1. Fig. 2 shows the nominal response as well as the worst upper and lower outcomes based on all 2^7 vertices.

A more detailed verification of the results was made. 60 logarithmically spaced points were taken from the critical region 630-680 Hz as well as 40 from 600-630 Hz. All the vertices were checked plus the nominal point, followed by 4000 Monte Carlo simulations uniformly distributed in the effective tolerance region. No violations were detected, and the upper and lower limits of response given by the vertices bounded the results from the Monte Carlo analysis except at 638.2 Hz, where the lowest relative loss obtained from the vertices was -0.0243 dB, whereas the Monte Carlo analysis yielded -0.0246 dB.

Problem 3: 3% Tuning for L_5 and L_7

Table 7 indicates further improvement in all tolerances. 51 constraints were used.

Problem 4: 3% Tuning for L_7

Table 7 shows slightly worse results than those for Problem 2. 69 constraints were used. We conclude that if only one inductor is to be tuned, L_5 should be chosen.

Conclusions

As expected, the inclusion of tunable elements can increase the tolerances on the components. A cost function tending to maximize tolerances and minimizing tuning has been implemented successfully in this context. Zero tuning ranges were indicated when the cost became too high. For the highpass filter the 3% tuning range on the inductors was considered free, thus tuning did not enter into the objective function. A reduced problem involving effective tolerances was found adequate.

Table 1. L_1 tuned, C and L_2 toleranced .

Parameters	$t_r = 0.2$	$t_r = 0.1$	$t_r = 0.05$
L_1^o	2.0932	2.2442	2.1953
C^o	0.9360	0.9059	0.9062
L_2^o	1.7718	1.7569	1.7920
$100\ t_1/L_1^o$	20.00 %	10.00 %	5.00 %
$100\ \varepsilon_2/C^o$	15.99 %	14.23 %	12.60 %
$100\ \varepsilon_3/L_2^o$	21.62 %	18.41 %	16.23 %

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References

1 J.W. Bandler, "Optimization of design tolerances using nonlinear programming", J. Optimization Theory and Applications, vol. 14, July 1974, pp. 99-114.

2 J.W. Bandler and P.C. Liu, "Automated network design with optimal tolerances", IEEE Trans.Circuits and Systems, vol. CAS-21, Mar. 1974, pp. 219-222.

3 J.F. Pinel and K.A. Roberts, "Tolerance assignment in linear networks using nonlinear programming", IEEE Trans. Circuit Theory, vol. CT-19, Sept. 1972, pp. 475-479.

4 J.W. Bandler, P.C. Liu and J.H.K. Chen, "Worst case network tolerance optimization", IEEE Trans. Microwave Theory Tech., to be published.

5 J.W. Bandler and P.C. Liu, "The tolerance-tuning problem: a nonlinear programming approach", Proc. 12th Allerton Conf. Circuit and System Theory (Urbana, Ill., Oct. 1974).

6 C. Charalambous, "Nonlinear least pth optimization and nonlinear programming", Dept. of Combinatorics and Optimization, Univ. of Waterloo, Waterloo, Canada, Rep. CORR 74-3, Jan. 1974.

7 R. Fletcher, "FORTRAN subroutines for minimization by quasi-Newton Methods", Atomic Energy Research Establishment, Harwell, Berkshire, England, Report AERE-R7125, 1972.

8 P.E. Gill and W. Murray, "Quasi-Newton methods for unconstrained optimization", J. Inst. Maths. and its Applications, vol. 9, 1972, pp. 91-108.

9 J.F. Pinel, Bell-Northern Research, Ottawa, Canada, Aug. 30, 1974, private communication.

10 K.A. Roberts, "The impact of design centering and tolerance assignment on network design", IEEE Circuits and Systems Society Computer-Aided Network Design Committee Workshop, Kennebunkport, Maine, Sept. 1974.

11 J.W. Bandler and J.H.K. Chen, "DISOPT - a general program for continuous and discrete nonlinear programming problems", Int. J. Systems Science, to be published. Documented listing available through Faculty of Engineering, McMaster Univ., Hamilton, Canada, Rep. SOC-29, March 1974.

12 J.W. Bandler and W.Y. Chu, "Computational merits of extrapolation in least pth approximation and nonlinear programming", Proc. 12th Allerton Conf. Circuit and System Theory (Urbana, Ill., Oct. 1974).

Table 2. C tuned, L_1 and L_2 toleranced .

Parameters	$t_r = 0.2$	$t_r = 0.1$	$t_r = 0.05$
L_1^o, L_2^o	1.8664	1.9536	2.0002
C^o	1.1336	1.0077	0.9546
$100\ \varepsilon_1/L_1^o, 100\ \varepsilon_3/L_2^o$	27.54 %	21.84 %	19.00 %
$100\ t_2/C^o$	20.00 %	10.00 %	5.00 %

Table 3. C toleranced and tuned, L_1 and L_2 toleranced.

Parameters	$t_r = 0.2$	$t_r = 0.1$	$t_r = 0.05$
L_1^o, L_2^o	2.0178	2.0380	2.0209
C^o	0.9366	0.9061	0.9040
$100\ \varepsilon_1/L_1^o, 100\ \varepsilon_3/L_2^o$	17.96 %	14.81 %	12.41 %
$100\ \varepsilon_2/C^o$	16.83 %	11.66 %	9.64 %
$100\ t_2/C^o$	20.00 %	10.00 %	5.00 %
$100/C^o x$	$t_2' = 3.17$ %	$\varepsilon_2' = 1.66$ %	$\varepsilon_2' = 4.64$%

Table 4. Optimal tuning.

Parameters	c=1	c=10	c=20	c=50	c=1000
L_1^o, L_2^o	1.8440	1.8440	1.9221	2.0492	1.9990
C^o	1.1730	1.1730	1.0486	0.9069	0.9056
$100\ \varepsilon_1/L_1^o$, $100\ \varepsilon_3/L_2^o$	29.08 %	29.08 %	23.84 %	16.15 %	9.89 %
$100\ \varepsilon_2/C^o$	100.00 %	31.62 %	22.36 %	14.14 %	7.60 %
$100\ t_2/C^o$	122.69 %	54.31 %	35.88 %	14.14 %	0.00 %
$100/C^o x$	t_2'=22.69%	t_2'=22.69%	t_2'=13.52%	t_2'=0.00%	ε_2'=7.60%

Table 5. Optimal Tuning.

Parameters	c = 10	c = 20	c = 50
L_1^o, L_2^o	1.8440	1.9221	2.0492
C^o	1.1730	1.0486	0.9069
$100\ \varepsilon_1/L_1^o, 100\ \varepsilon_3/L_2^o$	31.62 %	23.84 %	16.15 %
$100\ \varepsilon_2/C^o$	31.62 %	22.36 %	14.14 %
$100\ t_1/L_1^o, 100\ t_3/L_2^o$	2.54 %	0.00 %	0.00 %
$100\ t_2/C^o$	54.31 %	35.89 %	14.14 %
$100\ \varepsilon_1'/L_1^o, 100\ \varepsilon_3'/L_2^o$	29.08 %	23.84 %	14.14 %
$100\ t_2'/C^o$	22.69 %	13.53 %	0.00 %

Table 6. Highpass Filter Specifications.

Frequencies (Hz)	Sample Points* (Hz)	Relative Loss (dB)	Weight w
170	170	45.	-1
360	360	49.	-1
440	440	42.	-1
630-680	630	4.	+1
680-1800	680,710,725,740	1.75	+1
630-1800	630,650,680,860,910,930, 1050	-0.05	-1

Reference Frequency: 990 Hz. R_5, R_7 related to L_5^o and L_7^o through $Q = 2\pi 990 L_5^o/R_5 = 2\pi 990 L_7^o/R_7 = 1456$.

*Additional ones were used when necessary.

Table 7. Percentage Tolerances for Highpass Filter.†

Parameters	No Tuning	L_5 Tuned	L_5 and L_7 Tuned	L_7 Tuned
C_1	5.71(+18.1)	6.77(+17.8)	7.90(+18.3)	6.63(+17.6)
C_2	4.33(+16.2)	4.97(+15.2)	5.32(+14.4)	4.77(+15.3)
C_3	4.72(+16.6)	5.81(+18.0)	7.23(+18.8)	5.83(+17.8)
C_4	4.54(-3.8)	5.03(-2.2)	5.15(-1.2)	4.78(-3.1)
L_5	3.29(-3.0)	3.95(-3.0)	4.44(-4.3)	3.82(-4.1)
C_6	6.32(-7.3)	7.05(-5.1)	7.27(-3.6)	6.66(-6.0)
L_7	3.64(-6.4)	4.34(-7.9)	5.04(-7.9)	4.32(-6.3)
Cost	157	135	121	138*

*Violation of specifications. Relative Loss = -0.052 dB at 658 Hz.

†Numbers in brackets are nominal shift (%).

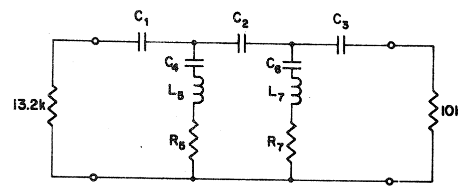

Fig. 1. The LC lowpass filter.

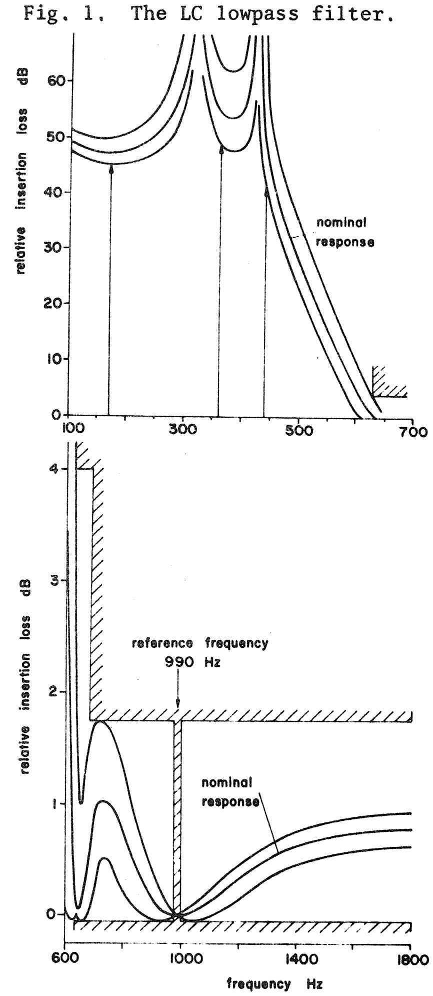

Fig. 2. Response of the optimized highpass filter.